PA199
Advanced Game Design

Lecture 4

Physics for Game Design

Dr. Fotis Liarokapis

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Basics

Units

- The numbers that specify a point have units
- The spaces we will deal with are mostly regular kinds of space
  - i.e. Units of distance
    - i.e. meters and kilometres
- Almost all physics applications use metric units
  - Called SI (Système International)

Some Basic SI Units

Quantity	Base Unit	Derived Units
Distance	Meter (m)	1 kilometer (km) = 1000 m 1 m = 100 centimetres (cm)
Mass	Kilogram (kg)	1 kg = 1000 grams (g) 1 g = 1000 milligrams (mg)
Time	Seconds (s)	
Temperature	Kelvin (K)	

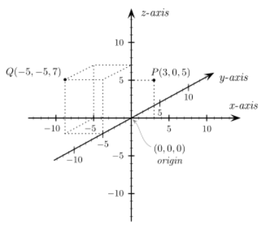
Prefixes

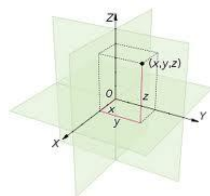
Power of 10	Prefix Name	Prefix Code	English Meaning	Original Language
+18	exa	E	outside	Greek
+15	peta	P	spread	Greek
+12	tera	T	monster	Greek
+9	giga	G	giant	Greek
+6	mega	M	large	Greek
+3	kilo	k	thousand	Greek
+2	hecto	h	hundred	Greek
+1	deka	D	ten	Greek
- 1	deci	d	ten	Latin
- 2	centi	c	hundred	Latin
- 3	milli	m	thousand	Latin
- 6	micro	μ	small	Latin
- 9	nano	n	dwarf	Greek
- 12	pico	p	tiny	Italian
- 15	femto	f	fifteen	Old Norse
- 18	atto	a	eighteen	Old Norse

Derived Units

Quantity	Unit Name	Unit Symbol	Expression in Terms of Other SI Units
Celsius temperature	Degree Celsius	degree C	K
Electric capacitance	farad	F	C/V
Electric charge, quantity of electricity	coulomb	C	A•s
Electric potential, potential difference, lectromotive force	volt	V	W/A
Electric resistance	ohm	Omega	V/A
Energy, work, quantity of heat	joule	J	N•m
Force	newton	N	kg•m/s^2
Frequency (of a periodic phenomenon)	hertz	Hz	1/s
Magnetic flux	weber	Wb	V•s
Magnetic flux density	tesla	T	Wb/m^2
Plane angle	radian	rad	m/m
Power, radiant flux	watt	W	J/s
Pressure, stress	pascal	Pa	N/m^2

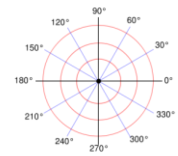
Cartesian Coordinate System

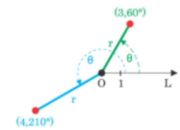




Polar Coordinate System

- In the polar coordinate system each point on a plane is determined by an *angle* and a *distance*
- Useful in situations where the relationship between two points is most easily expressed in terms of angles and distance
  - In the Cartesian coordinate system this can be found through trigonometry
- Each point is determined by two polar coordinates:
  - The radial coordinate
  - The angular coordinate





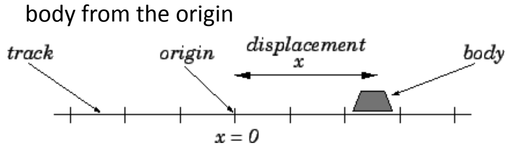
4D Coordinate Systems

- Well this is a bit hard to draw!
  - Even 3D is hard to draw
- Mathematically though it is easy to add more dimensions
  - Just name points with more numbers
- In 4D you need four numbers
  - The last dimension is usually called w
  - So points have coordinates (x, y, z, w)

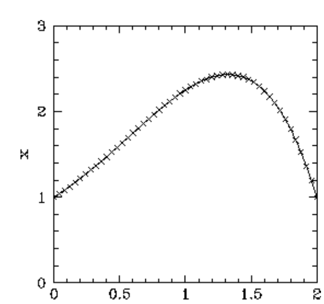
Motion

Displacement

- A positive **x** value implies that the body is located **x** meters to the *right* of the origin
- A negative **x** value implies that the body is located |**x**| meters to the *left* of the origin
- Here, **x** is termed as the displacement of the body from the origin



Displacement Over Time



$$x = 1 + t + \frac{t^2}{2} - \frac{t^4}{4}$$

Velocity 1D

- Determine the body's instantaneous velocity as a function of time:
  - Velocity is the rate of change of displacement with time
- This definition implies that:

$$v = \frac{\Delta x}{\Delta t}$$

- where **υ** is the body's velocity at time **t**, and **Δx** is the change in displacement of the body between times **t** and **t + Δt**

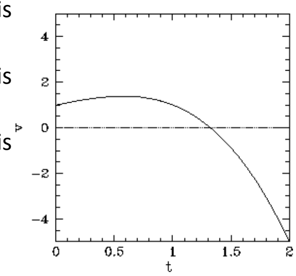
Velocity General Equation

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- where dx/dt represents the derivative of **x** with respect to **t**
- The above definition is particularly useful if we represent **x(t)** as an analytic function
  - It allows us to immediately evaluate the instantaneous velocity **υ(t)** via the rules of calculus

Velocity Graph

- When **υ>0** the body is moving to the right
- When **υ<0** the body is moving to the left
- When **υ=0** the body is instantaneously at rest



Velocity and Speed

- The terms velocity and speed are often confused with one another
- A velocity can be either **positive** or **negative**
  - Depending on the **direction** of motion
- The conventional definition of speed is that it is the magnitude of velocity
  - A body can never possess a negative speed

Vector Velocity

- Consider a body moving in 3D, and we know the Cartesian coordinates, x, y, and z, of this body as time, t, progresses
  - How can we use this information to determine the body's instantaneous velocity as functions of time?
- The vector displacement of the body is given by:

$$\mathbf{r}(t) = [x(t), y(t), z(t)]$$

Vector Velocity .

- The body's vector velocity **v = (u_x, u_y, u_z)** is simply the derivative of **r** with respect to **t**

$$\mathbf{v}(t) = \lim_{\Delta t \to 0} \frac{\mathbf{r}(t + \Delta t) - \mathbf{r}(t)}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

- When written in component form, the above definition yields to:

$$u_x = \frac{dx}{dt} \qquad u_y = \frac{dy}{dt} \qquad u_z = \frac{dz}{dt}$$

Acceleration 1D

- The definition of acceleration is as follows:
  - Acceleration is the rate of change of velocity with time
- This definition implies that:

$$a = \frac{\Delta v}{\Delta t}$$

- where **α** is the body's acceleration at time **t**, and **Δυ** is the change in velocity of the body between times **t** and **t + Δt**

Acceleration General Equation

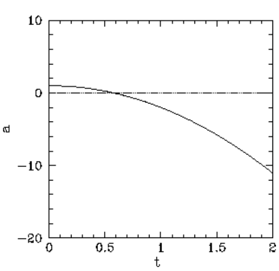
- A general expression for instantaneous acceleration can be obtained by taking the limit of the acceleration equation as **Δt** approaches zero:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

- which is valid irrespective of how rapidly or slowly the body's acceleration changes in time

Acceleration Graph

- When **α** is positive the body is accelerating to the right
- When **α** is negative the body is decelerating



Vector Acceleration

- The body's vector acceleration a = (a_x, a_y, a_z) is simply the derivative of v with respect to t

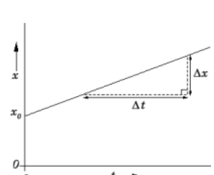
$$\mathbf{a}(t) = \lim_{\Delta t \to 0} \frac{\mathbf{v}(t + \Delta t) - \mathbf{v}(t)}{\Delta t} = \frac{d\mathbf{v}}{dt} = \frac{d^2 \mathbf{r}}{dt^2}$$

- When written in component form, the above definition yields

$$a_x = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2} \qquad a_y = \frac{dv_y}{dt} = \frac{d^2 y}{dt^2} \qquad a_z = \frac{dv_z}{dt} = \frac{d^2 z}{dt^2}$$

Motion with Constant Velocity 1D

- Simplest type of motion
  - Excluding the trivial case in which the body under investigation remains at rest
- Occurs in everyday life whenever an object slides over a horizontal, low friction surface



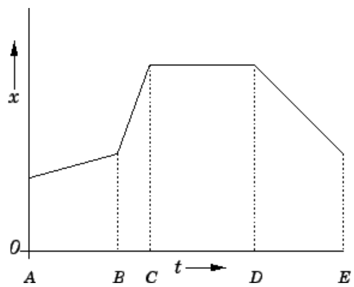
Motion with Constant Velocity 1D .

