$n C_n - (n-1) C_{n-1} = (n-1)(n-n+2) + 2 C_{n-1}$

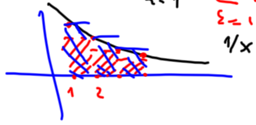
$n C_n = (n+1) C_{n-1} + 2(n-1)$

$: n(n+1)$

$$\underbrace{\frac{C_n}{n+1}}_{= B_n} = \underbrace{\frac{C_{n-1}}{n}}_{= B_{n-1}} + \frac{2(n-1)}{n(n+1)}$$

$$\sum_{k=1}^{n} \frac{1}{k} > \int_1^n \frac{1}{x}\,dx > \sum_{k=2}^{n+1} \frac{1}{k}$$

$\int_1^n \frac{1}{x}\,dx = \ln n$



4 7-13:56

a, b, c $\qquad n = 3$

$k = 9$

* * * * * * * * * * *

aaa|bbbb|ccccc

aaaa||ccccc

$\binom{3+9-1}{3}$

$\binom{15}{4}$

vždy

4 7-14:34

$1+2+3+\dots+98+99+100$

101

101

$1+2+3 = (1+3+2+2+3+1)\cdot 1/2 = \frac{n(n+1)}{2}$

$(x-1)(x^n + x^{n-1} + \dots + x + 1) = x^{n+1} - 1$

$n \to \infty \qquad \frac{1}{1-x}$

4 7-14:43

$$\underbrace{(x+y)(x+y)\cdot\ldots\cdot(x+y)}_{n\text{-krát}} = \boxed{\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}}$$

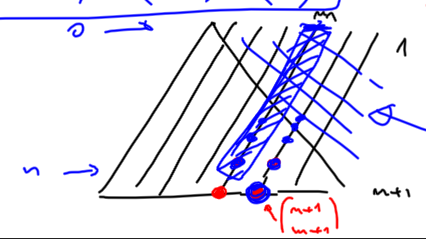
$$\boxed{\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}}$$

$n \to$ 1

1 1

1 2 1

1 3 3 1



$\binom{n+1}{n+1}$

4 7-14:50

$r = k + r - k$

$r(m) \qquad r(n)$

4 7-14:58

každý: 1/2

dohromady: $(1/2)^{100}$

Špatně!

$a, \sigma(a), \sigma^2(a), \dots$ $\quad \square \xrightarrow{\sigma} \square$

referenční strategie:

stačí jít po číslech, která ukáží!

$\boxed{1}$ 2 3 4 ... 99 100

2 3 4 5 ... 100 1

permutací na 100 prvcích: n prvků $\qquad n!$

permutace s cyklem > 50: cyklus > $n/2$

cyklů délky $r > n/2$: $\binom{n}{r}(r-1)!\,(n-r)! =$

$$= \frac{n!\,\cancel{(r-1)!}\,\cancel{(n-r)!}}{\cancel{r!}\,\cancel{(n-r)!}} = \frac{n!}{r} \qquad \boxed{\frac{1}{r}}$$

4 7-15:12

$\Rightarrow$ po naší větě:

$$1-\sum_{k=51}^{100}\frac{1}{k} \cong 0{,}312$$

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 7 & 1 & 2 & 6 & 5 & 4 \end{array}$$

Šnekce

4 7-15:25

$$(x+y)^\alpha = \sum_{k=0}^{\infty}\binom{\alpha}{k}x^k y^{\alpha-k} \qquad x+y>0$$

předp. $y>0$.

$$\cancel{y^\alpha}\cdot\left(\frac{x}{y}+1\right)^\alpha = \sum_{k=0}^{\infty}\binom{\alpha}{k}\left(\frac{x}{y}\right)^k\cdot\cancel{y^\alpha}$$

$$\frac{d}{dx}(1+x)^\alpha = \alpha\cdot(1+x)^{\alpha-1}$$

$$\frac{d}{dx}\left(\sum_{k=0}^{\infty} b_k x^k\right) = \sum_{k=1}^{\infty} b_k\cdot k\, x^{k-1}$$

$$\alpha\cdot(1+x)^{\cancel{\alpha-1}} = \left(\sum_{k=1}^{\infty} k\cdot b_k x^{k-1}\right)(1+x)$$

$$\alpha\sum_{k=0}^{\infty} b_k x^k \quad\Rightarrow\quad \alpha b_k = (k+1)b_{k+1} + k\,b_k$$

$$b_{k+1} = \frac{\alpha-k}{k+1}b_k$$

$b_0 = 1$

$b_1 = \frac{\alpha}{1}\cdot 1$

$b_2 = \frac{(\alpha-1)}{2}\cdot\frac{\alpha}{1}$

$\vdots$

$b_n = \binom{\alpha}{n}$

4 7-15:37