Introduction to Satisfiability Modulo Theories

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IA072 – Seminar on Concurrency

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First-Order Logic

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- $\forall s.\, \mathrm{Human}(s) \,\rightarrow\, \mathrm{Mortal}(s)$.
- $\exists x \exists y.\, x < 5 \,\wedge\, y < 3 \,\wedge\, 2 \cdot (x + y) > 20$.

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In addition to logical symbols, first-order formulas contain variables, constant symbols, function symbols, and predicate symbols.

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1. a Boolean combination of literals – $(R(x) \vee \neg R(y)) \wedge S(f(x), y), \dots$
2. a quantifier applied to a formula – $\forall x.\, R(x), \dots$.

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The set $\Sigma = \Sigma^F \cup \Sigma^P$ is called a signature.

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Is the following formula true?

$$\forall x \exists y. <(x, y) \ \land \ <(y, +(x, 1))$$

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- What does the function symbol $+$ mean?
- What does the relation symbol $<$ mean?

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Meaning of these three things is given by a Σ-structure.

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A Σ-structure $\mathcal{A}$ is a pair of

1. a non-empty set A called the universe,
2. a map $(\_)^{\mathcal{A}}$ that
   - to each $f \in \Sigma^F$ assigns a function $f^{\mathcal{A}} \colon A^{\mathrm{ar}(f)} \to A$,
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- $A = \mathbb{Z}$
- $+^{\mathcal{A}}(x, y) = x + y$
- $<^{\mathcal{A}} (x, y) \iff x < y$
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Solution

Consider only well-behaved structures

This gives rise to the Satisfiability Modulo Theories

Satisfiability Modulo Theories

Definition

A (Σ-)theory is a set of Σ-structures.

Definition

A formula φ is satisfiable modulo theory T if it evaluates to true for some structure $\mathcal{A} \in T$ and a variable assignment μ.

Satisfiability Modulo Theories – Example

Consider the structure $\mathcal{Z}$ with the universe $\mathbb{Z}$ and the standard interpretation of operations $+$, $<$, and 1.

The formula $(x < y) \;\wedge\; (y + 1 < x)$ is unsatisfiable modulo theory $T = \{\mathcal{Z}\}$.

The formula $(x < y) \;\wedge\; (y < x + 2)$ is satisfiable modulo theory $T = \{\mathcal{Z}\}$.

Theories – Tour d’horizon

Theory of equality and uninterpreted functions

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- satisfiability of conjunctions of literals is in $\mathcal{O}(n \cdot \log(n))$

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Theory of linear integer arithmetic

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- complexity of satisfiability of arbitrary formulas is in $\Omega(2^{2^n})$ (Fischer, Rabin, 1974)

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- complexity of satisfiability of arbitrary formulas is in $\mathcal{O}(2^{2^{2^{kn}}})$ (Oppen, 1978)

Theories – Tour d'horizon

Theory of linear integer arithmetic

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Theories – Tour d’horizon

Theory of non-linear real arithmetic

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Theories – Tour d’horizon

Theory of arrays

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- $\Sigma = \{\text{read}, \text{write}, =\}$
- T_A is a set of structures, where A is a set of arrays and elements and
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Theories – Tour d'horizon

More theories:

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Theories – Tour d’horizon

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A theory can be also viewed as a set of closed Σ-formulas (axioms).

A formula is then satisfiable modulo T iff it is true for some structure that satisfies all axioms in T.

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Example

- The theory of uninterpreted functions with equality is $T_= = \emptyset$
- The theory of arrays is

$$\begin{aligned} T_A = \{ & \forall a, i, j.\, (i = j \ \rightarrow\ read(a, i) = read(a, j)), \\ & \forall a, v, i, j.\, (i = j \ \rightarrow\ read(write(a, i, v), j) = v), \\ & \forall a, v, i, j.\, (i \neq j \ \rightarrow\ read(write(a, i, v), j) = read(a, j))\} \end{aligned}$$

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Deciding satisfiability modulo theories

Two approaches to SMT

- eager
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Deciding satisfiability modulo theories

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Eager approach
Encode the formula to SAT

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Lazy approach
Use a SAT solver to reason about Boolean structure of the formula and a specialized T-solver to reason about the constraints imposed by the theory.

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Lazy SMT solvers can be further divided to:

- offline
- online

Offline SMT solving

Suppose the formula is in conjunctive normal form – a conjunction of disjunctions of Σ-literals.

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- Use a SAT solver to get a Boolean model of the formula.
- Use a T-solver to check whether the model is satisfiable in the theory (T-consistent).
- If not, add a clause that prohibits this Boolean model and repeat.