- The graph consists of a straight-line which can be represented as:

$$x = x_0 + v\,t$$

- Here, $\mathbf{x_0}$ is the displacement at time **t=0**
  - This can be determined from the graph as the intercept of the straight-line with the –axis
- υ = dx/dt is the constant velocity of the body
  - Can be determined from the graph
- Note that: $a = d^2x/dt^2 = 0$

Displacement vs Time Graph



Motion with Constant Velocity 3D

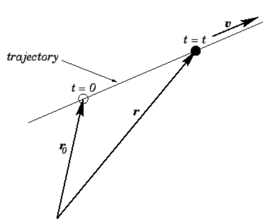
- An object moving in 3D with constant velocity possesses a vector displacement of the form

$$r(t) = r_0 + vt$$

- where the constant vector $\mathbf{r_0}$ is the displacement at time **t = 0**
  - Note that $\mathbf{dr/dt = v}$ and $\mathbf{d^2r/dt^2 = 0}$ as expected

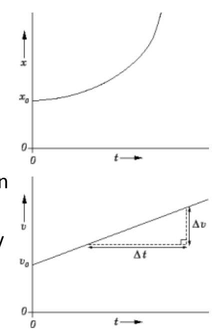
Motion with Constant Velocity 3D .

- The object's trajectory is a *straight-line* which passes through point $\mathbf{r_0}$ at time **t = 0** and runs parallel to vector **v**



Motion with Constant Acceleration 1D

- Motion with constant acceleration occurs in everyday life whenever an object is dropped
  - The object moves downward with the constant acceleration **9.81ms^{-2}**
  - Under the influence of gravity



Motion with Constant Acceleration 1D .

- The displacement-time graph consists of a curved-line whose gradient is increasing in time and represented as:

$$x = x_0 + v_0\,t + \frac{1}{2}\,a\,t^2$$

- Here, $\mathbf{x_0}$ is the displacement at time **t=0**
  - This can be determined from the graph as the intercept of the curved-line with the -axis
- Likewise, $\mathbf{\upsilon_0}$ is the body's instantaneous velocity at time **t=0**

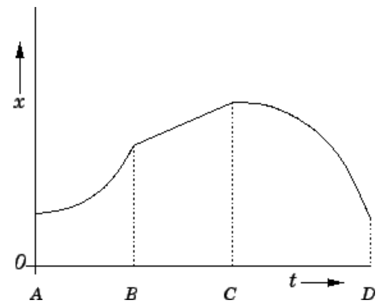
Motion with Constant Acceleration 1D ..

- The velocity-time graph consists of a straight-line which can be represented as:

$$v = \frac{dx}{dt} = v_0 + a\,t$$

- The quantity $\mathbf{\upsilon_0}$ is determined from the graph as the intercept of the straight-line with the –axis
- The quantity **α** is the constant acceleration
  - Can be determined as the gradient of the straight-line
- Note that: $dv/dt = a$

Motion with Constant Acceleration Graph



Motion with Constant Acceleration 3D

- An object moving in 3D with constant acceleration **a** possesses a vector displacement of the form

$$\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}_0\, t + \frac{1}{2}\,\mathbf{a}\, t^2$$

- Hence, the object's velocity is given by:

$$\mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = \mathbf{v}_0 + \mathbf{a}\, t$$

- Note that **dv/dt = a**
  - The constant vectors $\mathbf{r_0}$ and $\mathbf{v_0}$ are the object's displacement and velocity at time **t = 0**, respectively

Motion with Constant Acceleration 3D .

- These equation fully characterize 3D with constant acceleration:

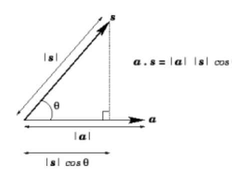
s = $\mathbf{v}_0\, t + \frac{1}{2}\,\mathbf{a}\, t^2$

v = $\mathbf{v}_0 + \mathbf{a}\, t$

v^2 = $v_0^2 + 2\,\mathbf{a}\cdot\mathbf{s}$

Motion with Constant Acceleration 3D ..

- Here, $\mathbf{s = r - r_0}$ is the net displacement of the object between times **t = 0** and **t**
- The quantity **a●s** is termed the scalar product of vectors **a** and **s**, and is defined as:



$$\mathbf{a}\cdot\mathbf{s} = a_x\, s_x + a_y\, s_y + a_z\, s_z$$

Free-fall Under Gravity

- All bodies in free-fall close to the Earth's surface accelerate vertically downwards with the same acceleration
  - **g = 9.81ms^{-2}**
- The neglect of air resistance is a fairly good approximation for large objects which travel slowly
  - i.e. Basketball game
- Becomes a poor approximation for small objects which travel relatively rapidly
  - i.e. Golf-ball game

Free-fall Under Gravity .

- Motion with constant acceleration equations can be modified as shown below:

s = $v_0\, t - \frac{1}{2}\, g\, t^2$

u = $v_0 - g\, t$

- where:
  - **g = 9.81ms^{-2}** is the downward acceleration due to gravity
  - **s** is the distance the object has moved vertically between times
  - $\mathbf{u_0}$ is the object's instantaneous velocity at **t=0**
  - **u** is the object's instantaneous velocity at time **t**

Free-fall Under Gravity ..

- Using the above equations we can calculate time as:

$$t = \sqrt{\frac{2\,h}{g}}$$

- Also can calculate the maximum height if we through an object to the air by:

$$h = \frac{u^2}{2\,g}$$

Example 1

- You drive a car for 2.0 h at 40 km/h, then for another 2.0 h at 60 km/h
  - What is your average speed?
  - Do you get the same answer if you drive 100 km at each of the two speeds?

Example 1 Solution

- Part A
  - Total distance driven = [(2 h)(40 km /h) + (2 h)(60 km/h)] = 200 km
  - Total time = 2 + 2 = 4 h
  - Average speed = (200 km)/(4 h) = 50km/h
- Part B
  - Total distance driven = 100 + 100 = 200 km
  - Total time = [(100 km)/(40 km/h) + (100 km)/ 60 km/h)] = 4.17 h
  - Average speed = (200 km)/(4.17 h) = 48 km/h

Example 2

- In a speed trap, two pressure-activated strips are placed **120 m** apart on a highway on which the speed limit is **85 km/h**
- A driver going **110 km/h** notices a police car just as he/she activates the first strip and immediately slows down
  - What deceleration is needed so that the car's average speed is within the speed limit when the car crosses the second strip?

Example 2 Solution

- Let **υ_1 = 110 Km/h** be the speed of the car at the first strip
- Let **Δx** be the distance between the two strips, and let **Δt** be the time taken by the car to travel from one strip to the other
- The average velocity of the car is:

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

Example 2 Solution .

