

$$b_n = b_0 b_{n-1} + \dots + b_{n-1} b_0$$

$$b_i b_j \quad i+j = n-1$$

$$\Rightarrow b_n = \sum_{i+j=n-1} b_i b_j$$

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$$\left(\sum g_n \frac{x^n}{n!} \right)^2$$

$$= \sum g_n \frac{n x^{n-1}}{n!}$$

$$= \sum g_n \frac{x^{n-1}}{(n-1)!}$$

$$\left(\sum a_i \frac{x^i}{i!} \right) \left(\sum b_j \frac{x^j}{j!} \right) \quad n=i+j \rightarrow \frac{x^n}{n!}$$

$$= \sum a_i b_j \frac{x^{i+j}}{i! j!} = \sum a_i b_j \frac{(i+j)!}{i! j!} \frac{x^{i+j}}{(i+j)!}$$

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$$m_n = \sum_{k=1}^n \frac{1}{k!} \frac{n!}{(n-k)!} (k, k_1, \dots, (k, k_n))$$

$$u_n = \sum_{k=1}^n \frac{1}{k!} \frac{n!}{(n-k)!} u_{n-k}$$

$$\frac{u_n}{n!} = \sum_{k=1}^n \frac{1}{k!} \frac{u_{n-k}}{(n-k)!}$$

$$\hat{u}(x) = \sum_{n=0}^{\infty} \frac{u_n}{n!} x^n$$

$$\hat{u}(x) = e^{2x} \hat{u}(x)$$

$$\hat{u}(x) = x e^{2x}$$

$$u_n = \sum_{k=0}^n \binom{n}{k} 2^k k!$$

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$$0 = c_1$$

$$3 = c_2$$

$$0 = c_3$$

$$c_n$$

$$r_{n-1}$$

$$r_{n-1}$$

$$c_{n-2}$$

$$c_{n-1}$$

$$r_{n-2}$$

$$r_n$$

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$$C_n = 2r_{n-1} + C_{n-2} + [n=0]$$

$$r_n = C_{n-1} + r_{n-2}$$

$$C_n x^n = 2r_{n-1} x^n + C_{n-2} x^n + C_{n-2} x^{n-1} x^2$$

$$C(x) = 2R(x) + C(x) x^2 + 1$$

$$R(x) = C(x) x + R(x) x^2$$

$$R(x)(1-x^2) = C(x) x$$

$$R(x) = \frac{C(x) x}{1-x^2}$$

$$C(x) = \frac{2x}{1-x^2} C(x) + C(x) x^2 + 1$$

$$(1-x^2) C(x) = 2x^2 C(x) + C(x) x^2 + 1$$

$$(1-4x^2+x^4) C(x) = 1-x^2$$

$$C(x) = \frac{1-x^2}{1-4x^2+x^4}$$

$$D(x) = \frac{1}{1-4x^2+x^4} \dots C(x) = (1-x^2) D(x)$$

$$C(x) = (1-x^2) \sum d_n x^{2n}$$

$$= \sum d_n x^{2n} - \sum d_n x^{2n+2}$$

$$= \sum d_n x^{2n} - \sum d_{n-1} x^{2n}$$

$$= \sum (d_n - d_{n-1}) x^{2n}$$

$$C_{2n} = d_n - d_{n-1} \quad C_{1n} = 0$$

$$D(x) = \frac{1}{1-4x^2+x^4}$$

$$(1-4x^2+x^4) = (1-\alpha x^2)(1-\beta x^2)$$

$$\alpha = 2+\sqrt{3}, \beta = 2-\sqrt{3}$$

$$= \frac{A}{1-\alpha x^2} + \frac{B}{1-\beta x^2} \quad A, B = ?$$

$$= \frac{A(1-\beta x^2) + B(1-\alpha x^2)}{(1-\alpha x^2)(1-\beta x^2)} = \frac{1}{(1-\alpha x^2)(1-\beta x^2)}$$

Teď dle čísel

$$A(1-\beta) + B(1-\alpha) = 1$$

$$A\beta + B\alpha = 0$$

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$$A+B=1 \quad \alpha = 2+\sqrt{3}$$

$$-A\beta - B\alpha = 0 \quad \beta = 2-\sqrt{3}$$

$$B = 1-A$$

$$-A\beta - (1-A)\alpha = 0$$

$$-A\beta - \alpha + A\alpha = 0$$

$$A(\alpha - \beta) = \alpha$$

$$A = \frac{\alpha}{\alpha - \beta} = \frac{2+\sqrt{3}}{2\sqrt{3}}$$

$$B = 1-A = \frac{-2+\sqrt{3}}{2\sqrt{3}}$$

$$D(x) = \frac{1}{1-4x^2+x^4} = \frac{2+\sqrt{3}}{2\sqrt{3}} \frac{1}{1-(2+\sqrt{3})x^2} + \frac{-2+\sqrt{3}}{2\sqrt{3}} \frac{1}{1-(2-\sqrt{3})x^2}$$

$$= \frac{2+\sqrt{3}}{2\sqrt{3}} \sum (2+\sqrt{3})^n x^{2n} - \frac{2-\sqrt{3}}{2\sqrt{3}} \sum (2-\sqrt{3})^n x^{2n}$$

$$d_n = [x^{2n}] D(x) = \frac{2+\sqrt{3}}{2\sqrt{3}} (2+\sqrt{3})^n - \frac{2-\sqrt{3}}{2\sqrt{3}} (2-\sqrt{3})^n$$

$$C_{2n} = d_n - d_{n-1} = \dots$$

$$= \frac{(2+\sqrt{3})^n}{3-\sqrt{3}} + \frac{(2-\sqrt{3})^n}{3+\sqrt{3}}$$

$$= \left[\frac{2+\sqrt{3}}{3-\sqrt{3}} \right]^n$$

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