Similarity Search by Aggregation of Multiple Pivot-Permutation Ranking

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Outline of the Talk

1. Approximate Distance-based Similarity Search
2. PPP-Codes Approach
   - Multiple Pivot Space Partitioning
   - Ranking of the Data Objects
   - Indexing and Searching
3. Efficiency of our Approach
4. Summary

Preliminaries

- generic similarity search
- data modeled as metric space $(\mathcal{D}, \delta)$, where $\mathcal{D}$ is a *domain* of objects and δ is a total *distance function* $\delta : \mathcal{D} \times \mathcal{D} \longrightarrow \mathbb{R}_0^+$ satisfying postulates of identity, symmetry, and triangle inequality

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- distance function δ may be time consuming
- requires approximate search

Motivation

current indexes for large-scale approximate search:

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reading and refinement of S_C form majority of the search costs

- accuracy of the candidate set is key

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4. the final candidate set is retrieved and refined

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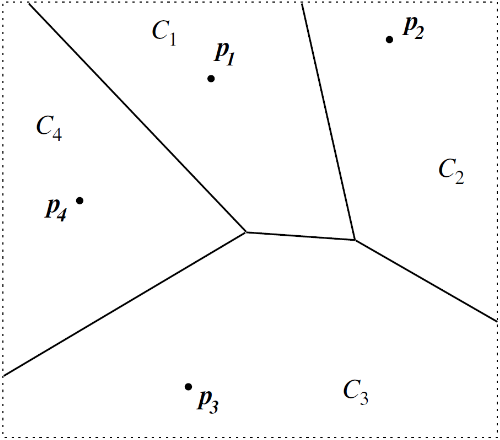

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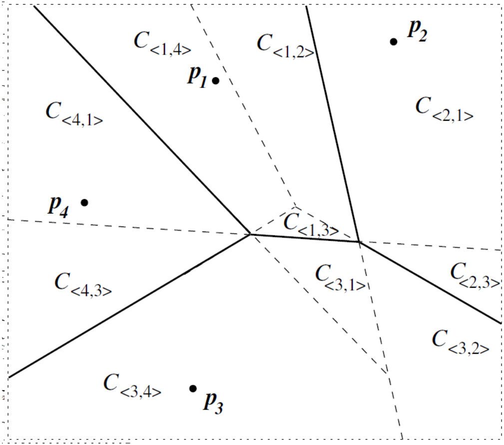

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Formally:
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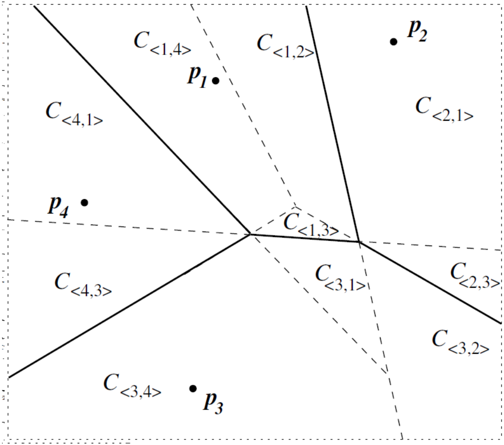
Π_x on $\{1, \ldots, k\}$ such that $\Pi_x(i)$ is the i-th closest pivot from x



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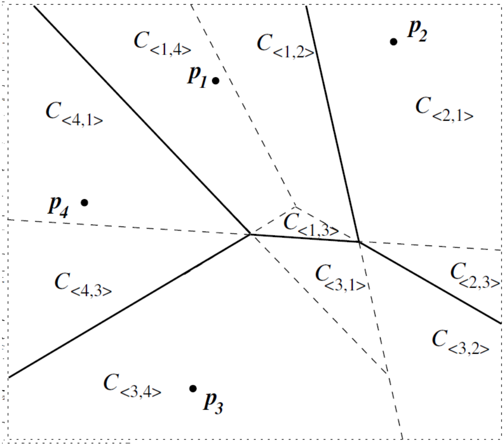
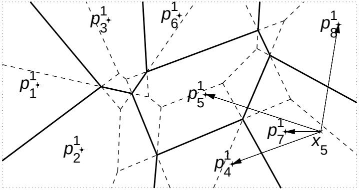
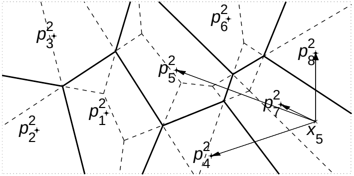
each Voronoi cell corresponds to a pivot permutation prefix (PPP) of length l: $\Pi_x(1..l)$



