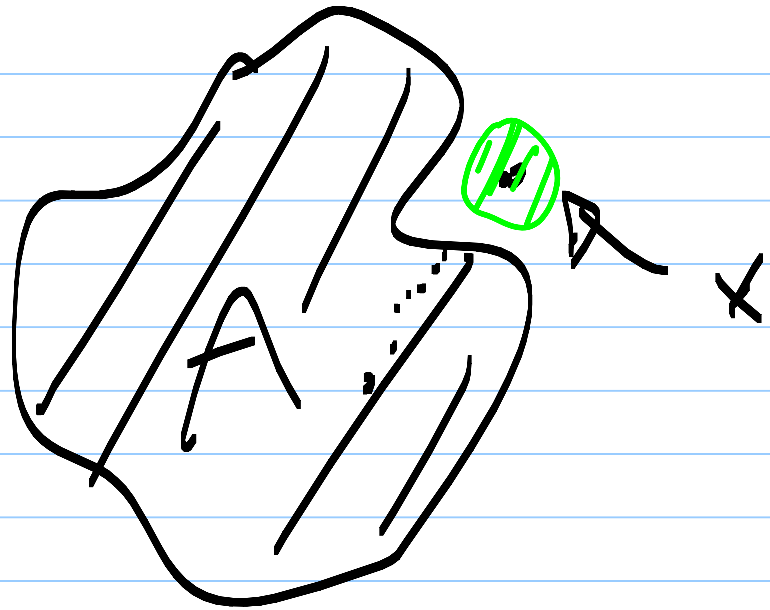
$$\|x-z\| = \|x-y+y-z\|$$

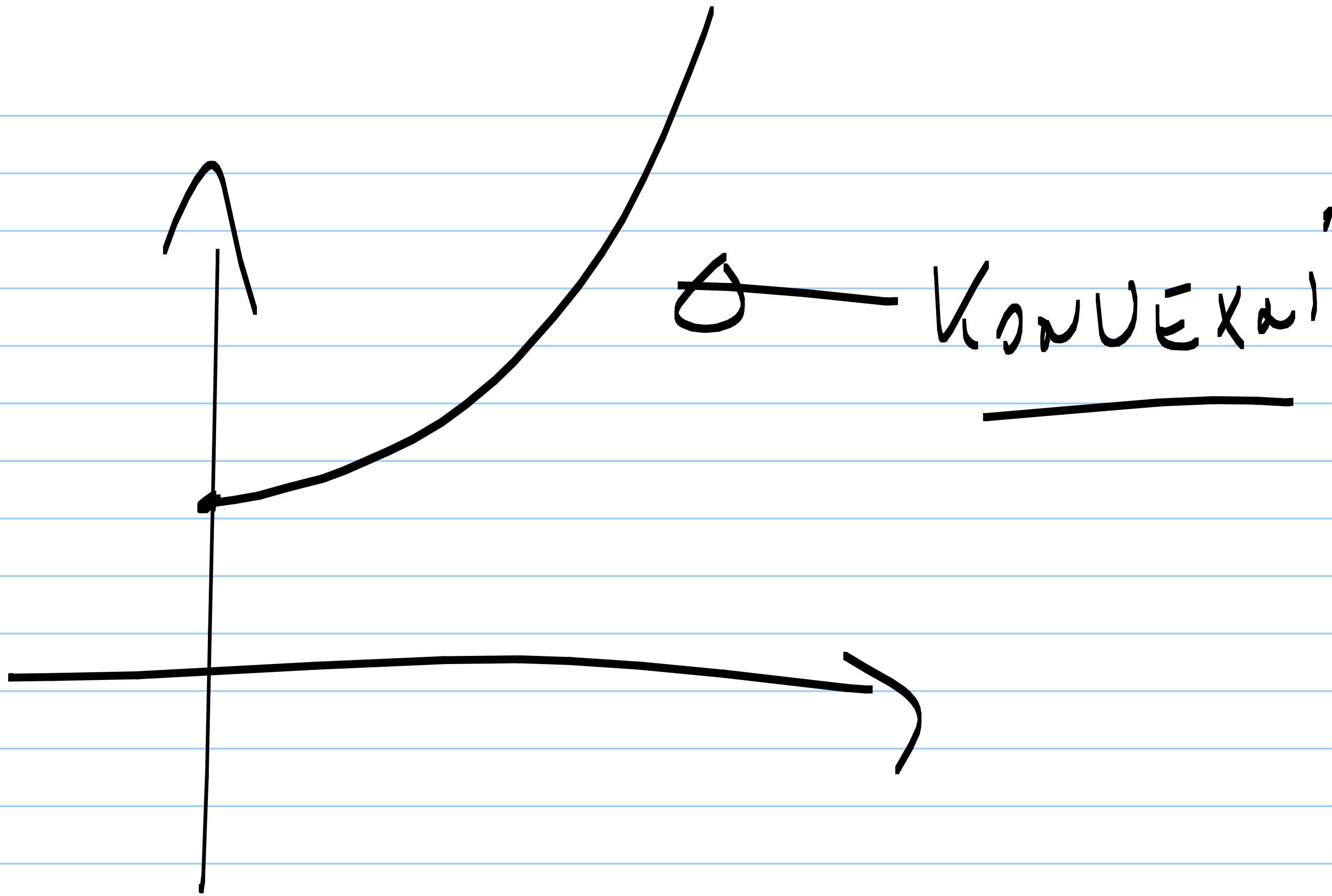
$$\leq \|x-y\| + \|y-z\|$$

$$\|f\|_2 = \left(\int_a^b |f(x)|^2 \, dx\right)^{1/2}$$

$$\|f\|_1 = \int_a^b |f(x)| \, dx$$

obě normy na $S[a,b]$


A
x


KONVEXNÍ

$$X = \left(\sum_{i=1}^{n} x_i^p\right)^{1/p} \qquad Y = \left(\sum_{i=1}^{n} y_i^q\right)^{1/q}$$

$$\cancel{\ne} 0 \qquad\qquad \cancel{\ne} 0$$

$x_i, y_i > 0$

$\frac{1}{p} + \frac{1}{q} = 1$

$$\boxed{x_i = X \cdot e^{v_i/p}} \qquad \boxed{y_i = Y \cdot e^{w_i/q}}$$

$$\frac{1}{p} v_i + \frac{1}{q} w_i$$

$$e^{v_i/p + w_i/q} \le \frac{1}{p} e^{v_i} + \frac{1}{q} \cdot e^{w_i}$$

$$\Rightarrow \frac{1}{XY} x_i y_i \le \frac{1}{p}\left(\frac{x_i}{X}\right)^p + \frac{1}{q}\left(\frac{y_i}{Y}\right)^q \qquad \sum_{i=1}^{n}$$

$$\Rightarrow \frac{1}{XY} \sum_{i=1}^{n} x_i y_i \le \frac{1}{p X^p} \sum_{i=1}^{n} x_i^p + \frac{1}{q Y^q} \sum_{i=1}^{n} y_i^q = 1$$

$X \cdot Y$

$$\left(\sum_{i=1}^{n}|x_i+y_i|^p\right)^{\boxed{1-1/q}} = 1/p$$