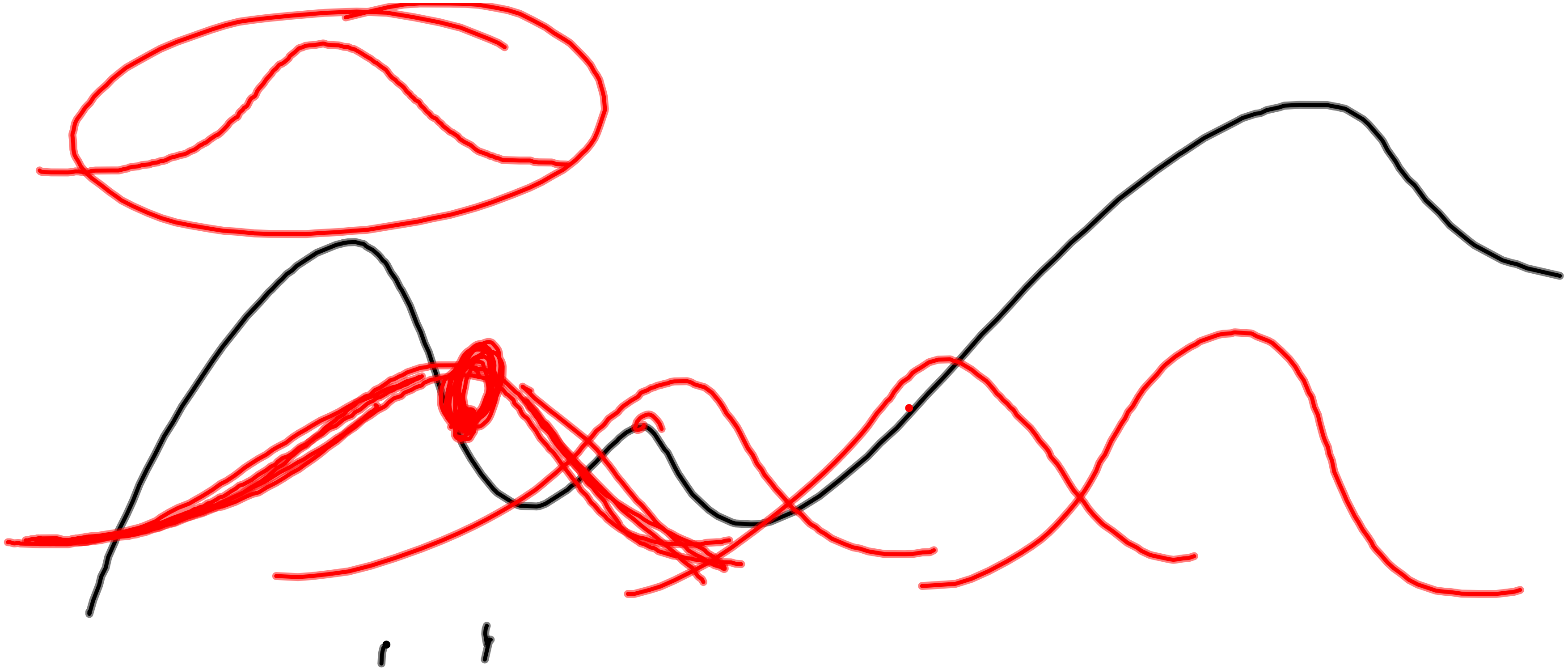