Multiple Pivot Space Partitioning

We propose to create λ independent pivot space partitionings





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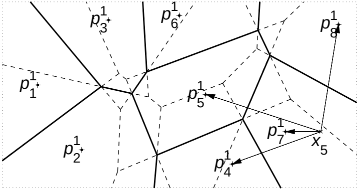

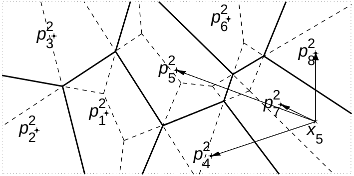

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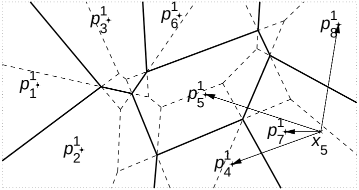

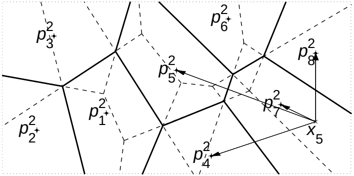

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in the example above $\lambda = 2,\ k = 8,\ l = 4$:

$$PPP_4^{1..2}(x_5) = \langle \langle 7,4,8,5 \rangle, \langle 7,8,4,6 \rangle \rangle$$

Ranking within a Single Pivot Space

Task: Having data $x \in \mathcal{X}$ encoded by PPP $\Pi_x(1..l)$ (single recursive Voronoi partitioning), define ranking of the PPPs with respect to $q \in \mathcal{D}$

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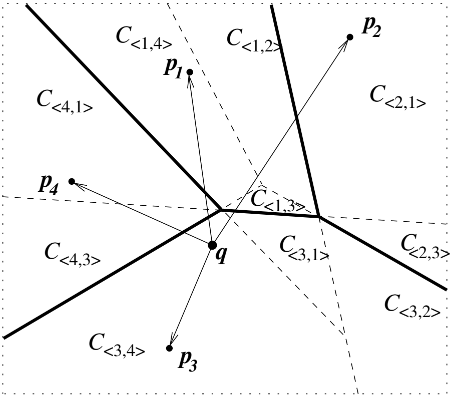
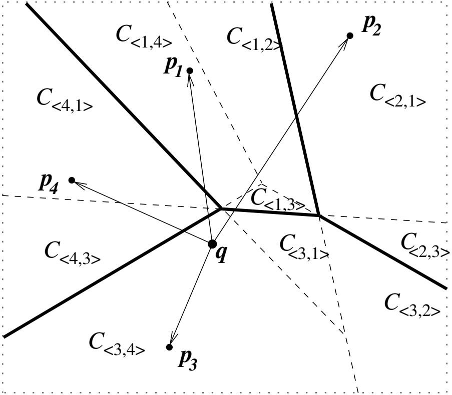


Ranking within a Single Pivot Space

Solution: We define distance between Voronoi cell $C_{\langle i_1,\dots,i_l\rangle}$ and query q as a weighted arithmetic mean of distances $\delta(q, p_{i_1}), \dots, \delta(q, p_{i_l})$



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$q \in \mathcal{D}$

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\begin{aligned}
\psi_q^{1}&: \overbrace{\{\boldsymbol{x}\; y_1\, y_2\}}^{\text{objects with the rank '1'}}\; \overbrace{\{y_3\, y_4\, y_5\}}^{\text{rank '2'}}\; \overbrace{\{y_6\}}^{\text{rank '3'}} \ldots\\
\psi_q^{2}&: \{y_3\, y_2\}\; \{y_1\, y_4\, y_6\, y_7\}\; \{\boldsymbol{x}\; y_8\} \ldots\\
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\end{aligned}
$$

Ranking using Multiple Pivot Spaces

Solution: Ranking of object x is p-percentile (e.g. median) of its λ ranks

$$\Psi_{\mathbf{p}}(q,x) = percentile_{\mathbf{p}}(\psi_q^1(x), \psi_q^2(x), \ldots, \psi_q^\lambda(x))$$

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$$\psi_q^5: \{y_1\ y_2\}\ \{y_4\ y_5\}\ \{y_3\}\ \{\boldsymbol{x}\ y_7\}\ \ldots$$

$$\Psi_{0.5}(q,x) = percentile_{0.5}\{1, 1, \mathbf{3}, 4, ?\} = 3$$

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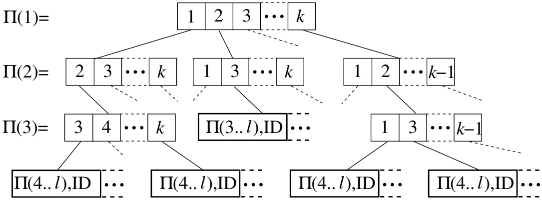
- the percentile-based aggregation increases probability that query-relevant objects are ranked higher than the distant ones