- We need this velocity to be **85 Km/h**, hence:

$$\Delta t = \frac{\Delta x}{\bar{v}} = \frac{120}{85 \times (1000/3600)} = 5.082\,\mathrm{s}$$

  - Note units changed from **km/h** to **m/s**
- Assuming that acceleration **α** of the car is uniform, we have: $\Delta x = v_1\,\Delta t + \frac{1}{2}\,a\,(\Delta t)^2 \rightarrow$

$$a = \frac{2\,(\Delta x - v_1\,\Delta t)}{(\Delta t)^2} = \frac{2\,(120 - 110 \times (1000/3600) \times 5.082)}{(5.082)^2} = -2.73\,\mathrm{m\,s^{-2}}$$

Projectile Motion

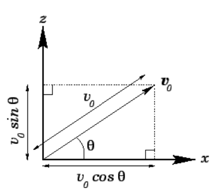
Projectile Motion

- As a simple illustration of the concepts introduced in the previous subsections, let us examine the following problem
  - Suppose that a projectile is launched upward from ground level, with speed $\mathbf{u_0}$, making an angle $\boldsymbol{\theta}$ with the horizontal
  - Neglecting the effect of air resistance, what is the subsequent trajectory of the projectile?

Projectile Motion .

- First task is to set up a suitable Cartesian coordinate system
- The projectile is subject to a constant acceleration g = 9.81 ms^{-1}
  - Due to gravity
    - Neglecting air resistance
- Thus, the projectile's vector acceleration is written as:

a = (0, 0, -g)



Projectile Motion ..

- What is the initial vector velocity $\mathbf{v_0}$ with which the projectile is launched into the air at **t = 0**?
- Given that the magnitude of this velocity is $\mathbf{u_0}$, its horizontal component is directed along the x-axis, and its direction subtends an angle $\boldsymbol{\theta}$ with this axis, the components of $\mathbf{v_0}$ take the form:

$$\mathbf{v}_0 = (v_0 \cos\theta,\ 0,\ v_0 \sin\theta)$$

Projectile Motion ...

- Since the projectile moves with constant acceleration, its vector displacement **s = (x, y, z)** from its launch point is:

$$\mathbf{s} = \mathbf{v}_0\, t + \frac{1}{2}\,\mathbf{a}\, t^2$$

- Making use of the previous equations the x-, y-, and z-components of the above equation are written as:

x = $v_0 \cos\theta\ t$

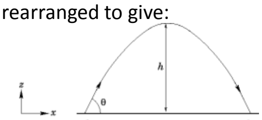
y = 0

z = $v_0 \sin\theta\ t - \frac{1}{2}\, g\, t^2$

Projectile Motion Equations

- Equations of **x** and **z** can be rearranged to give:

$$z = x \tan\theta - \frac{1}{2}\frac{g\, x^2}{v_0^2} \sec^2\theta$$



  - This is the equation of a parabola
    - No need to remember it
- When z=0 (strikes to ground) then:

$$R = \frac{2\, v_0^2}{g} \sin\theta \cos\theta = \frac{v_0^2}{g} \sin 2\theta$$

Projectile Motion Maximum Value

- The range attains its maximum value at:

$$R_{\max} = \frac{v_0^2}{g}$$

  - when **θ = 0**
- Neglecting air resistance, a projectile travels furthest when it is launched into the air at 45° to the horizontal

Projectile Motion Altitude

- The maximum altitude **h** of the projectile is attained when $\mathbf{u_z = dz/dt = 0}$
  - i.e. when the projectile has just stopped rising and is about to start falling
- It follows from z= $v_0 \sin\theta \, t - \frac{1}{2} g \, t^2$ that the maximum altitude occurs at time $\mathbf{t_0 = u_0 \sin\theta / g}$, so:

$$h = z(t_0) = \frac{v_0^2}{2\,g} \sin^2\theta$$

Projectile Motion Maximum Altitude

- Obviously, the largest value of h is:

$$h_{\max} = \frac{v_0^2}{2\,g}$$

- This is obtained when the projectile is launched vertically upwards
  - i.e. **θ = 90°**

Example 3

- Two equal weights are thrown simultaneously from a height of **100 m** above the ground
- The first weight is dropped straight down, and the second weight horizontally with a velocity of **5 m/s**
  - Which weight struck the ground first?
  - How long, after it was thrown, did it take to do this?
  - Finally, what horizontal distance was traveled by the second weight before it hit the ground?
- Neglect the effect of air resistance

Example 3 Solution

- Both weights strike the ground simultaneously
  - Because both weights start off traveling with the same initial velocities in the vertical direction
    - i.e. Zero
  - Accelerate vertically downwards at the same rate
- The time of flight of each weight is simply the time taken to fall h = 100m, starting from rest, under the influence of gravity is given by:

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 100}{9.81}} = 4.515\,\mathrm{s}$$

Example 3 Solution .

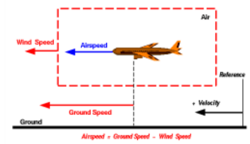
- The horizontal distance **R** traveled by the second weight is simply the distance traveled by a body moving at a constant velocity **u = 5m/s** during the time taken by the weight to drop **100m**
  - Note that gravitational acceleration does not affect horizontal motion
- Thus:

$$R = u\,t = 5 \times 4.515 = 22.58\,\mathrm{m}$$

Relative Velocity

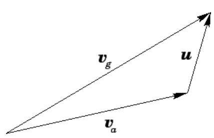
Introduction to Relative Velocity

- A relatively difficult concept
  - In 1D, it's straight-forward
  - In 2D, the equations look identical with 1D
- The main difference is that it's harder to add and subtract the vectors



Relative Velocity

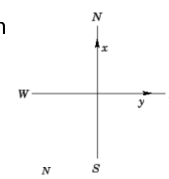
- Suppose that, on a windy day, an airplane moves with constant velocity $\mathbf{v}_a$ with respect to the air, and that the air moves with constant velocity **u** with respect to the ground
- What is the vector velocity $\mathbf{v}_g$ of the plane with respect to the ground?
- Solution: $\mathbf{v}_g = \mathbf{v}_a + \mathbf{u}$

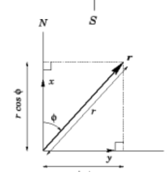


Note that (in general) $\mathbf{v}_g$ is parallel to neither $\mathbf{v}_a$ nor **u**

Relative Velocity Implementation

- Firstly set up a suitable Cartesian coordinate system
  - In this coordinate system, it is conventional to specify a vector **r** in term of its magnitude, r, and its *compass bearing*, ϕ
- A compass bearing is the angle subtended between the direction of a vector and the direction to the North pole
  - i.e. x-direction





A Compass Bearing

- Compass bearings run from 0° to 360°
  - By convention
- The compass bearings of North, East, South and West are 0°, 90°, 180°, and 270°, respectively
- According to the previous figure, the components of a general vector **r**, whose magnitude is **r** and whose compass bearing is **ϕ**, are simply:

r = (x, y) = (rcosϕ, rsinϕ)

Relative Velocity Simple Example

- Suppose that the plane's velocity relative to the air is 300Km/h, at a compass bearing of 120°, and the air's velocity relative to the ground is 85Km/h, at a compass bearing of 225°
- It follows that the components of v_a and u (measured in units of km/h) are:

v_a = $(300\cos 120^\circ,\ 300\sin 120^\circ) = (-1.500\times 10^2,\ 2.598\times 10^2)$

u = $(85\cos 225^\circ,\ 85\sin 225^\circ) = (-6.010\times 10^1,\ -6.010\times 10^1)$

Relative Velocity Simple Example .