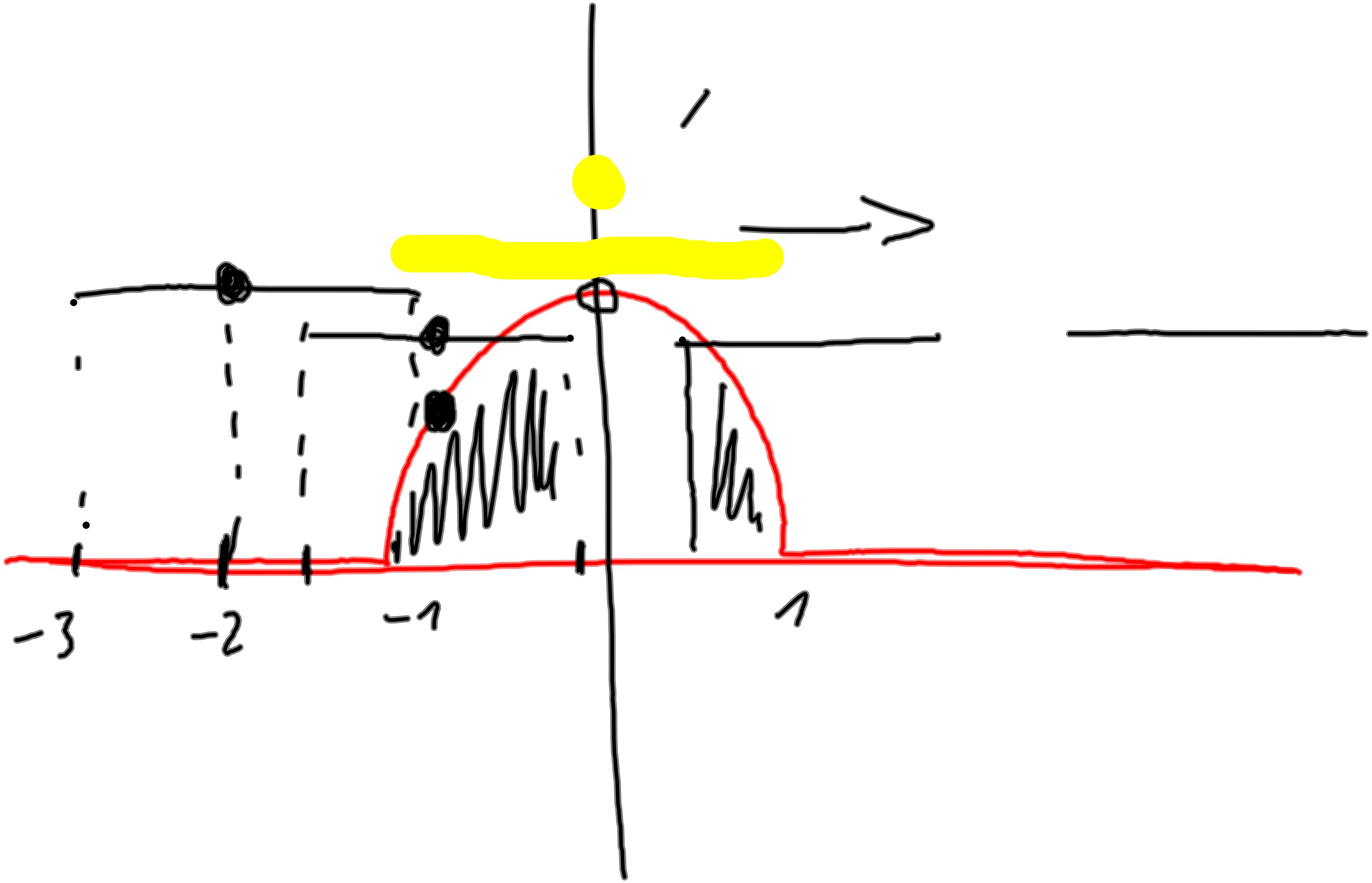