Indexing the PPP-Codes

We build trie-like structure for each pivot space

- leafs: only suffixes of PPPs (spare memory)
- dynamic splits to optimize the memory usage
- possible grouping and delta-encoding of IDs in leaves



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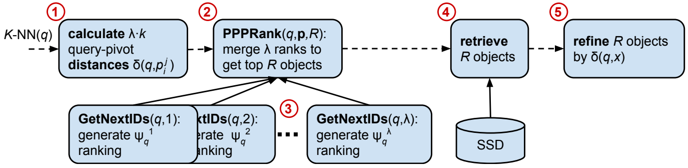

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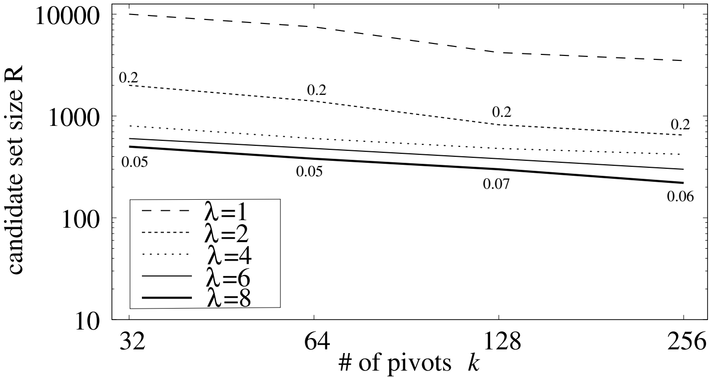

Candidate set size R necessary to achieve 80% of 1-NN recall

Settings: 1M CoPhIR dataset, $l = 8$ and $\mathbf{p} = 0.75$

Experimental Evaluation Criteria

three datasets:

- 100M CoPhIR (280-dim, complex metric, obj.: 600 B, δ time 0.01 ms)
- 1M SQFD (quadratic form distance, obj.: 2 kB, δ time 0.5 ms)
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technical evaluation of our approach:

- mutual influence of various parameters to recall
  - k, l, λ, $\mathbf{p}$, size of the PPP-Code representation
- $k \in \{64, 128, 256, 512\}$, $l = 8$, $\lambda = 5$, $\mathbf{p} = 0.5$ (3rd rank out of 5)

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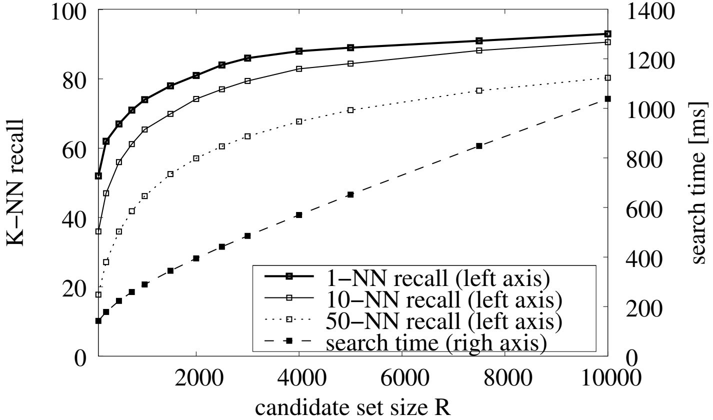

Recall and search time on while increasing candidate set size R.

Settings: 100M CoPhIR dataset, $k = 512$

Evaluation: Tradeoff

complexity of the PPPRank algorithm vs. candidate set reduction

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	PPP-Code / M-Index [ms]	size of an object [bytes] 512	1024	2048	4096
δ time	0.001 ms	370 / **240**	**370** / 410	**370** / 1270	**370** / 1700
	0.01 ms	**380** / 660	**380** / 750	**380** / 1350	**380** / 1850
	0.1 ms	**400** / 5400	**400** / 5400	**420** / 5500	**420** / 5700
	1 ms	**1100** / 52500	**1100** / 52500	**1100** / 52500	**1100** / 52500

Search times [ms] of PPP-Codes / M-Index smaller search times are in boldface.

Settings: 10M ADJUSTABLE dataset, 10-NN recall = 85 %, $k = 128$;
PPP-Codes: $R = 1000$; M-Index: $R = 400000$

Conclusions

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- use multiple pivot spaces to encode data objects
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The results show that

- even two pivot spaces help, more than five do not help much
- the candidate set is reduced by one–two orders of magnitude
- the rank & merge algorithm is complex but usually worth