# Digital Signal Processing

## Decibels (dB and dBm)

Moslem Amiri, Václav Přenosil
Masaryk University

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## Using Logarithms to Determine Relative Signal Power

- Decibels evolution
  - When comparing analog signal levels, early specialists found it useful to define a measure of difference in powers of two signals
  - If that difference was treated as logarithm of a ratio of powers, it could be used as a simple additive measure to determine overall gain or loss of cascaded electronic circuits
  - Positive logarithms associated with system components having gain could be added to negative logarithms of those components having loss quickly to determine overall gain or loss of system

## Using Logarithms to Determine Relative Signal Power

- Difference between two signal power levels

$$\text{Power difference} = \log_{10}\left(\frac{P_1}{P_2}\right) \text{bels}$$

  - Measured power differences smaller than one bel were so common that it led to the use of decibel (bel/10)

$$\text{Power difference} = 10 \cdot \log_{10}\left(\frac{P_1}{P_2}\right) \text{dB}$$

# Using Logarithms to Determine Relative Signal Power

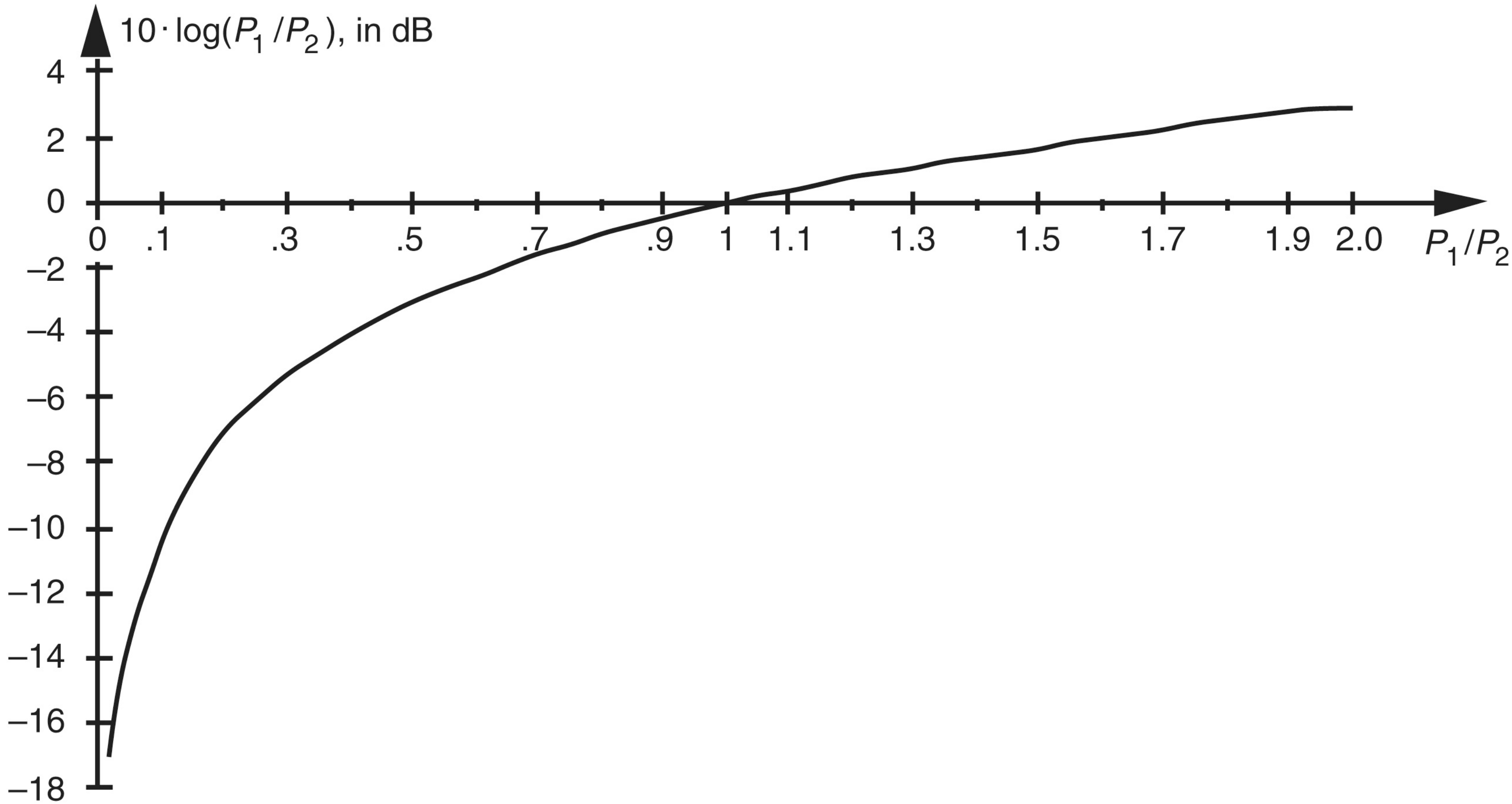


**Figure E–1** Logarithmic decibel function of Eq. (E–2).

# Using Logarithms to Determine Relative Signal Power

- Fig. E-1
  - Plot of logarithmic function $10 \cdot \log_{10}(P_1/P_2)$