-3
-2
-1
1

$$f(x) = \begin{cases} x^2 \cdot \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

a) SP. ~~AB~~ $x_0 \Leftrightarrow \lim\limits_{x \to x_0} f(x) = f(x_0)$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(x^2 \cdot \sin \frac{1}{x}\right) = |0 \cdot \text{OHR}| = \underline{\underline{0}}$$

$$\| \Leftarrow f(0)$$

JE SP.

b) $f'(x_0) = \lim\limits_{x \to x_0} \dfrac{f(x) - f(x_0)}{x - x_0}$

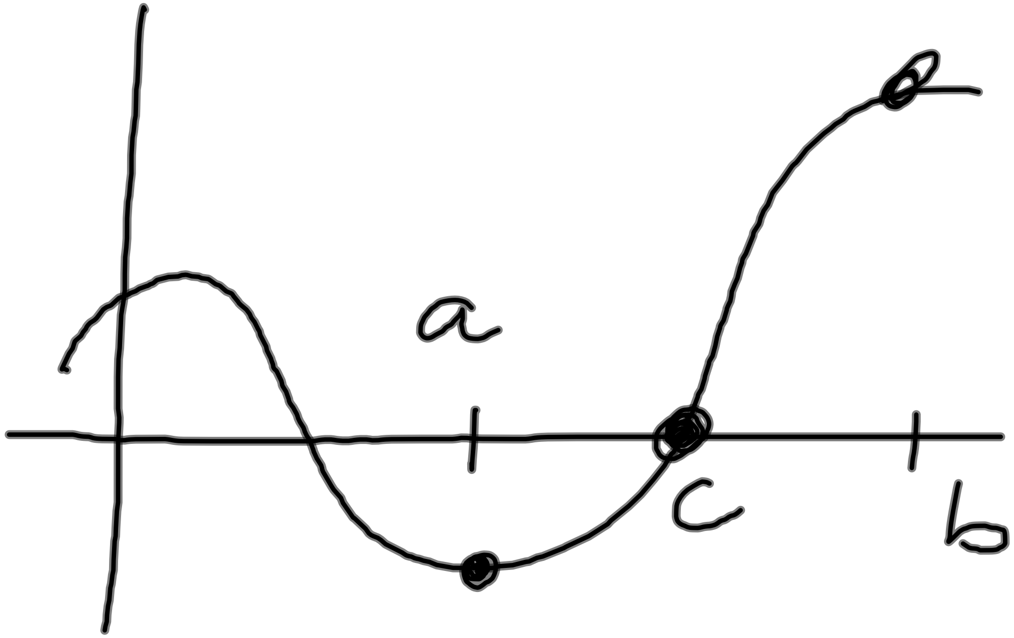
$$f'(0) = \lim_{x \to 0} \frac{x^2 \cdot \sin \frac{1}{x} - f(0)}{x - 0} = \lim_{x \to 0} \left(x \cdot \sin \frac{1}{x}\right) =$$

