$$\langle f, g \rangle = \int_a^b f(x)\,\overline{g(x)}\,dx$$

$$f, g : [a, b] \to \mathbb{C}$$

$$\mathbb{R}_2[x] = \langle 1, x, x^2 \rangle \quad \text{su} \quad [-1,1]$$

$$g_1 = 1$$

$$g_2 = x - \frac{1}{\|g_1\|^2} \int_{-1}^{1} x \cdot 1 \, dx \cdot 1 = x - \underbrace{\frac{1}{\|g_1\|^2} \int_{-1}^{1} x \cdot 1 \, dx}_{a_1} \cdot 1 =$$

$$= x$$

$$g_3 = x^2 - \frac{1}{\|g_1\|^2} \int_{-1}^{1} x^2 \cdot 1 \, dx \cdot 1 - \frac{1}{\|g_2\|^2} \int_{-1}^{1} x^2 \cdot x \, dx \cdot x$$

$$= 0$$

$$= x^2 - \frac{1}{3}$$

$$\mathbb{R}_2[x] = \left\langle \frac{1}{\sqrt{\frac{1}{2}}}, \frac{x}{\sqrt{\frac{1}{2}}}, x^2 - \frac{1}{3} \right\rangle$$

$\mathbb{R}_2[x] \Rightarrow f$

$\mathbb{R}_2[x] = \langle 1, x, x^2 - \frac{1}{3}, \dots \rangle = \langle h_1, h_2, h_3, \dots \rangle$

$f = c_1 h_1 + c_2 h_2 + c_3 h_3 + \cancel{\dots}$

$\langle f, h_i \rangle = c_i \langle h_i, h_i \rangle = c_i$

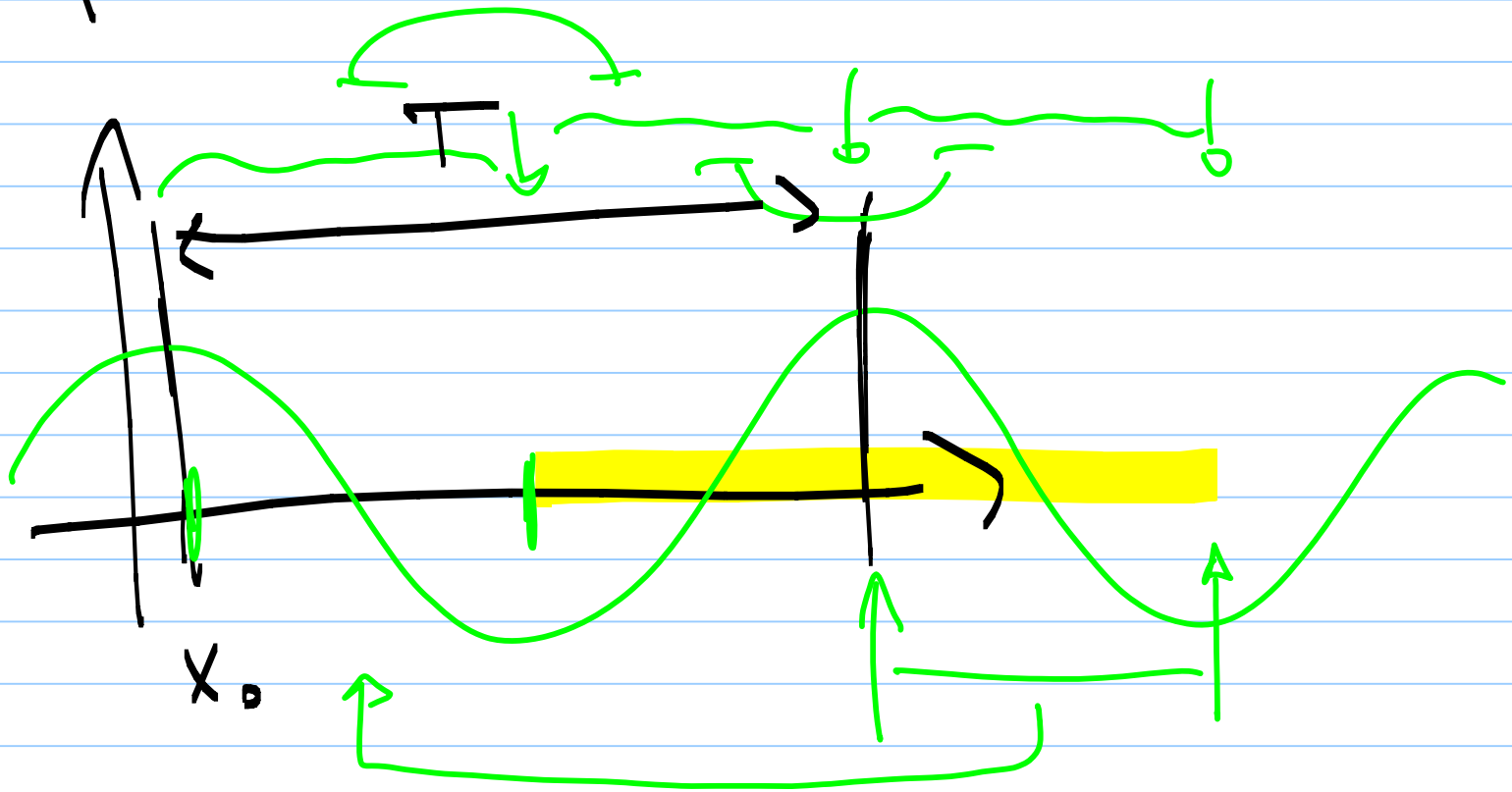
$$\left\langle g - \sum_{i=1}^{n} a_i f_i \,,\, g - \sum_{j=1}^{n} a_j f_j \right\rangle =$$