- According to ($\mathbf{v_g} = \mathbf{v_a} + \mathbf{u}$), the components of the plane's velocity $\mathbf{v_g}$ relative to the ground are simply the algebraic sums of the corresponding components of $\mathbf{v_a}$ and $\mathbf{u}$

$$v_g = (-1.500\times10^2 - 6.010\times10^1,\ 2.598\times10^2 - 6.010\times10^1)$$
$$= (-2.101\times10^2,\ 1.997\times10^2)$$

- The task is to reconstruct the magnitude and compass bearing of vector $\mathbf{v_g}$, given its components ($\mathbf{v_{gx}}$, $\mathbf{v_{gy}}$) based on Pythagoras' theorem

$$v_g = \sqrt{(v_{g\,x})^2 + (v_{g\,y})^2}$$
$$= \sqrt{(-2.101\times10^2)^2 + (1.997\times10^2)^2} = 289.9\,\mathrm{km/h}$$

Relative Velocity Simple Example ..

- The compass bearing of $\mathbf{v_g}$ is given by:

$$\phi = \tan^{-1}\left(\frac{v_{g\,y}}{v_{g\,x}}\right)$$

- Because: $v_{g\,x} = v_g \cos\phi$ and $v_{g\,y} = v_g \sin\phi$
  - Remember that: **r = (x, y) = (rcosϕ, rsinϕ)**
- Unfortunately, the above expression becomes a little difficult to interpret if $\mathbf{v_{gx}}$ is negative
- An unambiguous pair of expressions for **ϕ** is:

$$\phi = \tan^{-1}\left(\frac{v_{g\,y}}{v_{g\,x}}\right), \qquad v_{g\,x} \geq 0$$
$$\phi = 180^\circ - \tan^{-1}\left(\frac{v_{g\,y}}{|v_{g\,x}|}\right), \quad v_{g\,x} < 0$$

Relative Velocity Simple Example …

- These expressions can be derived from simple trigonometry:

$$\phi = 180^\circ - \tan^{-1}\left(\frac{1.997\times10^2}{2.101\times10^2}\right) = 136.5^\circ$$

- Thus, the plane's velocity relative to the ground is **289km/h** at a compass bearing of **136.5°**

Example 4 - 1D Relative Velocity

- A train travels at 60 m/s to the east with respect to the ground
- A businessman on the train runs at 5 m/s to the west with respect to the train
- Find the velocity of the man with respect to the ground

Example 4 Solution

- The velocity of the train with respect to the ground is: v_{TG} = 60 m/s
- The velocity of the man with respect to the train is: v_{MT}= -5 m/s
- Putting these together, we get:

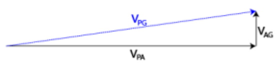
$$v_{MG} = v_{MT} + v_{TG} = -5\,\tfrac{m}{s} + 60\,\tfrac{m}{s} = 55\,\tfrac{m}{s}$$

Example 5 - 2D Relative Velocity

- An airplane flies at 250 m/s to the east with respect to the air
- The air is moving at 35 m/s to the north with respect to the ground
- Find the velocity of Air Force One with respect to the ground

Example 5 Solution

- Both v_{PA} and v_{AG} are two-dimensional vectors
- You can find v_{PG} by vector addition:
  - $v_{PG} = v_{PA} + v_{AG}$



- From Pythagoras:

$$v_{PG}^2 = v_{PA}^2 + v_{AG}^2 \Rightarrow v_{PG} = \sqrt{v_{PA}^2 + v_{AG}^2} \Rightarrow$$
$$v_{PG} = \sqrt{(250\,m/s)^2 + (35\,m/s)^2} = 252\,m/s$$

Example 5 Solution .

- Can find the angle of using basic trigonometric functions:

$$\tan\theta = \frac{opp}{adj} = \frac{v_{AG}}{v_{PA}} \Rightarrow \theta = \tan^{-1}\left(\frac{v_{AG}}{v_{PA}}\right) \Rightarrow$$
$$\theta = \tan^{-1}\left(\frac{35\,m/s}{250\,m/s}\right) = 8^\circ$$

- Therefore, the velocity with respect to the ground is 252 m/s at an angle of 8° north of east

Newtonian Physics

Newton's Laws of Motion

- Every body continues in its state of rest, or uniform motion in a straight line, unless compelled to change that state by forces impressed upon it
- The change of motion of an object is proportional to the force impressed upon it, and is made in the direction of the straight line in which the force is impressed
- To every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal and directed to contrary parts

Newton's First Law of Motion

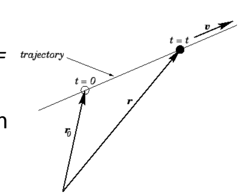
- Newton's first law was actually discovered by Galileo and perfected by Descartes
- This law states that if the motion of a given body is not disturbed by external influences then that body moves with constant velocity
- In other words, the displacement **r** of the body as a function of time **t** can be written

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}\,t$$

- where $\mathbf{r}_0$ and $\mathbf{v}$ are constant vectors

Newton's First Law of Motion .

- The body's trajectory is a straight-line which passes through point r_0 at time $t = 0$ and runs parallel to v
- In the special case in which $v = 0$ the body simply remains at rest



Newton's Second Law of Motion

- Newton used the word **motion** to mean what we nowadays call momentum
- The momentum **p** of a body is simply defined as the product of its mass **m** and its velocity: p = mv
- Newton's second law of motion is summed up in the equation:

$$\frac{d\mathbf{p}}{dt} = \mathbf{f}$$

  - where the vector **f** represents the net influence, or force, exerted on the object, whose motion is under investigation, by other objects

Newton's Second Law of Motion .

- For the case of a object with **constant mass**, the above law reduces to its more conventional form:

$$\mathbf{f} = m\,\mathbf{a}$$

- The net force exerted on a given object by other objects equals the product of that object's mass and its acceleration
  - Note that this law is entirely devoid of content unless we have some independent means of quantifying the forces exerted between different objects

Example 6

- A railway engine pulls a wagon of mass 10 000 kg along a straight track at a steady speed. The pull force in the couplings between the engine and wagon is 1000 N
  - A) What is the force opposing the motion of the wagon?
  - B) If the pull force is increased to 1200 N and the resistance to movement of the wagon remains constant, what would be the acceleration of the wagon?

Example 6 Solution (A)

- When the speed is steady, by Newton's first law, the resultant force must be zero
- The pull on the wagon must equal the resistance to motion
- So the force resisting motion is 1000 N

Example 6 Solution (B)

- The resultant force on the wagon is:
  - F_{total} = 1200 – 1000 = 200 N
- From Newton's 2^{nd} law of motion:

$$F = ma$$
$$200 = 10000a$$
$$a = 0.02 m/s^2$$

Example 7

- Find the acceleration of a 20 kg crate along a horizontal floor when it is pushed with a resultant force of 10 N parallel to the floor
- How far will the crate move in 5s
  - Starting from rest?

Example 7 Solution

- The acceleration can be calculated using Newton's 2nd law of motion:

$$F = ma$$
$$10 = 20a$$
$$a = 0.5 m/s^2$$

- Distance travelled is given by:

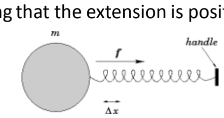
$$s = ut + \frac{1}{2}at^2$$
$$s = 0 + \frac{1}{2}0.5 \times 5^2$$
$$s = 6.25m$$

Hooke's Law

- Method of quantifying the force exerted on an object
- This law states that the force **f** exerted by a coiled spring is directly proportional to its extension **Δx**
- The extension of the spring is the difference between its actual length and its natural length
- The force acts parallel to the axis of the spring
- Hooke's law only holds if the extension of the spring is sufficiently small
  - If the extension becomes too large then the spring deforms permanently, or even breaks
    - Such behavior lies beyond the scope of Hooke's law

Hooke's Law .

- Use Hooke's law to quantify the force we exert on a body of mass **m** when we pull on the handle of a spring attached to it
- The magnitude **f** of the force is proportional to the extension of the spring:
  - Twice the extension means twice the force
- The direction of the force is towards the spring, parallel to its axis
  - Assuming that the extension is positive

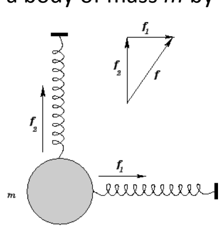


Hooke's Law ..