$$= |0 \cdot \text{OHR}| = \underline{\underline{0}}$$

$\Rightarrow f$ JE SP. NA $[a,b]$ & $f(a)\cdot f(b)<0$

$\Rightarrow \exists c \in (a,b): f(c)=0$

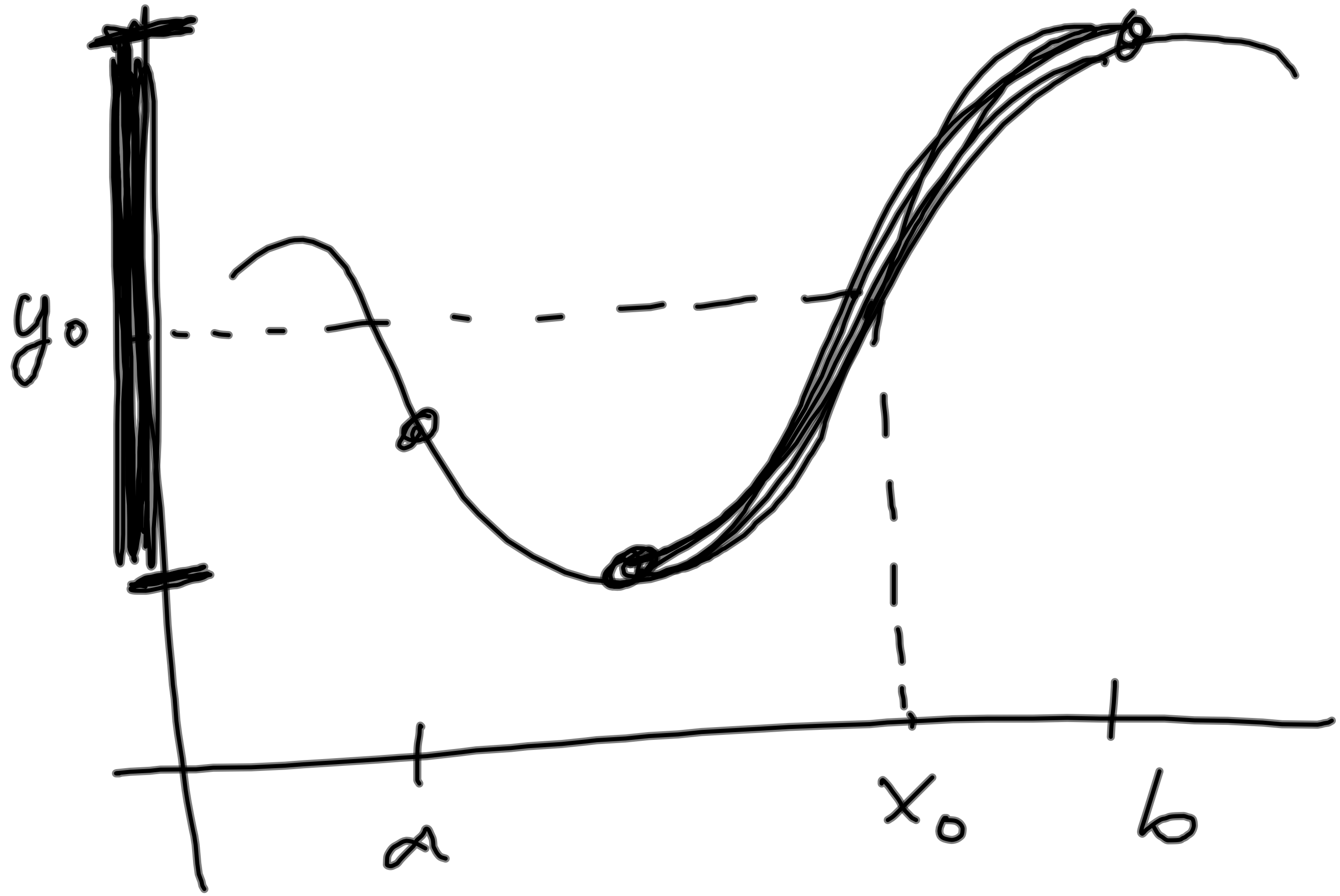
$\Rightarrow f$ SP. NA $[a,b] \Rightarrow$ NAB. V HODNOI NEZI SVOJ NEJV. A NEJM. H.



$f(a)<0$

$f(b)>0$

$\Rightarrow$ .....



y_0
a
x_0
b

$[-2, 3]$

$3-(-2)=5$

$$\int_{-2}^{3} |g(x)|\,dx = 17$$

$\frac{17}{5}$

$$g(x) = \begin{cases} \frac{17}{5}, & x \in \mathbb{Q} \\ -\frac{17}{5}, & x \in \mathbb{I} \end{cases}$$

$$|g(x)| = \frac{17}{5}, \quad \int_{-2}^{3} \frac{17}{5}\,dx = \left[\frac{17}{5}x\right]_{-2}^{3} =$$

$$= \underline{\underline{17}} \quad //$$

$[-2, 3]$

$$\int_{-2}^{3} |h(x)|\, dx = -3$$

NELZE ... INT. Z NEZÁP. F.
NA $[-2,3]$ JE VŽDY
≥ 0

REL. KONV. : $\sum a_n$ KONV.

$\sum |a_n|$ DIV. $(+\infty)$

(i) ABY SOUČET DIL < −1

(ii) + JEDEN KL.

(iii) SOUČET < −2

(iiii) + 1 KL.