$$= \|g\|^2 - 2\sum_{i=1}^{n} a_i \underbrace{\langle f_i, g \rangle}_{= c_i} + \sum_{i,j=1}^{n} a_i a_j \underbrace{\langle f_i, f_j \rangle}_{\begin{cases} 0 & i \neq j \\ 1 & \end{cases}}$$

$$+ \sum_{i=1}^{n} |a_i|^2 \underbrace{\|f_i\|^2}_{1}$$

$f : \mathbb{R} \to \mathbb{C}$ $(\mathbb{R})$, $f(x+T) = f(x)$, $T > 0$.

$\int_{x_0}^{x_0+T} f(x)\,dx$ mesmo $\forall\, x_0$



$$e^{ix} = \cos x + i \sin x$$

$$\left(z(x) \cdot e^{i\varphi(x)}\right)' = z'(x) \cdot e^{i\varphi(x)} + i z(x) \varphi'(x) \cdot e^{i\varphi(x)}$$

$$\int_{-\pi}^{\pi} e^{imx} \cdot e^{-inx} \, dx = \int_{-\pi}^{\pi} e^{i(m-n)x} \, dx = \frac{1}{i(m-n)} \left[e^{i(m-n)x} \right]_{-\pi}^{\pi}$$

$m \neq n \Rightarrow 0 = \langle e^{imx}, e^{inx} \rangle$

pro $m = n$ je $\int_{-\pi}^{\pi} dx = 2\pi$

$$\langle e^{imx}, e^{inx} \rangle = \langle \cos mx + i \sin mx, \cos nx + i \sin nx \rangle$$

$$= \langle \cos mx, \cos nx \rangle + \langle \sin mx, \sin nx \rangle$$

$$+ i \left(\langle \sin mx, \cos nx \rangle - \langle \cos mx, \sin nx \rangle \right)$$