Offline SMT solving – example

The formula φ over linear integer arithmetic:

$$x = 1 \;\wedge\; (y < 3 \vee y > 5) \;\wedge\; (x + y = 4 \vee y = 6)$$

Offline SMT solving – example

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Boolean model
$x = 1,\; y < 3,\; x + y = 4$

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Not satisfiable in the theory

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The formula φ over linear integer arithmetic:

$$x = 1 \;\wedge\; (y < 3 \vee y > 5) \;\wedge\; (x + y = 4 \vee y = 6) \;\wedge$$
$$(\neg(x = 1) \vee \neg(y < 3) \vee \neg(x + y = 4))$$

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$$\begin{aligned} &p \;\wedge\; (q \vee r) \;\wedge\; (s \vee t) \;\wedge \\ &(\neg p \vee \neg q \vee \neg s) \end{aligned}$$

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Boolean model
$x = 1,\; \neg(y < 3),\; y > 5,\; \neg(x + y = 4),\; y = 6$
Satisfiable in the theory! ($x = 1, y = 6$)

Online SMT solving – towards CDCL(T)

Online SMT
Integrate the CDCL SAT solver and the T-solver more tightly.

After a T-conflict, the T-solver provides the conflict clause and the search backtracks.

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$$x = 1 \;\wedge\; (y < 3 \vee y > 5) \;\wedge\; (x + y = 4 \vee y = 6)$$

Partial assignment

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The formula φ over linear integer arithmetic:

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Partial assignment
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Partial assignment

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Partial assignment

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The T-solver can guide the search, if a value of a literal is implied by the current partial assignment.

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Further enhancements

Online approach can be further improved:

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- early pruning,
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CDCL(T)

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- perform theory propagation (identify implied literals),
- perform early pruning (identify theory conflicts during the search).

CDCL($T_=$)

Let's consider CDCL($T_=$) with the details of the $T_=$-solver.

$$x = f(y) \ \wedge\ (y = z \vee x = y) \ \wedge\ (g(f(z)) \neq g(x) \vee f(z) \neq f(y))$$

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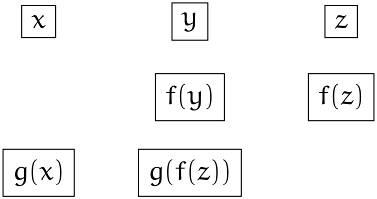
Partial assignment

Equality graph



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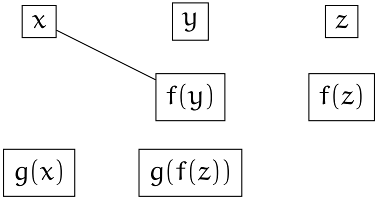

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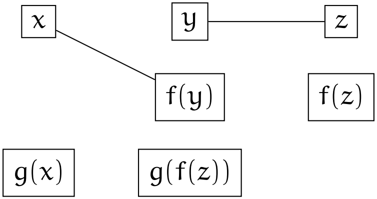

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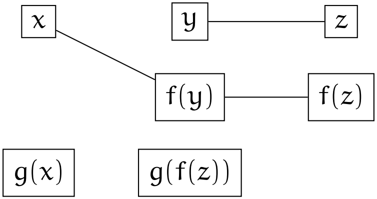

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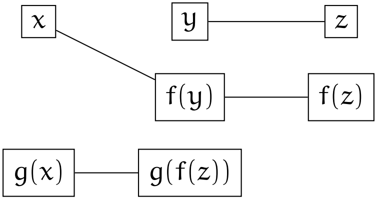
Equality graph



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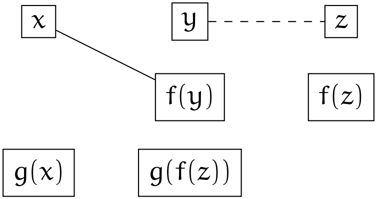
Equality graph



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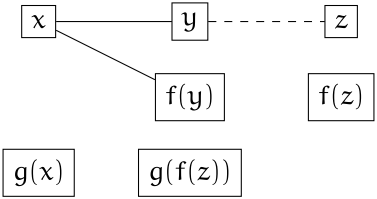

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Partial assignment

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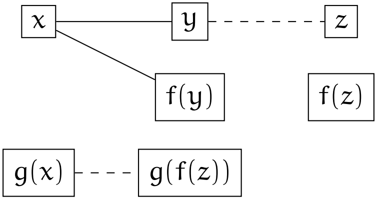

Further schedule

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- 19. 5. – An Approximation Framework for Solvers and Decision Procedures (Katka)