  - Large change in function's value when power ratio ($P_1/P_2$) is small, and gradual change when ratio is large
  - Effect of this nonlinearity is to provide greater resolution when ratio $P_1/P_2$ is small, giving us a good way to recognize very small differences in power levels of signal spectra, digital filter responses, and window function frequency responses

# Using Logarithms to Determine Relative Signal Power

- For any frequency-domain sequence *X(m)*

$$\text{discrete power spectrum of } X(m) = |X(m)|^2$$

$$X_{dB}(m) = 10 \cdot \log_{10}(|X(m)|^2) = 20 \cdot \log_{10}(|X(m)|)\ dB$$

  - These expressions are used to convert a linear magnitude axis to a logarithmic magnitude-squared, or power, axis measured in dB
  - Without the need for squaring operation, we calculate $X_{\text{dB}}(m)$ power spectrum sequence from $X(m)$ sequence

# Using Logarithms to Determine Relative Signal Power

- Normalized log magnitude spectral plots

$$\text{normalized } X_{dB}(m) = 10 \cdot \log_{10}\left(\frac{|X(m)|^2}{|X(0)|^2}\right) = 20 \cdot \log_{10}\left(\frac{|X(m)|}{|X(0)|}\right)\ dB$$

  - Division by $|X(0)|^2$ or $|X(0)|$ value forces the first value in normalized log magnitude sequence $X_{\text{dB}}(m)$ equal to 0 dB
    - This makes it easy to compare multiple log magnitude spectral plots

# Using Logarithms to Determine Relative Signal Power

(a)

(b)

(c)

