$$(x+x^2+x^3+x^4+x^5)(x^2+\dots+x^{10})(x^3+\dots+x^{15}) = \dots + ⓟ x^7 + \dots$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

$$3^n = \sum_{k=0}^{n} \binom{n}{k} \cdot 2^k$$

$$n(1+x)^{n-1} = \binom{n}{1} + 2\binom{n}{2}x + \dots + \binom{n}{n} n x^{n-1}$$

$$x=1: \quad \boxed{n 2^{n-1} = \sum_{k=0}^{n} k\binom{n}{k}.}$$

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$$A(x) = \sum_{i=0}^{\infty} a_i x^i \qquad B(x) = \sum_{j=0}^{\infty} b_j x^j$$

$$(A+B)(x) = \sum_{i=0}^{\infty} (a_i+b_i)x^i$$

$$(A\cdot B)(x) = \sum_{k=0}^{\infty} c_k x^k, \quad c_k = \sum_{i+j=k} a_i b_j$$

$$y = A(x) \quad B(y) = \sum_{i=0}^{\infty} b_i \left(\sum_{j=0}^{\infty} a_j x^j\right)^i$$

$$\cancel{(1-x)}\cdot(1+x+\dots+x^k) = \frac{1-x^{k+1}}{1-x} \xrightarrow[|x|<1]{k\to\infty} \boxed{\frac{1}{1-x}}$$

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$$\limsup_{k\to\infty} \sqrt[n]{|a_n|} = \rho$$

$$\left(-\frac{1}{\rho}, \frac{1}{\rho}\right)$$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

$$(e^x)' = \sum_{n=1}^{\infty} \frac{1}{n!}\cdot n x^{n-1} = \sum_{n=0}^{\infty} \frac{1}{n!} x^n = e^x$$

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$$a_k = (A(x))^{(k)}\Big|_0 \cdot \frac{1}{k!} \qquad A(x) = \frac{1}{1-x} = \sum a_i x^i$$

$$\left(\ln\frac{1}{1-x}\right)' = (1-x)(1-x)^{-2} = \frac{1}{1-x}$$

$$B(x) = \ln\frac{1}{1-x} \qquad b_k = \underbrace{(A(x))^{(k-1)}\cdot\frac{1}{(k-1)!}}_{a_{k-1}}\cdot\frac{1}{k}$$

$$e^{ix} = \cos x + i\sin x$$

$$(-1)^k \cdot \frac{-n(-n-1)\cdot\dots\cdot(-n-k+1)}{k!}$$

$$(1+x)^{-n} = \sum_{k\ge 0} \binom{-n}{k} x^k$$

$$= \frac{n\cdot(n+1)\cdot\dots\cdot(n+k-1)}{k!}$$

$n \in \mathbb{N}$ $\quad (-1)^k$

$$= \binom{n+k-1}{n-1}$$

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$$\frac{1}{(1-x)^2} = 1 + \binom{2}{1}x + \binom{3}{1}x^2 + \binom{4}{1}x^3 + \dots$$

$$= \sum_{k=0}^{\infty} \binom{k+1}{1} x^k = \sum_{k=0}^{\infty} (k+1)x^k$$

$$\frac{1}{(1-x)^3} = \sum_{k=0}^{\infty} \binom{k+2}{2} x^k$$

$$(a_0, a_1, \dots) \xmapsto{\text{v.f.p.}} A(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$A(x)\cdot x^k = \sum_{n=0}^{\infty} a_n x^{n+k}$$

$$(0, 0, \dots, 0, a_0, a_1, \dots)$$

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$$\left(\frac{1}{k+1} x^{k+1}\right)' = x^k$$

$$\boxed{\frac{1}{1-x}\cdot A(x)} \qquad \frac{1}{1-x} \sim (1,1,1,1,\dots)$$

$$c_k = \sum_{i+j=k} a_i = \sum_{i=0}^{k} a_i$$

$$H_n = \sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \dots \frac{1}{n} \qquad \frac{3}{2} + \frac{1}{3} = \frac{9+2}{6}$$

$$1, \frac{3}{2}, \frac{11}{6}, \dots \quad \text{harmonická čísla}$$

v.f.p. pro H_n je $\boxed{\frac{1}{1-x}\ln\frac{1}{1-x}}$

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$$\frac{1}{1-x} = 1 + x + x^2 + \dots$$

$$\frac{1}{1-x} \cdot \frac{1}{1-x} = 1 + 2x + 3x^2 + \dots$$

$$(1 + x + x^2 + \dots + x^{40}) = \frac{1-x^{41}}{1-x}.$$

$$(x^{10} + \dots + x^{40}) = \frac{x^{10} - x^{41}}{1-x}$$

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$$H_n \xrightarrow{\text{v.f.g.}} \frac{1}{1-x} \ln \frac{1}{1-x}$$

$$\underbrace{\frac{1}{(1-x)^2}}_{b_k = (k+1)} \cdot \underbrace{\ln \frac{1}{1-x}}_{a_k = \frac{1}{k} \quad a_0 = 0} \leftarrow \text{koef } u\ x^n\ ?$$

$$c_k = \sum_{i+j}^{k} a_i b_j = \sum_{i=1}^{k} \frac{1}{i} (k - i + 1)$$

$$c_n = (n+1) \sum_{k=1}^{n} \frac{1}{k} - n = (n+1) H_{n+1}$$

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