Hornerova shéma

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Hornerova schéma

Veta:

Majme polynóm stupňa n nad poľom $\mathbb{K}$ $P(x) = a_n x^n + a_{n-1}x^{n-1} + \dots a_1 x + a_0$. Podelíme ho polynómom $x - c$, $c \in \mathbb{K}$ a dostaneme polynóm $Q(x)$ stuňa najviac $n - 1$ a zvyšok $\frac{r}{x-c}$, $r \in \mathbb{K}$. Koeficienty polynómu $Q(x) = b_{n-1}x^{n-1} + b_{n-2}x^{n-2} + \dots + b_0$ a r máme dostať cez Hornerovu schému.

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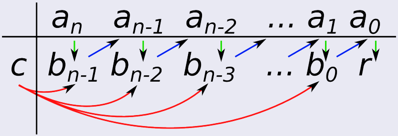

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Ak Hornerovou schémou delíme polynóm $P(X)$ polynómom $(x - c)$, tak posledný koeficient (r) je rovný $P(c)$.

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Dôsledok:

Ak dostaneme dostaneme pre nejaké c, že $r = 0$, tak c je koreň polynómu P.