integrje liché f-ce $\Rightarrow 0$

$$\cos(mx - \pi/2) = \sin(mx)$$

$$\| \cos nx \| = \pi \qquad \| \sin nx \| = \pi \qquad n \neq 0$$

$$\| 1 \| = 2\pi \,.$$

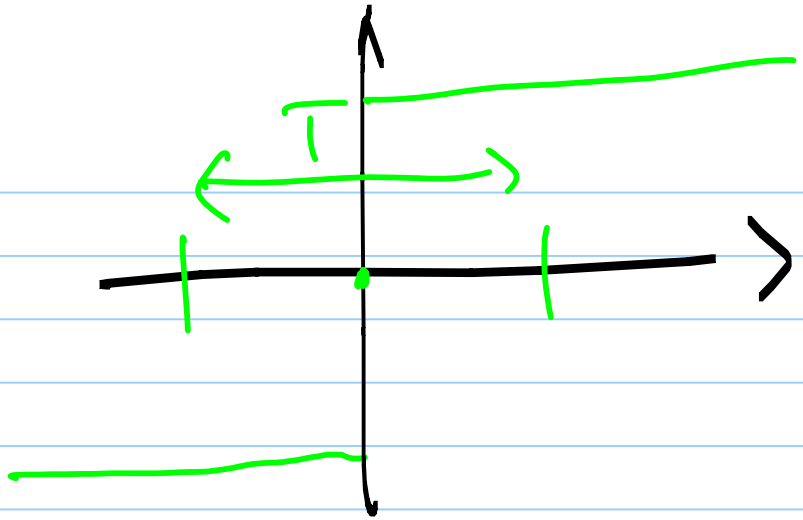
$\cos\left(\frac{2\pi}{T} n x\right), \sin\left(\frac{2\pi}{T} n x\right)$ $\qquad \omega = \frac{2\pi}{T}$

$\| \omega$ $\qquad$ $t = \omega x$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega x) + b_n \sin(n\omega x)$$

$$a_n = \frac{2}{T} \int_{x_0}^{x_0+T} g(x) \cos(n\omega x)\, dx$$

$$b_n = \frac{2}{T} \int_{x_0}^{x_0+T} g(x) \sin(n\omega x)\, dx$$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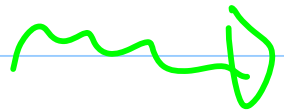

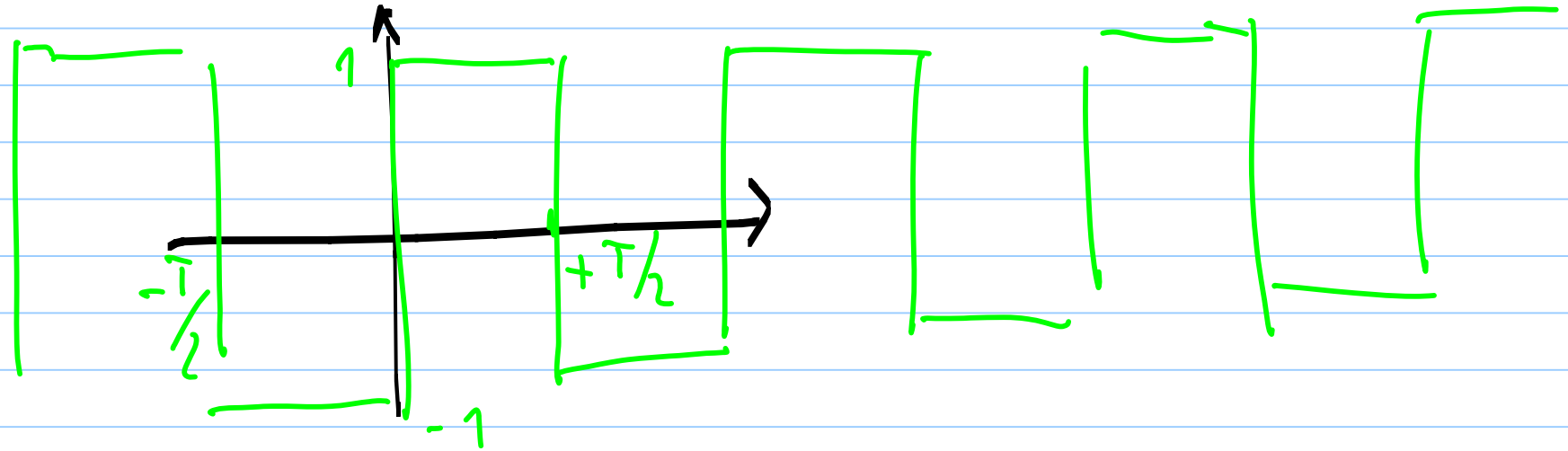
1
-1
$-\frac{T}{2}$
$+\frac{T}{2}$