⋮

LZE PROTOŽE $\sum$ ZÁP $= -\infty$

$\sum$ KL $= \infty$

-1	5			
0	10	5		
2	2	-4	-3	
3	1	-1	1	1

$$L(x) = 5 + 5\cdot(x+1) - 3\cdot(x+1)\cdot x + 1\cdot(x+1)\cdot x\cdot(x-2) = x^3 - 4x^2 + 10$$

$$\frac{2-10}{2-0} \qquad \frac{-4-5}{2+1} \qquad \frac{1+3}{3+1}$$

$$\frac{1-2}{3-2} \qquad \frac{-1+4}{3-0}$$

$$L\left(-\tfrac{1}{2}\right) = 5 + 5\cdot\frac{1}{2} - 3\cdot\frac{1}{2}\cdot\left(-\frac{1}{2}\right) + \frac{1}{2}\cdot\left(-\frac{1}{2}\right)\cdot\left(-\frac{5}{2}\right) = \underline{\underline{71/8}}$$

$$\lim_{n\to\infty}\left(3n-\sqrt{9n^2-3}\right)\cdot\frac{3n+\sqrt{9n^2-3}}{\sim\ ''\ \sim}=$$

$$=\lim_{n\to\infty}\frac{\cancel{9n^2}-\cancel{9n^2}+3}{3n+\sqrt{9n^2-3}}=\left|\frac{3}{\infty+\infty}\right|=\underline{\underline{0}}$$

$$\lim_{x\to 1^-}\left(\cos\frac{\pi x}{2}\right)^{\ln x}=\left|\left(\cos\frac{\pi}{2}\right)^{\ln 1}=0^0\right|=$$

$$=\lim_{x\to 1^-}e^{\ln\left(\cos\frac{\pi x}{2}\right)^{\ln x}}=\lim_{x\to 1^-}e^{\ln x\cdot\ln\cos\frac{\pi x}{2}}\overset{(*)}{=}$$

$$\lim_{x\to 1} \ln x \cdot \ln\cos\frac{\pi x}{2} = \left|0\cdot\underline{(-\infty)}\right| =$$

$$= \lim_{x\to 1^-} \frac{\ln\cos\frac{\pi x}{2}}{(\ln x)^{-1}} = \left|\frac{-\infty}{-\infty}\right| \overset{L'H}{=}$$

$$= \lim_{x\to 1^-} \frac{\frac{1}{\cos\frac{\pi x}{2}}\cdot\left(+\sin\frac{\pi x}{2}\right)\cdot\frac{\pi}{2}}{+1\cdot(\ln x)^{-2}\cdot\frac{1}{x}} = \frac{\pi}{2}\cdot\lim_{x\to 1^-}\frac{\quad}{\cos\frac{\pi x}{2}}$$

$$= \left|\frac{\pi}{2}\cdot\frac{0\cdot 1\cdot 1}{0} = \frac{0}{0}\right| \overset{L'H}{=} \frac{\pi}{2}\cdot\lim_{x\to 1^-}\frac{2\cdot\ln x\cdot\frac{1}{x}\cdot\sin\frac{\pi x}{2}\cdot x + \ln^2 x\cdot\cos\frac{\pi x}{2}\cdot\frac{\pi}{2}\cdot x + \ln^2 x\cdot\sin\frac{\pi x}{2}\cdot 1}{-\sin\frac{\pi x}{2}\cdot\frac{\pi}{2}} = \left|\frac{\pi}{2}\cdot\frac{0+0+0}{-\frac{\pi}{2}}\right| =$$

$$= 0 \Rightarrow (*) = e^0 = \underline{1}$$

$$\lim_{x\to 0} \frac{e^{-1/x^2}}{x^{100}} = \left|\frac{0}{0}\right| \to \lim_{x\to 0} \frac{e^{-1/x^2}\cdot\left(-\frac{-2}{x^3}\right)}{100\cdot x^{99}} =$$

$$= 2\lim_{x\to 0} \frac{e^{-1/x^2}}{100\cdot x^{102}} =$$

$$\lim_{x\to 0} \frac{x^{-100}}{e^{1/x^2}} = \left|\frac{\infty}{\infty}\right| \overset{l'H}{=} \lim_{x\to 0} \frac{-100\cdot x^{-101}}{e^{1/x^2}\cdot\frac{-2}{x^3}} =$$

