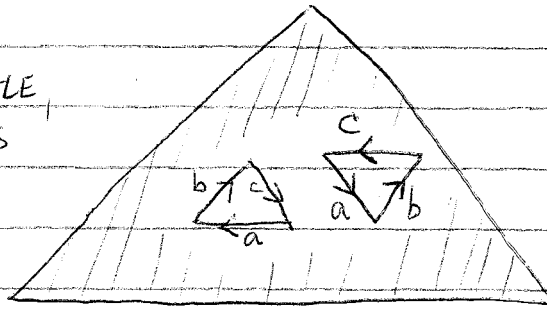


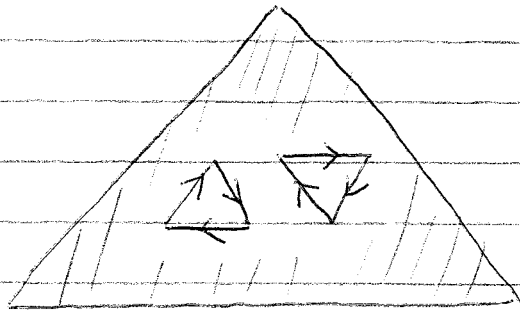
DEF: (On triangulated surfaces)
 - adding a handle =

TAKE A TRIANGLE,
 CUT 2 TRIANGLES
 FROM IT AND
 GLUE THEM
 AS IN PICTURE

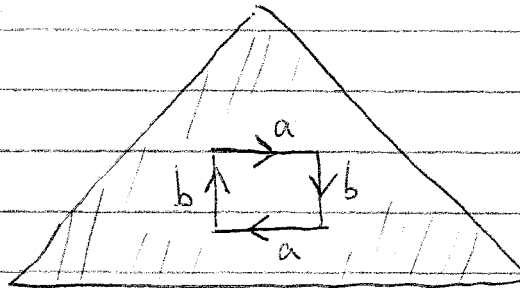


(THESE 2 TRIANGLES CAN BE FROM DIFFERENT TRIANGLES → SAME RESULT)

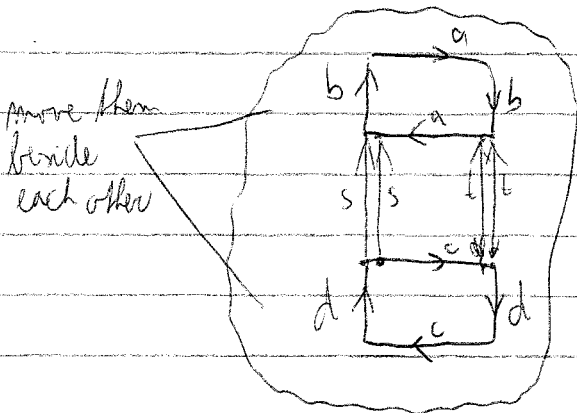
- adding an antihandle (twisted handle)



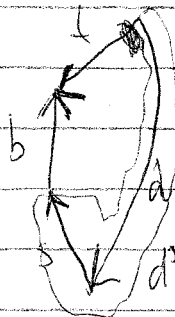
- adding a crosscap



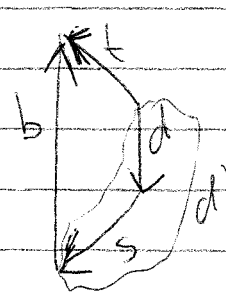
CLAIM: antihandle = 2x crosscap



FROM LEFT SIDE:



FROM RIGHT SIDE:



then this will result in an antihandle.

If there is a crosscap, then (disjoint) an ihandle = handle.

COROLLARY: Surfaces may be

- S_0 the sphere,
- S_h the sphere with h handles
- N_h the sphere with h crosscaps

THEOREM: A triangulated surface with f faces (triangles), g edges, and n vertices (?) is homeomorphic to either S_h or N_h where

$$n - g + f = 2 - 2h = 2 - h = \text{"Euler char. of the surface"}$$

Proof: Assume a counterexample such that it is a simple graph and

- Euler char. is max possible,
- among those n is minimal, and
- among those min. degree is minimum.