- The magnitude of the force can be quantified in terms of the critical extension required to impart a unit acceleration (i.e. $1 m/s^2$) to a body of unit mass (i.e. 1 Kg)
- According to **f = ma** the force corresponding to this extension is **1 newton**
- Thus, if the critical extension corresponds to a force of *1N* then half the critical extension corresponds to a force of *0.5N*, and so on

Hooke's Law Example

- Suppose that we apply two forces, f_1 and f_2
- The forces are acting in different directions, to a body of mass *m* by means of two springs



- According to Newton's second law of motion, the body does not accelerate
- It either remains at rest or moves with uniform velocity in a straight line

Newton's Third Law of Motion

- Suppose, for the sake of argument, that there are only two bodies in the Universe
- Let us label these bodies **a** and **b**
- Suppose that body **b** exerts a force $\mathbf{f_{ab}}$ on body **a**
- According to to Newton's third law of motion, body *a* must exert an *equal* and *opposite* force $f_{ba} = - f_{ab}$ on body *b*
- Thus, if we label f_{ab} the 'action' then, in Newton's language, f_{ba} is the equal and opposed 'reaction'

Newton's Third Law of Motion .

- Suppose, now, that there are many objects in the Universe
  - As is, indeed, the case
- According to Newton's third law, if object *j* exerts a force f_{ij} on object *i* then object *i* must exert an equal and opposite force $f_{ji} = - f_{ij}$ on object *j*
- It follows that all of the forces acting in the Universe can ultimately be grouped into equal and opposite action-reaction pairs
- Note, incidentally, that an action and its associated reaction always act on different bodies

Newton's Third Law of Motion ..

- Why do we need Newton's third law?
  - Actually, it is almost a matter of common sense
- Suppose that bodies *a* and *b* constitute an *isolated* system
- If $f_{ba} \neq - f_{ab}$ then this system exerts a *non-zero net force* $f = f_{ab} + f_{ba}$ on itself
  - Without the aid of any external agency
  - It will, therefore, accelerate forever under its own steam

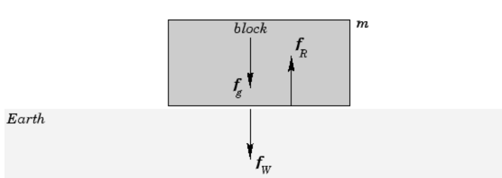
Mass, Weight and Strings

Mass and Weight

- The terms **mass** and **weight** are often confused with one another but in physics their meanings are quite distinct
- A body's mass is a measure of its inertia
  - i.e., its reluctance to deviate from uniform straight-line motion under the influence of external forces
- According to Newton's second law (**f = mα**) if two objects of differing masses are acted upon by forces of the same magnitude then
  - The resulting acceleration of the larger mass is less than that of the smaller mass

Mass and Weight Example

- Imagine a block of granite resting on the surface of the Earth



Mass and Weight Example .

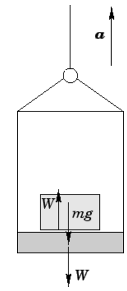
- The block experiences a downward force $\mathbf{f_g}$ due to the gravitational attraction of the Earth of magnitude **mg**
- The block exerts a downward force $\mathbf{f_W}$, of magnitude **mg**, on the ground beneath it
  - We refer to this force as the weight of the block
- According to Newton's third law, the ground below the block exerts an upward reaction force $\mathbf{f_R}$ on the block
  - This force is also of magnitude **mg**
- Thus, the net force acting on the block is:
  - $\mathbf{f_R} + \mathbf{f_g} = \mathbf{0}$ and thus the block remains stationary

Calculation of Weight

- So far, we have established that the weight **W** of a body is the magnitude of the downward force it exerts on any object which supports it
- Thus, **W = mg** where **m** is the mass of the body and **g** is the local acceleration due to gravity
  - Note that since weight is a force it is measured in Newton
- A body's weight is location dependent and is not an intrinsic property of that body
  - For instance, a body weighing **10N** on the surface of the Earth will only weigh about **3.8N** on the surface of Mars, due to the weaker surface gravity of Mars relative to the Earth

Weight in an Elevator

- Consider a block of mass **m** resting on the floor of an elevator
- Suppose that the elevator is accelerating upwards with acceleration **α**
- How does this acceleration affect the weight of the block?

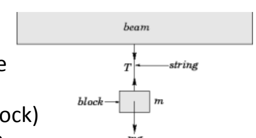


Weight in an Elevator .

- The block is subject to 2 forces:
  - A downward force **mg** due to gravity
  - An upward reaction force **W**
- Hence: **W – mg = mα** or **W = m(g + α)**
- The upward acceleration of the elevator has the effect of increasing the weight **W** of the block:
  - i.e. if the elevator accelerates upwards at **g = 9.81 m/s²** then the weight of the block is doubled
- if the elevator accelerates downward (i.e. if becomes negative) then the weight of the block is reduced

Strings

- Consider a block of mass **m** which is suspended from a fixed beam by means of a string
- The string is assumed to be light (its mass is negligible compared to that of the block) and inextensible (its length increases by a negligible amount because of the weight of the block)



Strings

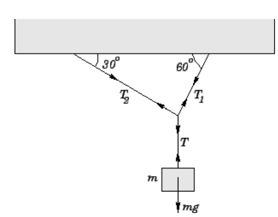
- If we apply Newton's second law to the block, the mass of the block is **m**, and its acceleration is zero
  - Since the block is assumed to be in equilibrium
- The block is subject to two forces, a downward force **mg** due to gravity, and an upward force due to the tension of the string so:

T – mg = 0

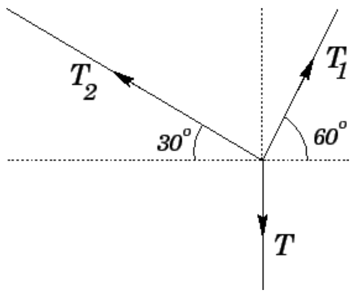
- In other words, in equilibrium, the tension of the string equals the weight of the block

Three Strings Example

- A slightly more complicated example in which a block of mass **m** is suspended by three strings is shown on the right
- What are the tensions, **T**, $\mathbf{T_1}$ and $\mathbf{T_2}$ in these strings, assuming that the block is in equilibrium?



Three Strings Example .



Horizontal Components

- The horizontal component of tension **T** is zero, since this tension acts straight down
- The horizontal component of tension $\mathbf{T_1}$ is: $\mathbf{T_1 cos60° = T_1/2}$
  - Since this force subtends an angle of with respect to the horizontal
- Likewise, the horizontal component of tension $\mathbf{T_2}$ is $\mathbf{T_2 cos30° = - \sqrt{3}\ T_2/2}$
- Since the knot does not accelerate in the horizontal direction, we can equate the sum of these components to zero: $\mathbf{T_1/2 = - \sqrt{3}\ T_2/2}$

Vertical Components

- Consider the vertical components of the forces acting on the knot
- Let components acting upward be positive, and vice versa
- Since the knot does not accelerate in the vertical direction, we can equate the sum of these components to zero

$$- mg + T_1/2 + \sqrt{3}\ T_2/2 = 0$$

Putting all Together

- From the previous equations:

$$T_1/2 = - \sqrt{3}\ T_2/2$$
$$- mg + T_1/2 + \sqrt{3}\ T_2/2 = 0$$

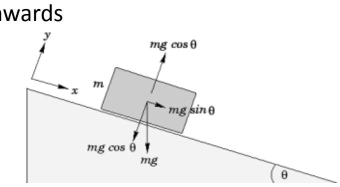
- We can calculate $\mathbf{T_1}$ and $\mathbf{T_2}$

$$T_1 = \sqrt{3}\ mg/2$$
$$T_2 = mg/2$$

Inclines, Pulleys and Friction

Inclines

- Consider a block of mass **m** sliding down a smooth frictionless incline which subtends an angle **θ** to the horizontal
- The weight **mg** of the block is directed vertically downwards



Incline Weight Analysis

- Weight can be resolved into components:
  - **mgcosθ** acting perpendicular (or normal) to the incline
  - **mgsinθ** acting parallel to the incline
- Note that the reaction of the incline to the weight of the block acts normal to the incline, and only matches the normal component of the weight
  - It is of magnitude **mgcosθ**
- In general the reaction of any unyielding surface is always locally normal to that surface
  - Directed outwards and matches the normal component of any inward force applied to the surface

Incline Weight Analysis .