$$= 50\cdot\lim_{x\to 0} \frac{x^{-98}}{e^{1/x^2}} = \left|49\times l'H\right| =$$

$$= 50!\cdot\lim_{x\to 0} \frac{1}{e^{1/x^2}} = \left|c\cdot\frac{1}{\infty}\right| = \underline{\underline{0}}$$

$$s(t) = v_0 t - \frac{1}{2} a t^2 = 90t - \frac{1}{2} a t^2$$

$$v(t) = s'(t) = 90 - \frac{1}{2} a \cdot 2t = \underbrace{90 - at = 0}$$

$$\boxed{a = \frac{90}{t}}$$

$$s(t) = 90t - \frac{1}{2} \cdot \frac{90}{t} \cdot t^2 = 90t - 45t =$$

$$= \boxed{45t = 1} \Rightarrow t = \frac{1}{45}\,[h] = 80\,[s]$$

$$v\left(\frac{1}{120}\right) = 90 - at = 90 - \frac{a}{120} = 90 - \frac{90 \cdot 45}{120}$$

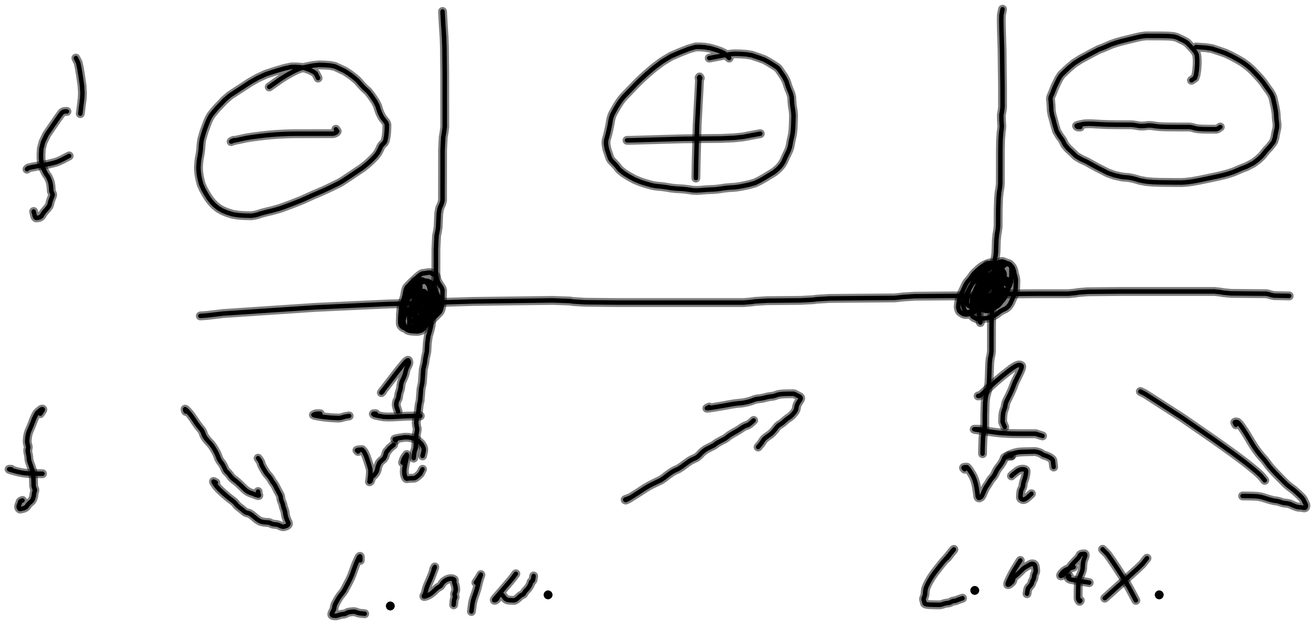
$$a = \frac{90}{\frac{1}{45}} = 90 \cdot 45 \quad \| = \frac{225}{4}\,[km/h]$$

