$e_G \in G$ $\quad f : G \to H \quad$ $f(a\cdot b) = f(a)\cdot f(b)$

$f(b) = f(a) + j\cdot$ ~~$f(a^{-1}\cdot b) = f(a^{-1})\cdot f(b)$~~

$(f(a))^{-1} \cdot f(a) = e_H \;\; = f$

2) $K \subset G$ podgrupa : $\underset{\overset{\cap}{f(K)}}{f(a)} \cdot \underset{\overset{\cap}{f(K)}}{f(b)} = \underset{\overset{\cap}{f(K)}}{f(a\cdot b)}$

$\Rightarrow x = f(\{e\}) \subset H$ je podgrupa, $x \cdot x = x \quad / \cdot x^{-1}$

$\Rightarrow \quad x = e$

(3) $K \subset H$ pokud $f(a), f(b) \in K$

$$K \ni f(a) \cdot f(b) = f(a \cdot b) \Rightarrow \underline{(a \cdot b) \in f^{-1}(K)}$$

(4) $K \ni f(a^{-1}) = (f(a))^{-1}$ protože $f(e_G) = e_H$

(5) $g = f^{-1} : H \to G$ $\quad y = f(a),\ x = f(b)$

$$f(a \cdot b) = f(a) \cdot f(b) = y \cdot x \quad / \ f^{-1} = g$$

$$\boxed{g(y \cdot x) = a \cdot b = g(y) \cdot g(x)}$$

b) $f(a) = f(b) \iff f(a \cdot b^{-1}) = f(x) \cdot (f(x))^{-1} = e_H$

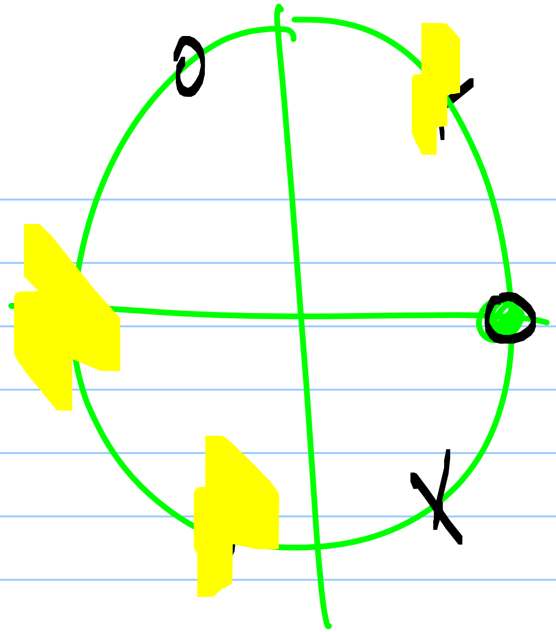
$$\Downarrow$$

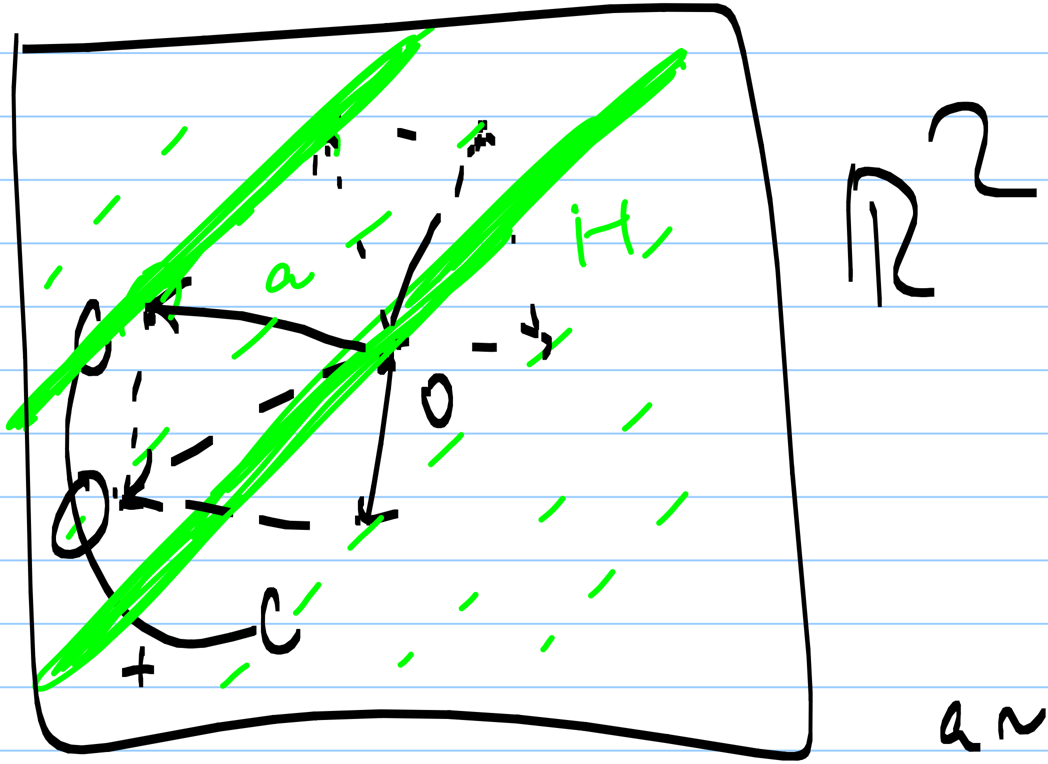
e_G [illegible] $f^{-1}(e_H)$ ✓

$$G \longrightarrow G \times H$$

$$a \longmapsto (a, e_H)$$

$$(a, e_H) \cdot (b, e_H) = (a \cdot b, e_H)$$





$a^{-1} \cdot b \in H$

$a \sim b, \; b^{-1} \cdot a \in H$

$$aH = \{a \cdot h \; ; \; h \in H\}$$

$$b = a \cdot h \quad / \cdot a^{-1}$$

$$b = \underbrace{a \cdot a^{-1}}_{e} \cdot b$$

$$= \quad e \quad h$$

1) $\forall a \in G, h \in H, \quad \underline{h \cdot a = a \cdot h'}, h' \in H$

$\Leftrightarrow \quad a^{-1} \cdot h \cdot a = h' \in H$

2) $a \cdot h = a \cdot h' \Leftrightarrow \underset{\overset{\|}{h}}{a^{-1} \cdot a h} = \underset{\overset{\|}{h'}}{a^{-1} \cdot a \cdot h'}$

$|aH| = |H|$

$$|aH| = |H|$$ a rög hű

$aH = bH$ $\qquad aH \cap bH = \emptyset$

$$\Rightarrow \left|\bigcup_{a \in G} aH\right| = |G/H| \cdot |H|$$

$f: G \to H \qquad K = f^{-1}(e)$

$$f(a^{-1} k \cdot a) = \cancel{(f(a))}^{-1} \cdot \underset{e}{\underset{\parallel}{\cancel{f(k)}}} \cdot \cancel{f(a)} = e \checkmark$$

(levá) Akce grupy : $\varphi : G \times S \to S$

$\varphi_g(x) = \varphi(g, x)$ $\qquad \varphi(a\cdot b, x) = \varphi(a, \varphi(b, x))$

$a\cdot(b\cdot x) = (a\cdot b)\cdot x$

$\forall x \quad S_x \subset S, \quad S_x = \{G\cdot x\}$ orbita akce bodu x

$G_x \subset G, \quad G_x = \{g \in G;\ g\cdot x = x\}$

izotropní podgrupa

tranzitivní akce ⟶ $\boxed{G \text{ působí } G/H}$