- Applying Newton's second law to this problem we obtain:

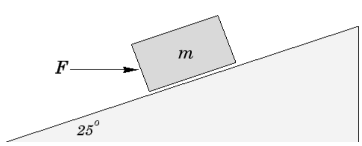
$$m\frac{d^2x}{dt^2} = m\,g\,\sin\theta$$

- which can be solved to give:

$$x = x_0 + v_0\,t + \frac{1}{2}\,g\,\sin\theta\,t^2$$

Accelerating up a Slope

- Suppose that the block, mass **m= 5 kg**, is subject to a horizontal force **F = 27 N**.
- What is the acceleration of the block up the (frictionless) slope?



Accelerating up a Slope .

- Only that component of the applied force which is parallel to the incline has any influence on the block's motion
  - The normal component of the applied force is canceled out by the normal reaction of the incline.
- The component of the applied force acting up the incline is **Fcos25º**
- The component of the block's weight acting down the incline is **mgsin25º**

Accelerating up a Slope ..

- Hence, using Newton's second law to determine the acceleration **α** of the block up the incline, we obtain:

$$a = \frac{F\,\cos 25^\circ - m\,g\,\sin 25^\circ}{m}$$

- Since m = 5 kg and F = 27 N, we have:

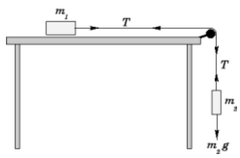
$$a = \frac{27 \times 0.9063 - 5 \times 9.81 \times 0.4226}{5} = 0.7483\,\mathrm{m/s^2}$$

Breakout Application of Inclines

- Tip:
  - You could to add 3 inclines to the ground so that the ball hits the upper bricks of the well
  - Ball will perform a projectile motion

Pulleys

- Consider two masses, $\mathbf{m_1}$ and $\mathbf{m_2}$ connected by a light inextensible string
- Suppose that the:
  - First mass slides over a smooth, frictionless, horizontal table
  - Second is suspended over the edge of the table by means of a light frictionless pulley



Pulleys Weight Analysis

- Since the pulley is light, we can neglect its rotational inertia in our analysis
- No force is required to turn a frictionless pulley
- Can assume that the tension **T** of the string is the same on either side of the pulley
- Again apply Newton's second law of motion to each mass in turn

Pulley - First Mass

- The first mass is subject to a downward force $\mathbf{m_1 g}$ due to gravity
  - However, this force is completely canceled out by the upward reaction force due to the table
- The mass $\mathbf{m_1}$ is also subject to a horizontal force **T** due to the tension in the string, which causes it to move rightwards with acceleration

$$a = \frac{T}{m_1}$$

Pulley - Second Mass

- The second mass is subject to a downward force $\mathbf{m_2 g}$ due to gravity, plus an upward force **T** due to the tension in the string
- These forces cause the mass to move downwards with acceleration:

$$a = g - \frac{T}{m_2}$$

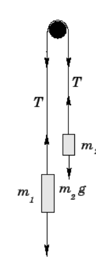
Putting Everything Together

- The rightward acceleration of the first mass must match the downward acceleration of the second, since the string which connects them is inextensible
- Equating the previous two expressions:

$$T = m_1 m_2 g / (m_1 + m_2)$$
$$a = m_2 g / (m_1 + m_2)$$

Another Pulleys Example

- Consider two masses $\mathbf{m_1}$ and $\mathbf{m_2}$ connected by a light inextensible string which is suspended from a light frictionless pulley
- Again must apply Newton's second law to each mass in turn



Another Pulleys Example .

- Without being given the values of m_1 and m_2 we cannot determine beforehand which mass is going to move upwards
- Let us assume that mass m_1 is going to move upwards:
  - Note that in this case we will obtain a negative acceleration for this mass

Another Pulleys Example - First Mass

- The first mass is subject to an upward force **T** due to the tension in the string, and a downward force $\mathbf{m_1 g}$ due to gravity
- These forces cause the mass to move upwards with acceleration:

$$a = \frac{T}{m_1} - g$$

Another Pulleys Example - Second Mass

- The second mass is subject to a downward force $\mathbf{m_2 g}$ due to gravity, and an upward force **T** due to the tension in the string
- These forces cause the mass to move downward with acceleration:

$$a = g - \frac{T}{m_2}$$

Putting Everything Together

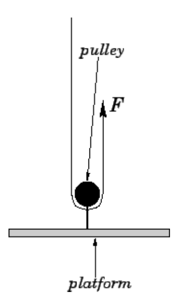
- The upward acceleration of the first mass must match the downward acceleration of the second, since they are connected by an inextensible string
- Hence, equating the previous two expressions, we obtain:

$$\mathbf{T = 2m_1m_2g / (m_1 + m_2)}$$

$$\mathbf{a = (m_2 - m_1)\, g / (m_1 + m_2)}$$

Example 8

- Consider the diagram
- The platform and the attached frictionless pulley weigh a total of 34N
- With what force **F** must the (light) rope be pulled to lift the platform at **3.2 m/s²** ?



Example 8 Solution

- Let **W** be the weight of the platform, **m = W/g** the mass of the platform, and **T** the tension in the rope
- From Newton's third law, it is clear that **T = F**
- Let us apply Newton's second law to the upward motion of the platform
- The platform is subject to two vertical forces:
  - A downward force **W** due to its weight, and
  - An upward force **2T** due to the tension in the rope
    - The force is **2T**, rather than **T**, because both the leftmost and rightmost sections of the rope, emerging from the pulley, are in tension and exerting an upward force on the pulley

Example 8 Solution .

- The upward acceleration **α** of the platform is:

$$a = \frac{2T - W}{m}$$

- Since **T = F** and **m = W/g**, we obtain:

$$F = \frac{W\,(a/g + 1)}{2}$$

- Finally, given that **W = 34N** and **α = 3.2 m/s²**, we have:

$$F = \frac{34\,(3.2/9.81 + 1)}{2} = 22.55\,\text{N}$$

Friction

- When a body slides over a rough surface a frictional force generally develops which acts to impede the motion
  - Friction, when viewed at the microscopic level, is actually a very complicated phenomenon
- The frictional force exerted on a body sliding over a rough surface is proportional to the normal reaction $\mathbf{R_n}$ at that surface, the constant of proportionality depending on the nature of the surface

Friction Definition

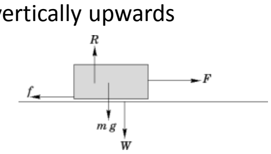
- Definition:

$$\mathbf{f = \mu R_n}$$

- where **μ** is termed the coefficient of (dynamical) friction
- For ordinary surfaces **μ** is generally of order unity

Friction Example

- Consider a block of mass **m** being dragged over a horizontal surface, whose coefficient of friction is **μ** by a horizontal force **F**
- The weight **W = mg** of the block acts vertically downwards, giving rise to a reaction **R = mg** acting vertically upwards



Friction Example .