$$f(x) = x \cdot e^{-x^2}$$

$$f'(x) = 1 \cdot e^{-x^2} + x \cdot e^{-x^2} \cdot (-2x) = e^{-x^2} \cdot (1 - 2x^2)$$

$$f'(x) = 0 \Leftrightarrow 2x^2 = 1 \Leftrightarrow x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

$$D(f) = \mathbb{R}, \quad D(f') = \mathbb{R}$$



$$\int_1^{e^8} \frac{7}{x\sqrt{1+\ln x}}\,dx = \left| \begin{array}{l|l} t = 1+\ln x & x = e^8 \Rightarrow t = 9 \\ dt = \frac{1}{x}\,dx & x = 1 \Rightarrow t = 1 \end{array} \right|$$

$$= \int_1^9 \frac{7}{\sqrt{t}}\,dt = 7 \cdot \int_1^9 t^{-1/2}\,dt =$$

$$= 7 \cdot \left[\frac{\sqrt{t}}{\frac{1}{2}} \right]_1^9 = 14 \cdot (3-1) = \underline{\underline{28}}$$

$$\int_0^{\frac{\pi}{2}} x \cdot \sin x \, dx = \left| \begin{matrix} u = x & u' = 1 \\ v' = \sin x & v = -\cos x \end{matrix} \right| =$$

$$= \left[-x \cdot \cos x \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x \, dx = \left[-x \cdot \cos x + \sin x \right]_0^{\frac{\pi}{2}} =$$

$$= \left(-\frac{\pi}{2} \cdot 0 + 1 \right) - \left(-0 \cdot 1 + 0 \right) = \underline{\underline{1}}$$

$$\int \underline{(3x^3 - x + 1)} \cdot \sin(-3x) \, dx = \ldots$$

$$\int_4^\infty \frac{-3}{\sqrt{x}\cdot e^{\sqrt{x}}}\,dx = \left| \begin{array}{l} t = \sqrt{x} \\ dt = \frac{1}{2\sqrt{x}}\,dx \\ \quad\quad\;\; \underset{2\,dt}{\shortparallel} \end{array} \right| \begin{array}{l} x = \infty \Rightarrow t = \infty \\ x = 4 \Rightarrow t = 2 \end{array} \Bigg|$$

$$= -3 \cdot 2 \int_2^\infty \frac{1}{e^t}\,dt = -6 \cdot \int_2^\infty e^{-t}\,dt =$$

$$= -6\left[-e^{-t}\right]_2^\infty = +6\cdot\left(\lim_{t\to\infty} e^{-t} - e^{-2}\right) =$$

$$= 6\cdot\left(0 - e^{-2}\right) = \underline{\underline{-6/e^2}}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (x+2)^n}{n+\sqrt{n}}, \qquad \boxed{X_0 = -2}$$

$$a_n = \frac{(-1)^n}{n+\sqrt{n}}$$

$$\frac{1}{R} = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\dfrac{1}{n+1+\sqrt{n+1}}}{\dfrac{1}{n+\sqrt{n}}} =$$

$$= \lim_{n\to\infty} \frac{n+\cancel{\sqrt{n}}}{n+\cancel{1}+\cancel{\sqrt{n+1}}} = 1 \Rightarrow R = 1$$

ABS. K. PRO $x \in (-3,-1)$

$$x=-1 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n \cdot (-1+2)^n}{n+\sqrt{n}} =$$

$$= \sum_{1}^{\infty} \frac{(-1)^n}{n+\sqrt{n}}$$ .. ALT. Ř.

$$\frac{1}{n+\sqrt{n}} \searrow 0 \Rightarrow$$ LEIBN. K.

