$$x^i\cdot x^j\cdot x^k = x^{22} \iff i+j+k=22$$

$$x^1\cdot x^6\cdot x^{15} + x^3\cdot x^4\cdot x^{15}$$
$$+\, x^2\cdot x^{10}\cdot x^{10} + x^4\cdot x^8\cdot x^{10}$$

$$(x^1+x^2+\dots+x^5)(x^2+x^3+\dots+x^{10})(x^3+\dots+x^{15})$$

$$(1+x)^n = \sum \binom{n}{k}x^k \qquad /\ ()'$$
$$n(1+x)^{n-1} = \sum\binom{n}{k}k\cdot x^{k-1} \qquad /\ x=1$$
$$n2^{n-1} = \sum\binom{n}{k}\cdot k$$

5 2-13:54

$$\int_0^x \frac{1}{1-t}\,dt = -\ln(1-x) = \ln\frac{1}{1-x} \leftarrow \begin{array}{l}-\ln(\,)\\ =\ln(\,)^{-1}\end{array}$$
$$\|$$
$$\int_0^x \sum t^n\,dt = \sum\left[\frac{t^{n+1}}{n+1}\right]_0^x = \sum_{n=0}^{\infty}\frac{x^{n+1}}{n+1} = \sum_{n=1}^{\infty}\frac{x^n}{n}$$

$$(1+(-x))^{-n} = \sum\binom{-n}{k}(-x)^k$$
$$=\sum\frac{(-n)(-n-1)\cdots(-n-k+1)}{k!}(-x)^k$$
$$=\sum\frac{(-1)^k\cdot n(n+1)\cdots(n+k-1)}{k!}(-x)^k$$
$$=\sum\binom{n+k-1}{k}x^k$$
$$=\sum\binom{n+k-1}{n-1}x^k$$

Př.

$n=1$: $\frac{1}{1-x} = \sum\binom{k}{0}x^k = \sum x^k$

$n=2$: $\frac{1}{(1-x)^2} = \sum\binom{k+1}{1}x^k = \sum(k+1)x^k$

$$\| $$
$$\left(\frac{1}{1-x}\right)' = \left(\sum x^k\right)' = \sum k\cdot x^{k-1} = \sum(k+1)x^k$$

$$\left((1-x)^{-1}\right)' = (-1)\cdot(1-x)^{-2}\cdot \underset{\uparrow\ (1-x)'}{(-1)} = (1-x)^{-2}$$

5 2-14:24

$$a_0+b_0+(a_1+b_1)x+(a_2+b_2)x^2+\dots$$
$$= a_0+a_1x+a_2x^2+\dots$$
$$+\,b_0+b_1x+b_2x^2+\dots$$
$$= a(x)+b(x)$$

$$x^k(a_0+a_1x+a_2x^2+\dots)$$
$$= a_0x^k+a_1x^{k+1}+a_2x^{k+2}+\dots$$
$$(\underbrace{0,\dots,}_{kx}0,a_0,a_1,a_2,\dots)$$

$$\frac{a(x)-(a_0+a_1x+\dots+a_{k-1}x^{k-1})}{x^k}$$

$$a(\alpha\cdot x) = a_0+a_1\cdot\alpha x+a_2\cdot(\alpha x)^2+\dots$$
$$= \underline{a_0}+\underline{a_1\alpha}\cdot x+\underline{a_2\alpha^2}\cdot x^2+\dots$$

$$\left(\sum a_k\cdot x^k\right)' = \sum a_k\cdot k\cdot x^{k-1}$$
člen u x^k: $a_{k+1}(k+1)$

$$\int_0^x\sum a_kt^k\,dt = \sum a_k\cdot\frac{x^{k+1}}{k+1}$$
člen u x^k: $a_{k-1}\cdot\frac1k$

$a(x)\,b(x)$ … člen s x^k je:
$$\sum_{i+j=k}(a_ix^i)\cdot(b_jx^j) = \Big(\underbrace{\sum_{i+j=k}a_ib_j}_{c_k}\Big)x^k$$

5 2-14:30

$$\frac{1}{1-x}\qquad a(x) \leftarrow a_0+a_1x+a_2x^2+\dots$$
$$\uparrow$$
$$1+x+x^2+\dots \multimap b_j=1$$

člen u x^k: $\sum_{i+j=k}a_ib_j = \sum_{i=0}^{k}a_i$ a $j=k-i$

$$\ln\frac{1}{1-x} = \sum_{k\ge1}\frac1kx^k = 0+\frac11x+\frac12x^2+\dots$$

$$\frac{1}{1-x}\ln\frac{1}{1-x}\quad \text{v.f.p.}\ \left(0,\frac11,\frac11+\frac12,\frac11+\frac12+\frac13,\dots\right)$$

5 2-14:30

$$1+x+x^2+\dots+x^n = \frac{1-x^{n+1}}{1-x}$$

$$(1+x+\dots+x^{30})(1+x+\dots+x^{40})(1+x+\dots+x^{50})$$
$$=\frac{1-x^{31}}{1-x}\cdot\frac{1-x^{41}}{1-x}\cdot\frac{1-x^{51}}{1-x}$$
$$=\frac{1}{(1-x)^3}(1-x^{31})(1-x^{41})(1-x^{51})$$
$$=\left(\binom22+\binom32x+\dots\right)\left(1-x^{31}-x^{41}-x^{51}+x^{72}+\dots\right)$$

