$$\underline{F_n = F_{n-1} + F_{n-2}} \quad n \geq 2 \qquad \sum_{n=0}^{\infty} F_n x^n =: F(x)$$

$$F(x) = xF(x) + x^2 F(x) + x$$

$$F_1 \overset{?}{=} F_0 + F_{-1}$$

$$\underline{F_1 = F_0 + F_{-1} + 1}$$

$$F_0 = F_2 + F_{-1} = 0$$

$$F(x) = \frac{x}{1-x-x^2}$$

$$= \frac{A}{x-x_1} + \frac{B}{x-x_2} = \underline{\frac{a}{1-\lambda_1 x} + \frac{b}{1-\lambda_2 x}}$$

$$x_1 = -\frac{1+\sqrt{5}}{2} \qquad x_2 = -\frac{1-\sqrt{5}}{2}$$

$$\lambda_1 = \frac{1}{x_1}, \quad \lambda_2 = \frac{1}{x_2}$$

$$F(x) = a\frac{1}{1-\lambda_1 x} + b \cdot \frac{1}{1-\lambda_2 x} =$$

$$= a\left(\sum_{n=0}^{\infty} \lambda_1^n x^n\right) + b\left(\sum_{n=1}^{\infty} \lambda_2^n x^n\right)$$

$$F_n = a\lambda_1^n + b\lambda_2^n$$

$$\lambda_1 = \frac{1}{x_1} = -\frac{2}{1+\sqrt{5}} = -\frac{2(1-\sqrt{5})}{-4} = \frac{1-\sqrt{5}}{2}$$

$$\lambda_2 = \frac{1}{x_2} = \frac{1+\sqrt{5}}{2}$$

$n=0$

$$0 = a + b \Rightarrow a = -b$$

$$1 = a\left(\frac{1-\sqrt{5}}{2}\right) - a\left(\frac{1+\sqrt{5}}{2}\right) = -\sqrt{5}a \Rightarrow a = -\frac{1}{\sqrt{5}}$$

$$b = \frac{1}{\sqrt{5}}$$

$$F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

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$$a(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots$$

$$a'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \dots$$

$$a''(x) = 2a_2 + 3\cdot 2 a_3 x + 4\cdot 3 a_4 x^2 + \dots$$

$$\vdots$$

$$a^{(n)}(x) = n!\,a_n + (n+1)!\,a_{n+1} x + \frac{(n+2)!}{2}\,a_{n+2} x^2 + \dots$$

$$\Downarrow$$

$$a^{(n)}(0) = n!\,a_n$$

$$p(x) = (x^2 + 3x + 2)(x^3 + 11x^2 + x + 1) =$$

$$[x^3]\,p(x) = 2 + 3\cdot 11 + 1\cdot 1$$

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$$\frac{1}{1-x} \cdot a(x) \qquad a(x) = a_0 + a_1 x + a_2 x^2 + \dots$$

$$\frac{1}{1-x} \sim (1, 1, 1, 1, \dots)$$

$$f(x) = \ln\left(\frac{1}{1-x}\right) \qquad f(0) = 0$$

$$f'(x) = (1-x) \cdot \frac{1}{(1-x)^2}(-1)(-1) = \frac{1}{1-x} \qquad f'(0) = 1$$

$$f''(x) = \frac{1}{(1-x)^2} \qquad f^{(n)}(0) = (n-1)!$$

$$f^{(3)}(x) = \frac{2}{(1-x)^3}$$

$$a_n = \frac{a^{(n)}(0)}{n!} \qquad \left(0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$$

$$H_n = \sum_{i=1}^{n} \frac{1}{i}$$

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$S_n := \sum_{k=1}^{n} H_n$, S_n je konvoluční posloupností

$(1, 1, 1, \dots)$ a $(H_0, H_1, H_2, \dots)$

$\rightsquigarrow \frac{1}{1-x} \cdot \frac{1}{1-x} \ln\left(\frac{1}{1-x}\right)$

v.f. konvoluce $(1, 1, \dots)$ a $(H_0, H_1, H_2, \dots)$

je $\frac{1}{(1-x)^2} \cdot \ln\left(\frac{1}{1-x}\right)$

je konvoluce $(1, 2, 3, \dots)$ a $(0, 1, \frac{1}{2}, \frac{1}{3}, \dots)$

$$[x^n] \frac{1}{(1-x)^2} \cdot \ln\left(\frac{1}{1-x}\right) = \sum_{k=1}^{n} \frac{1}{k}(n+1-k)$$

$$\sum_{k=1}^{n} \frac{1}{k}(n+1-k) = (n+1)\cdot\sum_{k=1}^{n}\frac{1}{k} + \sum_{k=1}^{n}(-1) =$$

$$= (n+1)H_n - 1 - n = (n+1)(H_{n+1} - 1)$$

$a_0 = 0$, $a_1 = 1$ $\quad n \geq 0$

$a_n = 5a_{n-1} - 6a_{n-2} + [n=1]$ $\quad a(x) = \sum_{i=0}^{\infty} a_i x^i$

$a(x) = 5xa(x) - 6x^2 a(x) + x$

$n=1$: $a_1 = 5a_0 - 6a_{-1} + 1$

$a_0 = 5a_{-1} - 6a_{-2}$ ✓

$$a(x) = \frac{x}{1-5x+6x^2} = \frac{x}{(1-2x)(1-3x)} =$$

$$= \frac{A}{1-2x} + \frac{B}{1-3x}$$

$$x = A(1-3x) + B(1-2x)$$

$$x=\tfrac{1}{3}: \quad \tfrac{1}{3} = \tfrac{1}{3}B \Rightarrow B = 1$$

$$x=\tfrac{1}{2}: \quad \tfrac{1}{2} = -\tfrac{1}{2}A \Rightarrow A = -1$$

$$\frac{x}{(1-2x)(1-3x)} = \frac{1}{1-3x} - \frac{1}{1-2x}$$

$$[x^n]\frac{x}{(1-2x)(1-3x)} = 3^n - 2^n$$

$$C_n = n-1 + 2\sum_{k=1}^{n-1}\frac{1}{n}C_k \qquad C(x) = \sum_{n=0}^{\infty} C_n x^n$$

$$nC_n = n(n-1) + 2\sum_{k=1}^{n-1} C_k$$

$$(n-1)C_{n-1} = (n-1)(n-2) + 2\sum_{k=1}^{n-2} C_k$$

$$\frac{1}{(1-x)^3} = \sum_{k=0}^{\infty} \binom{k+2}{2} x^k = \sum_{k=0}^{\infty} \frac{(k+2)(k+1)}{2} x^k$$

$$C'(x) = \frac{2x}{(1-x)^3} + 2\frac{1}{1-x} C(x)$$

Int. faktor $e^{-\int \frac{2}{1-x}} = e^{+2\ln(1-x)} = e^{\ln(1-x)^2} = (1-x)^2$

$$C'(x) - 2\frac{1}{1-x} C(x) = \frac{2x}{(1-x)}$$

$$\left((1-x)^2 C(x)\right)' = \frac{2x}{1-x} = 2\left(-1 + \frac{1}{1-x}\right)$$

$$(1-x)^2 C(x) = 2\left(\ln(1-x) - x\right)$$

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