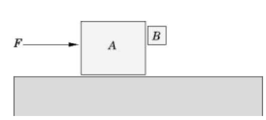
- The magnitude of the frictional force **f** which impedes the motion of the block, is simply times the normal reaction **R = mg**
- Hence **f = μmg**
- The acceleration of the block is, therefore:

$$a = \frac{F - f}{m} = \frac{F}{m} - \mu g$$

- Assuming that F > f

Example 9

- Consider the diagram where the mass of block A is 75kg and the mass of block B is 15 Kg
- The coefficient of static friction between the two blocks is μ = 0.45 and the horizontal surface is frictionless
- What minimum force F must be exerted on block A in order to prevent block B from falling?



Example 9 Solution

- Suppose that block A exerts a rightward force R on block B
- By Newton's third law, block B exerts an equal and opposite force on block A
- Applying Newton's second law of motion to the rightward acceleration α of block B, we obtain:

$$\alpha = R / m_B$$

- where m_B is the mass of block B

Example 9 Solution .

- The normal reaction at the interface between the two blocks is R
- Hence, the maximum frictional force that block A can exert on block B is μR
- In order to prevent block B from falling, this maximum frictional force (which acts upwards) must exceed the downward acting weight, $m_B g$, of the block
- Hence, we require:

$$\mu R > m_B g \quad \text{or} \quad \alpha > g / \mu$$

Example 9 Solution ..

- Applying Newton's second law to the rightward acceleration α of both blocks we obtain:

$$\alpha = F / (m_A + m_B)$$

- where m_A is the mass of block A
- It follows that: $F > (m_A + m_B)g / \mu$
- Substituting we get:

$$F > (75 + 15)9.81 / 0.45 = 1.962*10^3 \text{ N}$$

Breakout Application of Friction

- You can consider the ground to have friction
  - What will be the effects in the ball?
  - How would you start the ball if it stops?

Conservation of Energy

Conservation of Energy

- The conservation of energy is undoubtedly the single most important idea in physics
- Although the basic idea of energy conservation was familiar to scientists from the time of Newton onwards, this crucial concept only moved to centre-stage in physics in about 1850
  - When scientists first realized that heat was a form of energy

Forms of Energy

- Energy is the substance from which all things in the Universe are made up
- Energy can take many different forms:
  - Potential energy
  - Kinetic energy
  - Electrical energy
  - Thermal energy
  - Chemical energy
  - Nuclear energy
  - Etc

Energy Transformation

- Everything that we observe in the world around us represents one of the multitudinous manifestations of energy
- Various processes in the Universe transform energy from one form into another:
  - i.e. Mechanical (which are the focus of this course), thermal, electrical, nuclear, etc…
- However, all of these processes leave the total amount of energy in the Universe invariant

Energy in Closed Systems

- Whenever, and however, energy is transformed from one form into another, it is always conserved
- For a closed system the above law of universal energy conservation implies that the total energy of the system in question must remain constant in time
  - i.e. a system which does not exchange energy with the rest of the Universe

Energy Conservation During Free-Fall

- Consider a mass **m** which is falling vertically under the influence of gravity
  - Know how to analyze the motion of such a mass
- Let us employ this knowledge to search for an expression for the conserved energy during this process
  - This is clearly an example of a closed system, involving only the mass and the gravitational field

Energy Conservation During Free-Fall .

- The physics equations of free-fall under gravity is summarized by:

$$s = \upsilon_0 t - ½ gt^2$$
$$\upsilon = \upsilon_0 - gt$$
$$\upsilon^2 = \upsilon_0^2 - 2gs$$

Energy Conservation During Free-Fall ..

- Let us examine the last of these equations:
  - $\upsilon^2 = \upsilon_0^2 - 2gs$
- Suppose that the mass falls from height $\mathbf{h_1}$ to $\mathbf{h_2}$ its initial velocity is $\boldsymbol{\upsilon_1}$ and its final velocity is $\boldsymbol{\upsilon_2}$
- It follows that the net vertical displacement of the mass is:

$$s = h_2 - h_1$$

Energy Conservation During Free-Fall …

- Moreover, $\upsilon_0 = \upsilon_1$ and $\upsilon = \upsilon_2$
- Hence, the previous expression can be rearranged to give:

$$m\upsilon_1^2 + 2mgh_1 = m\upsilon_2^2 + 2mgh_2$$

or

$$\tfrac{1}{2} m\upsilon_1^2 + mgh_1 = \tfrac{1}{2} m\upsilon_2^2 + mgh_2$$

Kinetic and Gravitational Energy of Mass

- The kinetic energy of the mass can be defined as:

$$K = \tfrac{1}{2} m\upsilon^2$$

- The gravitational potential energy of the mass can be defined as:

$$U = mgh$$

Kinetic and Gravitational Energy of Mass .

- Note that kinetic energy represents energy the mass possesses by virtue of its motion
- Likewise, potential energy represents energy the mass possesses by virtue of its position
- Thus:

$$E = K + U = \text{constant}$$

Kinetic and Gravitational Energy of Mass ..

- Note that **E** is the total energy of the mass:
  - The sum of its kinetic and potential energies
- It is clear that **E** is a conserved quantity:
  - Although the kinetic and potential energies of the mass vary as it falls, its total energy remains the same
- The mks unit of energy is called the joule (symbol J)
  - 1 joule is equivalent to 1 kilogram meter-squared per second-squared, or 1 newton-meter

Free-Fall Energy Conservation Example

- Although we have already analyzed free-fall under gravity using Newton's laws of motion it is illuminating to re-examine this problem from the point of view of energy conservation
- Suppose that a mass **m** is dropped from rest and falls a distance **h**
  - What is the final velocity υ of the mass?

Free-Fall Energy Conservation Example .

- If energy is conserved then:

$$\Delta K = - \Delta U$$

- Any increase in the kinetic energy of the mass must be offset by a corresponding decrease in its potential energy

Free-Fall Energy Conservation Example ..

- The change in potential energy of the mass is simply **ΔU = mgs = - mgh**
  - Where **s = - h** is its net vertical displacement
- The change in kinetic energy is simply **ΔK = ½ mυ²** where **υ** is the final velocity
- This follows because the initial kinetic energy of the mass is zero
  - Since it is initially at rest

Free-Fall Energy Conservation Example ...

- Hence, the above expression yields:

$$\tfrac{1}{2} m\upsilon^2 = mgh$$

- Or

$$\upsilon = \text{sqrt}(2gh)$$

Free-Fall Energy Conservation Another Example

- Suppose that the same mass is thrown upwards with initial velocity **υ**
  - What is the maximum height **h** to which it rises?
- It is clear that as the mass rises its potential energy increases
  - It follows from energy conservation that its kinetic energy must decrease with height

Free-Fall Energy Conservation Another Example .

- Note that kinetic energy can never be negative
  - Since it is the product of the two positive definite quantities, **m** and **υ² ½**
- Hence, once the mass has risen to a height **h** which is such that its kinetic energy is reduced to zero it can rise no further
  - Must presumably start to fall

Free-Fall Energy Conservation Another Example ..

- The change in potential energy of the mass in moving from its initial height to its maximum height is **mgh**
- The corresponding change in kinetic energy is (**– ½ mυ²**)
  - Since **½ mυ²** is the initial kinetic energy
  - And the final kinetic energy is zero

Free-Fall Energy Conservation Another Example ...

- From:

$$-\tfrac{1}{2} m\upsilon^2 = -mgh \text{ (- since we are rising)}$$

- Which can be rearranged to give:

$$h = \upsilon^2/(2g)$$

Work

- We have seen that when a mass free-falls under the influence of gravity some of its kinetic energy is transformed into potential energy
  - Or vice versa
- The mass falls because it is subject to a downwards gravitational force of magnitude **mg**
  - The transformation of kinetic into potential energy is a direct consequence of the action of this force

Work .