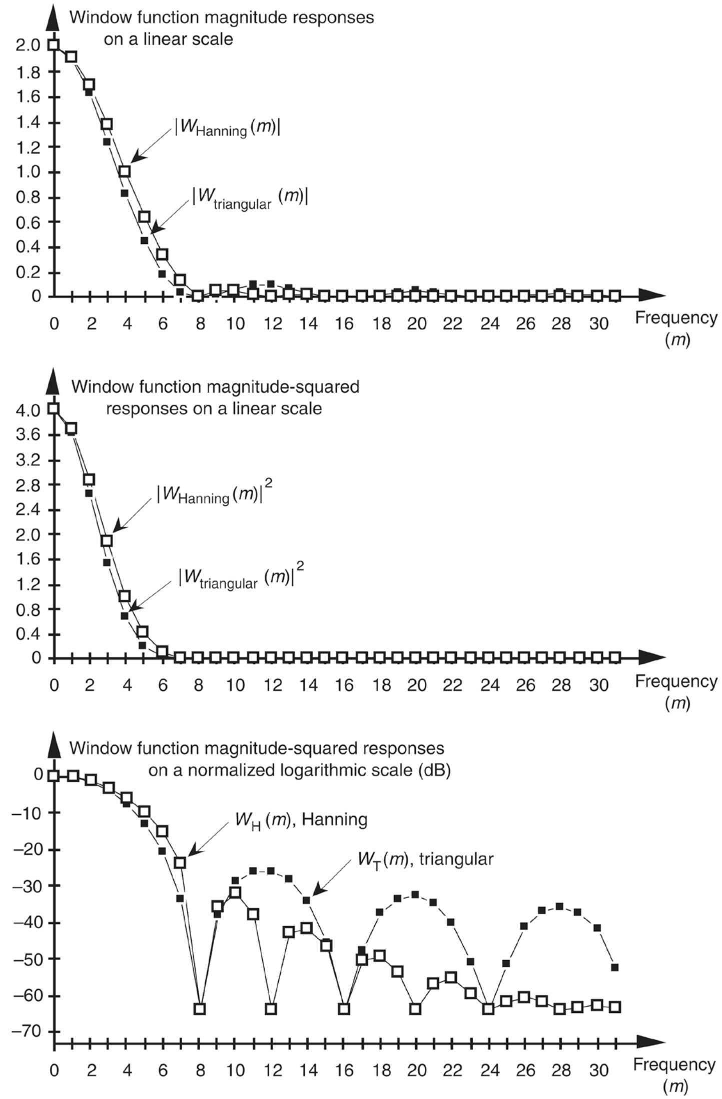


**Figure E–2** Hanning (white squares) and triangular (black squares) window functions in the frequency domain: (a) magnitude responses using a linear scale; (b) magnitude-squared responses using a linear scale; (c) log magnitude responses using a normalized dB scale.

# Using Logarithms to Determine Relative Signal Power

- Fig. E-2(c)
  - Normalization

$$W_H(m) = 10 \cdot \log_{10}\left(\frac{|W_{Hanning}(m)|^2}{|W_{Hanning}(0)|^2}\right) = 20 \cdot \log_{10}\left(\frac{|X_{Hanning}(m)|}{|X_{Hanning}(0)|}\right)\ dB$$

  - We can clearly see the difference in magnitude-squared window functions in (c) as compared to linear plots in (b)

