$y_{n+2} = f(y_{n+1}, y_n, n)$ rekurence 2. řádu

$\boxed{y_{n+2} = a\, y_{n+1} + b\, y_n}$ lineární! s konst. koef.

konstanty

$y_{n+1} = a\, y_n \qquad y_n = a^n y_0 = \underbrace{a \cdots a \cdot a}_{n\times}\, y_0$

$\lambda^{n+2} = a\lambda^{n+1} + b\lambda^n \Leftrightarrow \lambda^n(\boxed{\lambda^2 - a\lambda - b}) = 0$

$(a_0, a_1, a_2, \dots) \sim a_0 + a_1x + a_2x^2 + \dots = \sum_{i\geq 0} a_i x^i$

$a_n: [x^n]F(x) \qquad F(x)$

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$\boxed{a_{n+2} = a_{n+1} + a_n} \qquad a_0 = 0,\ a_1 = 1$

$(0, 1, 1, 2, 3, 5, 8, 13, \dots)$

$\boxed{F(x) = xF(x) + x^2F(x) + x} \qquad \boxed{a_0 + (a_1 - a_0)x}$

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \vdots \end{pmatrix} = \begin{pmatrix} 0 \\ a_0 \\ a_1 \\ a_2 \\ \vdots \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ a_0 \\ a_1 \\ \vdots \end{pmatrix} + \begin{pmatrix} a_0 \\ a_1 - a_0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}$$

$(-x^2 - x + 1)F(x) = x \Rightarrow F(x) = \boxed{\frac{x}{1 - x - x^2}}$

$F(x) = \frac{A}{x - x_1} + \frac{B}{x - x_2} \qquad x_{1,2} = \frac{1 \pm \sqrt{5}}{2}$

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$\frac{1}{1-x} = 1 + x + x^2 + \dots$

$\frac{1}{1-\lambda x} = 1 + \lambda x + \lambda^2 x^2 + \dots$

$\frac{1}{x - x_1} = \frac{-1/x_1}{1 - x/x_1}$

$\rightarrow x^2 - ax - b = 0$

$\rightarrow 1 - a\frac{1}{x} - b\,{}^1/{x^2}$

$\lambda_1 = \frac{1+\sqrt{5}}{2}$

$\lambda_2 = \frac{1-\sqrt{5}}{2}$

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$\frac{1}{(1-x)^k} = \binom{k-1}{k-1} + \binom{1+k-1}{k-1}x + \dots + \binom{n+k-1}{k-1}x^n$

$\boxed{\frac{1}{1-x}} = 1 + x + x^2 + \dots = \sum_{n\geq 0} x^n$

$\frac{1}{(1-x)^2} = \binom{1}{1} + \binom{2}{1}x + \binom{3}{1}x^2 + \dots = \boxed{\sum_{n\geq 0}(n+1)x^n}$

$\frac{1}{(1-x)^3} = \binom{2}{2} + \binom{3}{2}x + \binom{4}{2}x^2 + \dots = \frac{1}{2}\sum_{n\geq 0}(n+2)(n+1)x^n$

$\vdots$

$\boxed{\frac{A}{(x-\alpha)^k} = \frac{(-1)^k A/\alpha^k}{(1 - x/\alpha)^k}}$

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$(1, 1, 1, \dots)$

$1 + x + x^2 + \dots = \frac{1}{1-x}$

$1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots = e^x$

$\boxed{(e^x)' = e^x}$

$\frac{1}{1-x} = 1 + x + x^2 + \dots$

$\frac{1}{1-x} \cdot F(x) = c_0 + c_1x + c_2x^2 + \dots$

$c_k = \sum_{i=0}^{k} a_i$

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$\sum_{n\geq 0} a_n x^n = \sum_{n\geq 0} 5a_{n-1}x^n - \sum_{n\geq 0} 6a_{n-2}x^n + x$

$A(x) = 5xA(x) - 6x^2A(x) + x$

$\frac{x}{(1 - 5x + 6x^2)} = A(x) = \frac{a}{1-3x} + \frac{b}{1-2x}$

$(2x-1)(3x-1)$

$x = a(1-2x) + b(1-3x)$

$0 = a + b \qquad a = -b = 1$

$1 = -2a - 3b$

$\boxed{a_n = 3^n - 2^n}$

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$$x \cdot \frac{1}{1-x} \cdot C(x) = \sum_{n \geq 0} \left(\sum_{k=0}^{n} c_k \right) x^{n+1}$$

$$C'(x) = \frac{2x}{(1-x)^3} + 2\frac{1}{1-x} \cdot C(x)$$

$$\left((1-x)^2 C(x)\right)' = -2\cancel{(1-x)}\, C(x) + \underbrace{(1-x)^2 C'(x)}$$

$$(1-x)^2 C'(x) = \frac{2x}{1-x} + 2\cancel{(1-x)} \cdot C(x)$$

$$\frac{1}{1-x} \ln \frac{1}{1-x} \sim \text{v.f.p. } H_n$$

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