- The gravitational potential energy of a given body is stored in the gravitational field which surrounds it
  - Thus, when the body rises its potential energy consequently increases by an amount **ΔU**
- Thus, when we speak of a body's kinetic energy being transformed into potential energy, we are really talking about a flow of energy from the body to the surrounding gravitational field
  - This energy flow is mediated by the gravitational force exerted by the field on the body in question

Work Equations

- Suppose that a mass **m** falls a distance **h**
- During this process, the energy of the gravitational field decreases by a certain amount
  - Also the body's kinetic energy increases by a corresponding amount
  - This transfer of energy, from the field to the mass, is, presumably, mediated by the gravitational force **-mg** acting on the mass
- Given that **U = mgh** it follows from **ΔK = - ΔU** that:

$$\Delta K = f\,\Delta h$$

Work Equations .

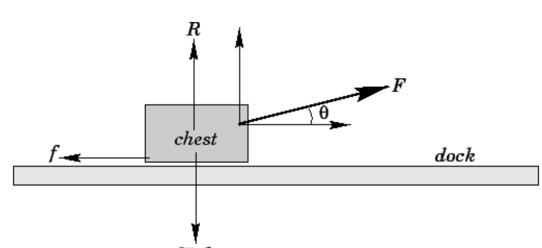
- We refer to the amount of energy transferred to a body, when a force acts upon it, as the amount of work **W** performed by that force on the body in question
- When a gravitational force **f** acts on a body, causing it to displace a distance **x** in the direction of that force, then the net work done on the body is:

$$W = fx$$

Example 10

- A man drags a 100kg treasure chest over the rough surface of a dock by exerting a constant force of 150N acting at an angle of 15º above the horizontal
- The chest moves 10m in a straight line, and the coefficient of kinetic friction between the chest and the dock is 0.15
  - Draw the resultant diagram including all forces that act on the treasure chest
  - Calculate how much work the man does
  - Calculate how much energy is dissipated as heat via friction
  - Calculate the final velocity of the chest

Draw the Resultant Diagram



Calculate Work

- The work W performed by the horizontal component is simply the magnitude of this component times the horizontal distance x moved by the chest:

W = Fcosθ x = 150*cos15*10 = 1448.8J

Calculate how much energy is dissipated as heat via friction

- Since the chest does not accelerate in the vertical direction, these forces must balance

R = mg – Fsinθ = 100*9.81 – 150sin15 = 942.8N

- The frictional force f is the product of the coefficient of kinetic friction μ_κ and the normal reaction R, so:

f = μ_κ R = 0.15*942.8 = 141.4N

Calculate how much energy is dissipated as heat via friction .

- The work W' done by the frictional force is

W' = -fx = -141.4*10 = -1414J

- The final kinetic energy K of the chest is the difference between the work W done by the man and the energy –W' dissipated as heat, hence:

K = W + W' = 1448.8 - 1414 = 34.8J

Calculate the Final Velocity

- Since K = ½ mu^2, the final velocity of the chest is:

u = sqrt(2K/m)=sqrt(2*34.8/100)

Rotational Motion

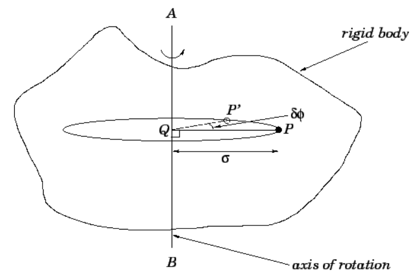
Introduction to Rotational Motion

- An extended object can exhibit another type of motion which remains located at the same spatial position
  - But constantly changes its orientation with respect to other fixed points in space
- This new type of motion is called rotation
  - An important aspect is rotational motion

Rigid Body Rotation

- Consider a rigid body executing rotational motion
  - i.e. Rotational motion with no translational component
- Must define an axis of rotation
  - Which is assumed to pass through the body
    - This axis corresponds to the straight-line which is the locus of all points inside the body which remain stationary as the body rotates
- A general point located inside the body executes circular motion which is:
  - Centred on the rotation axis
  - Orientated in the plane perpendicular to this axis

Rigid Body Rotation Example



Assume that the axis of rotation remains fixed

Rigid Body Rotation Analysis

- The axis of rotation is the line **AB**
- A general point **P** lying within the body executes a circular orbit, centred on **AB**
  - In the plane perpendicular to **AB**
- Let the line **QP** be a radius of this orbit which links the axis of rotation to the instantaneous position of **P** at time **t**
- Suppose that at time **t + δt** point **P** has moved to **P/** and the radius **QP** has rotated through an angle **δφ**

Rigid Body Rotation Analysis .

- The instantaneous angular velocity of the body **ω(t)** is defined:

$$\omega = \lim_{\delta t \to 0} \frac{\delta \phi}{\delta t} = \frac{d\phi}{dt}$$

- Note that if the body is indeed rotating rigidly, then the calculated value of **ω** should be the same for all possible points **P** lying within the body
  - Except for those points lying exactly on the axis of rotation, for which **ω** is ill-defined

Rigid Body Rotation Analysis ..

- The rotation speed **υ** of point **P** is related to the angular velocity **ω** of the body via:

$$\upsilon = \sigma\omega$$

  - where **σ** is the perpendicular distance from the axis of rotation to point **P**
- Angular acceleration **α(t)** of a rigidly rotating body is defined as the time derivative of the angular velocity:

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\phi}{dt^2}$$

  - where **ϕ** is the angular coordinate of some arbitrarily chosen point reference within the body, measured with respect to the rotation axis

Rigid Body Rotation Analysis ...

- For a body rotating with constant angular velocity **ω** the angular acceleration is zero, and the rotation angle **ϕ** increases linearly with time:

$$\phi(t) = \phi_0 + \omega t$$

  - where $\phi_0 = \phi(t = 0)$
- Likewise, for a body rotating with constant angular acceleration **α**:

$$\omega(t) = \omega_0 + \alpha t$$

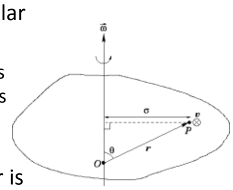
and the rotation angle satisfies:

$$\phi(t) = \phi_0 + \omega_0 t + \tfrac{1}{2} \alpha t^2$$

  - Here $\omega_0 = \omega(t = 0)$

Rigid Body Rotation Vectors

- Rigid body rotating with angular velocity **ω** with assumption:
  - The axis of rotation, which runs parallel to **ω** is assumed to pass through the origin **O** of our coordinate system
- Point **P** whose position vector is **r** represents a general point inside the body
  - What is the velocity of rotation **υ** at point **P**?



Rigid Body Rotation Vectors .

- The magnitude of this velocity is simply:

$$υ = σω = ωr \sin θ$$

- where **σ** is the perpendicular distance of point **P** from the axis of rotation and **θ** is the angle subtended between the directions of **ω** and **r**
- The direction of the velocity is into the page

Rigid Body Rotation Vectors ..

- Alternatively the direction of the velocity is mutually perpendicular to the directions of **ω** and **r** in the sense indicated by the right-hand grip rule when **ω** is rotated onto **r**
  - Through an angle less than 180

$υ = ω \times r$ (cross product)

Centre of Mass

- The co-ordinates of the centre of mass (or centre of gravity) of an extended object are the mass weighted averages of the coordinates of the elements which make up that object
- If the object has net mass **M** and is composed of **N** elements, such that the i_{th} element has mass $\mathbf{m_i}$ and position vector $\mathbf{r_i}$ then the position vector of the centre of mass is given by:

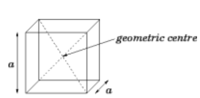
$$\mathbf{r}_{cm} = \frac{1}{M} \sum_{i=1,N} m_i \, \mathbf{r}_i.$$

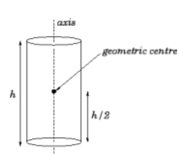
Centre of Mass .

- If the object is continuous:

$$m_i = \rho(\mathbf{r}_i) \, V_i,$$

  - where **ρ(r)** is the mass density of the object and $\mathbf{V_i}$ is the volume occupied by the i_{th} element





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Questions