KONV.

$$x=-3 \Rightarrow \sum_{1}^{\infty} \frac{(-1)^n \cdot (-3+2)^n}{n+\sqrt{n}} = \sum_{1}^{\infty} \frac{1}{n+\sqrt{n}} \geq$$

$$\geq \sum_{1}^{\infty} \frac{1}{n+n} = \frac{1}{2} \cdot \sum_{1}^{\infty} \frac{1}{n} = \infty$$

$\boxed{|x| < 1}$

$$\sum_{0}^{\infty} x^n = \left| \begin{matrix} a_0 = 1 \\ q = x \end{matrix} \right| = \frac{1}{1-x} \quad \Big/ \frac{d}{dx}$$

$$\left(\sum_{0}^{\infty} x^n\right)' = \sum_{0}^{\infty} n \cdot x^{n-1} = \sum_{n=1}^{\infty} n \cdot x^{n-1} = \frac{+1}{(1-x)^2} \quad \Big/ \cdot x^3$$

$$\sum_{n=1}^{\infty} n \cdot x^{n+2} = \frac{x^3}{(1-x)^2} \quad \Big/ \frac{d}{dx}$$

$$\sum_{1}^{\infty} n \cdot (n+2) \cdot x^{n} = \left(\frac{x^3}{(1-x)^2}\right)' \quad \Big/ \cdot x^{-1}$$

$$\sum^{\infty} \ldots = \frac{1}{x}\cdot\left(\frac{x^3}{(1-x)^2}\right)'$$

$$= \ldots$$

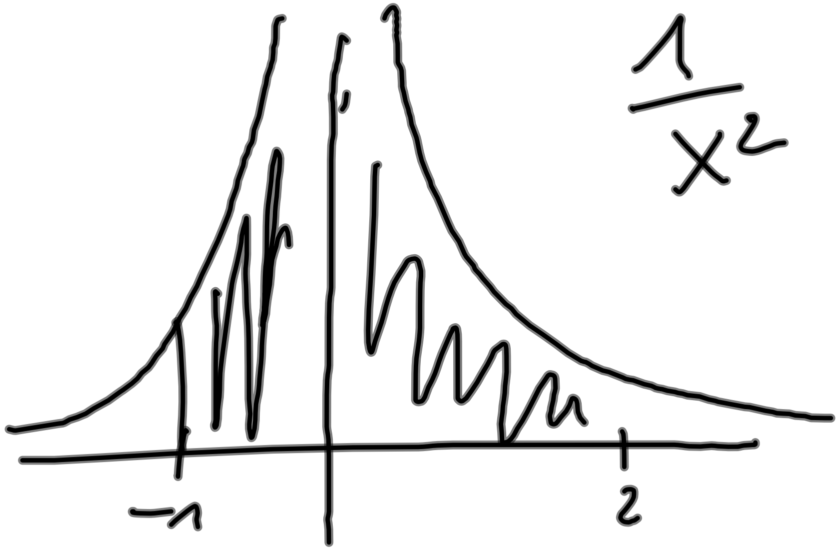
$$\sum_{n=1}^{\infty} n\cdot(n+2)\cdot\left(-\frac{1}{3}\right)^n, \quad x=-\frac{1}{3}\in(-1,1)$$

$$\|$$

$$\frac{3\cdot\left(-\frac{1}{3}\right)-\frac{1}{9}}{\left(1+\frac{1}{3}\right)^3} = \frac{-1-\frac{1}{9}}{\left(\frac{4}{3}\right)^3} = \ldots = -\frac{15}{32}$$

$$\int_{-1}^{2} \frac{1}{x^2}\, dx = \left[\frac{x^{-1}}{-1}\right]_{-1}^{2} = \left[\frac{-1}{x}\right]_{-1}^{2} =$$

$$= -\frac{1}{2} - 1 = -\frac{3}{2}$$



$$\int_{-1}^{2} \frac{1}{x^2}\,dx = \left| \text{EJHLE NULA} \right| =$$

$$= \int_{-1}^{0} \frac{1}{x^2}\,dx + \int_{0}^{2} \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_{-1}^{0} + \left[-\frac{1}{x}\right]_{0}^{2} =$$

$$= \lim_{x\to 0^-}\left(-\frac{1}{x}\right) - 1 + \left(-\frac{1}{2}\right) - \lim_{x\to 0^+}\left(-\frac{1}{x}\right) =$$

$$= +\infty - \frac{3}{2} + \infty = \underline{\underline{+\infty}}$$