člen s x^{70} je:
$$\binom{72}{2}x^{70}\cdot1+\binom{41}{2}x^{39}\cdot(-x^{31})+\binom{31}{2}x^{29}(-x^{41})$$
$$+\binom{21}{2}x^{19}(-x^{51})$$

5 2-14:53

$$H_k = \frac11+\frac12+\dots+\frac1k$$
je to koef. u x^k funkce $\frac{1}{1-x}\ln\frac{1}{1-x}$

$\sum_{n=1}^{n}H_k$ …. konvoluce $(1,1,\dots)$ $(0,H_1,H_2,\dots)$

– odpovídá součinu
$$\frac{1}{1-x}\cdot\left(\frac{1}{1-x}\ln\frac{1}{1-x}\right) = \frac{1}{(1-x)^2}\cdot\ln\frac{1}{1-x}$$

– odpovídá konvoluci

$a_i=i+1$ $(1,2,3,\dots)$ $\multimap$ $\frac{1}{(1-x)^2}$

$b_j=\frac1j$ $(0,\frac11,\frac12,\frac13,\dots)$ $\multimap$ $\ln\frac{1}{1-x}$

$\Rightarrow$ konvoluce má k-tý člen
$$\sum_{i+j=k}a_ib_j = \sum_{\substack{i+j=k\\ j\ge1}}(i+1)\frac1j = \sum_{\boxed{j=1}}^{k}(k-j+1)\frac1j \qquad i=k-j$$
$$=(k+1)\sum\frac1j-\sum j\cdot\frac1j$$
$$=(k+1)H_k-k = (k+1)(H_{k+1}-1)$$

$$H_{k+1} = H_k+\frac{1}{k+1}$$
$$(k+1)H_{k+1} = (k+1)H_k+1$$

$$(k+1)H_k-k = (k+1)H_{k+1}-1-k$$
$$=(k+1)(H_{k+1}-1)$$

5 2-15:01

$F_n = F_{n-1} + F_{n-2}$

přepsat jako rovnost pro v.f.

$0 \quad 1$

$F(x) = F_0 + F_1 x + F_2 x^2 + \cdots$

$x\,F(x) = \quad F_0 x + F_1 x^2 + \cdots$

$x^2 F(x) = \quad 0 \quad F_0 x^2 + \cdots$

$F(x) = x F(x) + x^2 F(x)$

! AŽ NA konst. a lin. člen

oprava: $+\,x$

$$F(x) = \underbrace{x\,F(x) + x^2 F(x)}_{\text{rekurentní vztah}} + \underbrace{x}_{\text{počáteční podmínky}} = F_0 + (F_1 - F_0)x$$

$$F(x) = \frac{x}{1-x-x^2}$$

5 2-15:15

$$\frac{A}{x-x_1} = \frac{-A/x_1}{-x/x_1 + 1} = \frac{a}{1-\underbrace{(1/x_1)}_{\lambda_1}x}$$

$$= \underbrace{\frac{a}{1-\lambda_1 x}}_{\text{kvocient geom. řady}}$$

$$= a\left(1 + \lambda_1 x + \lambda_1^2 x^2 + \cdots\right)$$

$$\frac{x}{1-x-x^2} = \frac{a}{1-\lambda_1 x} + \frac{b}{1-\lambda_2 x}$$

$x_{1,2} = (\text{kořeny } 1-x-x^2)$

$= \frac{1 \pm \sqrt{5}}{-2}$

$$\lambda_{1,2} = \frac{1}{x_{1,2}} = \frac{-2}{1 \pm \sqrt{5}} = \frac{-2\,(1 \mp \sqrt{5})}{(1 \pm \sqrt{5})(1 \mp \sqrt{5})}$$

$$= \frac{-2\,(1 \mp \sqrt{5})}{1-5} = \frac{1 \mp \sqrt{5}}{2}$$

$\lambda_1 = \frac{1+\sqrt{5}}{2} \quad \lambda_2 = \frac{1-\sqrt{5}}{2}$

$$\frac{x}{1-x-x^2} = \frac{a}{1-\lambda_1 x} + \frac{b}{1-\lambda_2 x} =$$

$$= \frac{a(1-\lambda_2 x) + b(1-\lambda_1 x)}{(1-\lambda_1 x)(1-\lambda_2 x)} =$$

$$= \frac{(a+b) - (a\lambda_2 + b\lambda_1)x}{1-x-x^2}$$

$a+b=0 \qquad a\lambda_2 + b\lambda_1 = -1$

$a = -b \qquad b\underbrace{(\lambda_1 - \lambda_2)}_{\sqrt{5}} = -1$

$\Rightarrow \; b = -\frac{1}{\sqrt{5}} \quad a = \frac{1}{\sqrt{5}}$

$$\frac{x}{1-x-x^2} = \frac{1}{\sqrt{5}}\left(\frac{1}{1-\lambda_1 x} - \frac{1}{1-\lambda_2 x}\right)$$

$$= \frac{1}{\sqrt{5}}\Big(1 + \lambda_1 x + \lambda_1 x^2 + \cdots - (1 + \lambda_2 x + \lambda_2 x^2 + \cdots)\Big)$$

$$= \sum \frac{1}{\sqrt{5}}\left(\lambda_1^k - \lambda_2^k\right)x^k$$

$$= \sum \underbrace{\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^k - \left(\frac{1-\sqrt{5}}{2}\right)^k\right)}_{F_k} x^k$$

5 2-15:25