

Seminar 7

Definition 1 (Naive Bayes Classifier) *Naive Bayes (NB) Classifier assumes that the effect of the value of a predictor x on a given class c is class conditional independent. Bayes theorem provides a way of calculating the posterior probability $P(c|x)$ from class prior probability $P(c)$, predictor prior probability $P(x)$ and probability of the predictor given the class $P(x|c)$*

$$P(c|x) = \frac{P(x|c)P(c)}{P(x)}$$

and for a vector of predictors $X = (x_1, \dots, x_n)$

$$P(c|X) = P(x_1|c) \dots P(x_n|c)P(c).$$

The class with the highest posterior probability is the outcome of prediction.

Exercise 1

Considering the table of observations, use the Naive Bayes classifier to recommend whether to *Play Golf* given a day with *Outlook = Rainy*, *Temperature = Mild*, *Humidity = Normal* and *Windy = True*. Do not deal with the zero-frequency problem.

Outlook	Temperature	Humidity	Windy	Play Golf
Rainy	Hot	High	False	No
Rainy	Hot	High	True	No
Overcast	Hot	High	False	Yes
Sunny	Mild	High	False	Yes
Sunny	Cool	Normal	False	Yes
Sunny	Cool	Normal	True	No
Overcast	Cool	Normal	True	Yes
Rainy	Mild	High	False	No
Rainy	Cool	Normal	False	Yes
Sunny	Mild	Normal	False	Yes
Rainy	Mild	Normal	True	Yes
Overcast	Mild	High	True	Yes
Overcast	Hot	Normal	False	Yes
Sunny	Mild	High	True	No

Table 1: Exercise.

First build the likelihood tables for each predictor

		Play Golf					Play Golf		
		Yes	No				Yes	No	
Outlook	Sunny	3/9	2/5	5/14	Temperature	Hot	2/9	2/5	4/14
	Overcast	4/9	0/5	4/14		Mild	4/9	2/5	6/14
	Rainy	2/9	3/5	5/14		Cool	3/9	1/5	4/14
		9/14	5/14				9/14	5/14	

		Play Golf					Play Golf		
		Yes	No				Yes	No	
Humidity	High	3/9	4/5	7/14	Windy	True	3/9	2/5	5/14
	Normal	6/9	1/5	7/14		False	6/9	3/5	9/14
		9/14	5/14				9/14	5/14	

We see that probability of *Sunny* given *Yes* is $3/9 = 0.33$, probability of *Sunny* is $5/14 = 0.36$ and probability of *Yes* is $9/14 = 0.64$. Then we count the likelihoods of *Yes* and *No*

$$\begin{aligned}
 P(Yes|Rainy, Mild, Normal, True) &= \\
 &= P(Rainy|Yes) \cdot P(Mild|Yes) \cdot P(Normal|Yes) \cdot P(True|Yes) \cdot P(Yes) \\
 &= \frac{2}{9} \cdot \frac{4}{9} \cdot \frac{6}{9} \cdot \frac{3}{9} \cdot \frac{9}{14} = 0.014109347 \\
 P(No|Rainy, Mild, Normal, True) &= \\
 &= P(Rainy|No) \cdot P(Mild|No) \cdot P(Normal|No) \cdot P(True|No) \cdot P(No) \\
 &= \frac{3}{5} \cdot \frac{2}{5} \cdot \frac{1}{5} \cdot \frac{3}{5} \cdot \frac{5}{14} = 0.010285714
 \end{aligned} \tag{1}$$

and suggest *Yes*. We can normalize the likelihoods to obtain the % confidence:

$$\begin{aligned}
 P(Yes|Rainy, Mild, Normal, True) &= \frac{0.014109347}{0.014109347 + 0.010285714} = 57.84\% \\
 P(No|Rainy, Mild, Normal, True) &= \frac{0.010285714}{0.014109347 + 0.010285714} = 42.16\%
 \end{aligned}$$

Definition 2 (Support Vector Machines Classifier (two-class, linearly separable))

Support Vector Machines (SVM) finds the hyperplane that bisects and is perpendicular to the connecting line of the closest points from the two classes. The separating (decision) hyperplane is defined in terms of a normal (weight) vector \mathbf{w} and a scalar intercept term b as

$$f(x) = \mathbf{w} \cdot \mathbf{x} + b$$

where \cdot is the dot product of vectors. Finally, the SVM classifier becomes

$$\text{class}(x) = \text{sgn}(f(x)).$$

Exercise 2

Build the SVM classifier for the training set $\{([1, 1], -1), ([2, 0], -1), ([2, 3], +1)\}$.

We first take the closest two points from the respective classes: $[1, 1]$ and $[2, 3]$. We have $\mathbf{w} = a \cdot ([1, 1] - [2, 3]) = [a, 2a]$. Now we calculate a and b

$$a + 2a + b = -1$$

$$2a + 6a + b = 1$$

for the points $[1, 1]$ and $[2, 3]$, respectively. The solution is

$$a = \frac{2}{5} \quad b = \frac{-11}{5}$$

building the weight vector

$$\mathbf{w} = \left[\frac{2}{5}, \frac{4}{5} \right]$$

and the final classifier becomes

$$\text{class}(x) = \text{sgn} \left(\frac{2}{5}x_1 + \frac{4}{5}x_2 - \frac{11}{5} \right).$$