# Using Logarithms to Determine Relative Signal Power

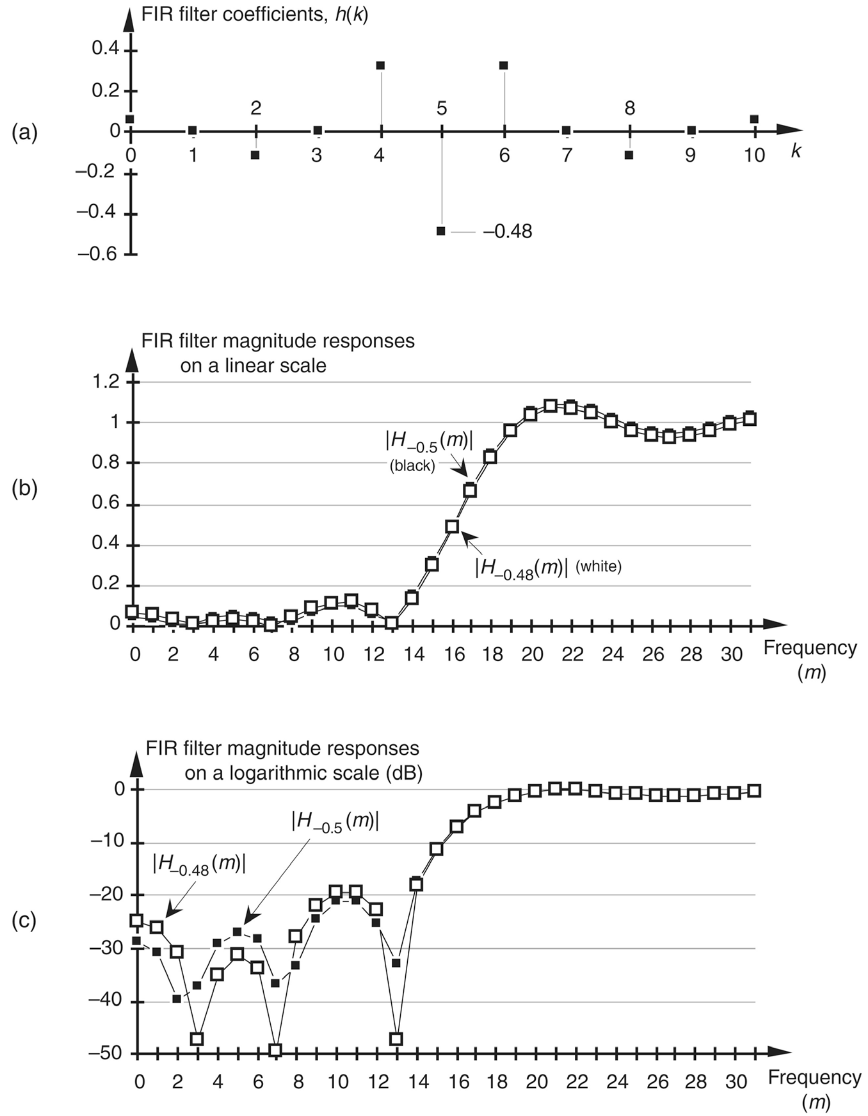


**Figure E-3** FIR filter magnitude responses: (a) FIR filter time-domain coefficients; (b) magnitude responses using a linear scale; (c) log magnitude responses using the dB scale.

# Using Logarithms to Determine Relative Signal Power

- Fig. E-3
  - We're designing an 11-tap highpass FIR filter whose coefficients are shown in (a)
  - If center coefficient $h(5)$ is –0.48, filter's frequency magnitude response $|H_{-0.48}(m)|$ can be plotted as white dots on linear scale in (b)
  - $h(5)$: –0.48 → –0.5, new frequency magnitude response $|H_{-0.5}(m)|$ are black dots in (b)
  - Difficult to see much difference on a linear scale
  - Calculating two normalized log magnitude sequences, filter sidelobe effects of changing $h(5)$ are now easy to see, as shown in (c)

# Some Useful Decibel Numbers

- A few constants to memorize
  - A power difference of 3 dB corresponds to a power factor of two
    - That is, if magnitude-squared ratio of two different frequency components is 2, then

$$\text{power difference} = 10 \cdot \log_{10}\left(\frac{2}{1}\right) = 10 \cdot \log_{10}(2) = 3.01 \approx 3\,\text{dB}$$

  - If magnitude-squared ratio of two different frequency components is 1/2

$$\text{power difference} = 10 \cdot \log_{10}\left(\frac{1}{2}\right) = 10 \cdot \log_{10}(0.5) = -3.01 \approx -3\,\text{dB}$$

# Some Useful Decibel Numbers

| **Magnitude ratio** | **Magnitude-squared power ($P_1/P_2$) ratio** | **Relative dB (approximate)** | |
|---|---|---|---|
| $10^{-1/2}$ | $10^{-1}$ | -10 | $P_1$ is one-tenth $P_2$ |
| $2^{-1}$ | $2^{-2} = 1/4$ | -6 | $P_1$ is one-fourth $P_2$ |
| $2^{-1/2}$ | $2^{-1} = 1/2$ | -3 | $P_1$ is one-half $P_2$ |
| $2^{0}$ | $2^{0} = 1$ | 0 | $P_1$ is equal to $P_2$ |
| $2^{1/2}$ | $2^{1} = 2$ | 3 | $P_1$ is twice $P_2$ |
| $2^{1}$ | $2^{2} = 4$ | 6 | $P_1$ is four times $P_2$ |
| $10^{1/2}$ | $10^{1} = 10$ | 10 | $P_1$ is ten times $P_2$ |
| $10^{1}$ | $10^{2} = 100$ | 20 | $P_1$ is one hundred times $P_2$ |
| $10^{3/2}$ | $10^{3} = 1000$ | 30 | $P_1$ is one thousands times $P_2$ |

# Absolute Power Using Decibels

- Another use of decibels
  - To measure signal-power levels referenced to a specific absolute power level
    - In this way, we can speak of absolute power levels in watts while taking advantage of convenience of decibels
  - The most common absolute power reference level used is milliwatt

$$\text{absolute power of } P_1 = 10 \cdot \log_{10}\left(\frac{P_1}{P_2}\right) = 10 \cdot \log_{10}\left(\frac{P_1 \text{ in watts}}{1 \text{ milliwatt}}\right) \text{dBm}$$

  - dBm = dB relative to a milliwatt