(1) CLASSIF. OF SURFACES

IN PREVIOUS LECTURE

I. Every surface can be triangulated

II. Every triang. surface is obtained from simplex by adding handles (S_h) or crosscaps (N_k).

III. Euler's formula for $\mathcal{S}$: $n - q + f = \chi(\mathcal{S}) = 2-2h = 2-k$ (invariant on all triang. of $\mathcal{S}$)

IV. No two of $S_h, S_{h'}, N_k, N_{k'}$ ($h \neq h'$, $k \neq k'$) are homeomorphic.

Proof (IV): assume $h' > h$, then the characteristics χ of those two are different. If they were homeomorphic, then we could embedd the higher surface ~~to~~ into the lower.

⋮

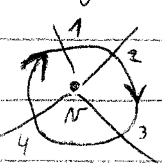
(2) EMBEDDINGS ON SURF.

GRAPH → SURFACE
- 2-cell embedding
  "making the surf. from triangles"

SURFACE → GRAPH
- cellular embedding
  "vertices to points on $\mathcal{S}$, edges to arcs on $\mathcal{S}$"

THM: 2-cell embedding ⇔ cellular embedding (every face hom. to disc)

I. The clockwise order of edges from a vertex is properly defined on $\mathcal{S}$.

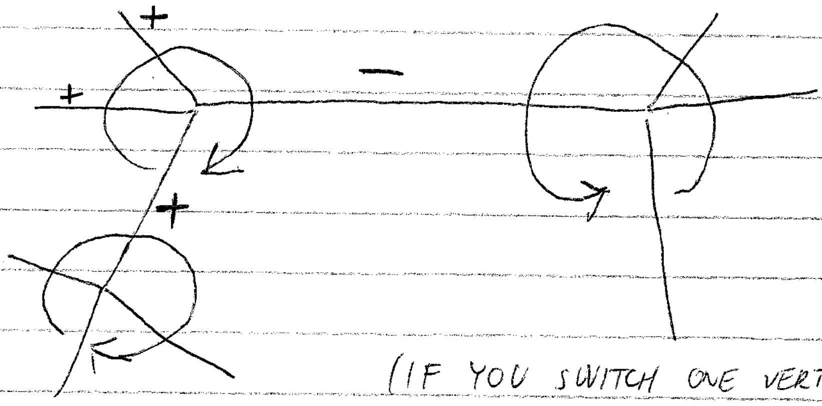


- Every face at v is homeomorphic to a disc also union of those faces is homeomorphic to a disc. It is orientable and we can say that v is always on the right

II. These clockwise orders $(\pi(v); v \in V(G))$ form a so called rotation scheme. For orientable $\mathcal{S}$, this gives a 2-cell embedding homeomorphic to G on $\mathcal{S}$.

FOR ORIENTABLE SURFACES

III. For nonorientable surface Σ, we use an embedding scheme = rotation scheme + signs on the edges.



(IF YOU SWITCH ONE VERTEX, SIGNS AT THIS VERTEX'S EDGES ARE SWITCHED).

(If it is not possible to perform schwitches to get only +'s, then it is nonorientable).