IA158 Real Time Systems

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Organization of This Course

Sources:

- Lectures (slides, notes)
  - based on several sources (hard to obtain)
  - slides are prepared for lectures, lots of stuff on whiteboard ($\Rightarrow$ attend the lectures)

Homework:

- a larger project, most probably with LEGO mindstorms

Evaluation:

- Homework project
  (have to do to be allowed to the exam)
- Oral exam

Real-Time Systems

Definition 1 (Time)

Mirriam-Webster: Time is the measured or measurable period during which an action, process, or condition exists or continues.

Definition 2 (Real-time)

Real-time is a quantitative notion of time measured using a physical clock.

Example: After an event occurs (eg. temperature exceeds 500 degrees) the corresponding action (cooling) must take place within 100ms.

Compare with qualitative notion of time (before, after, eventually, etc.)

Definition 3 (Real-time system)

A *real-time system* must deliver services in a timely manner.

Not necessarily fast, must satisfy some *quantitative* timing constraints

Real-time Embedded Systems

Definition 4 (Embedded system)

An *embedded system* is a computer system designed for specific control functions within a larger system, usually consisting of electronic as well as mechanical parts.

Most (not all) real-time systems are embedded

Most (not all) embedded systems are real-time

(Few) Examples of Real-time Embedded Systems

- Industrial
  - chemical plant control
  - automated assembly line (e.g. robotic assembly, inspection)
- Medical
  - pacemaker, medical monitoring devices
  - robots used to move radioactive materials
- Transportation systems
  - computers in cars (ABS, MPFI, cruise control, airbag ...)
  - aircraft (FMS, fly-by-wire ...)
- Military applications
  - controllers in weapons, missiles, fighter aircraft, ...
  - radar and sonar tracking
- Multimedia – video telephony, multimedia center, videoconferencing
- ...

(Non-)Real-time (non-)embedded systems

There are real time systems that are not embedded:

- trading systems
- ticket reservation
- multimedia (on PC)
- ...

There are embedded systems that are (possibly) not real-time

e.g. a weather station sends data once a day without any deadline – not really real-time system

Caveat: Aren't all systems real-time in a sense?

Characteristics of Real-Time Embedded Systems

Real-time systems often are

- safety critical
  - Serious consequences may result if services are not delivered on timely basis
  - Bugs in embedded real-time systems are often difficult to fix

  ... need to validate their correctness

- concurrent
  - Real-world devices operate in parallel – better to model this parallelism by concurrent tasks in the program

  ... validation may be difficult, formal methods often needed

- reactive
  - Interact continuously with their environment (as opposed to information processing systems)

  ... “traditional” validation methods do not apply

Validating Time Requirements and Predictability

- Given real-time requirements and an implementation on HW and SW, how to show that the requirements are met?

  ... testing might not suffice:

  Maiden flight of space shuttle, 12 April 1981: 1/67 probability that a transient overload occurs during initialization; and it actually did!

- We need a formal model and validation ...

- ... we need **predictable** behavior!
  It is difficult to obtain
  - caches, DMA, unmaskable interrupts
  - memory management
  - scheduling anomalies
  - difficult to compute worst-case execution time
  - ...

Types of Timing Requirements

Time sharing systems: minimize average response time

The goal of scheduling in standard op. systems such as Linux and Windows

Often it is **not** enough to minimize average response time!

(A man drowned crossing a stream with an average depth of 15cm.)

"hard" real-time tasks must be **always** finished before their deadline!

e.g. airbag in a car: whenever a collision is detected, the airbag must be deployed within 10ms

Not all tasks in a real-time system are critical, only the quality of service is affected by missing a deadline

Most "soft" real-time tasks should finish before their deadlines.

e.g. frame rate in a videoconf. should be kept above 15fps most of the time

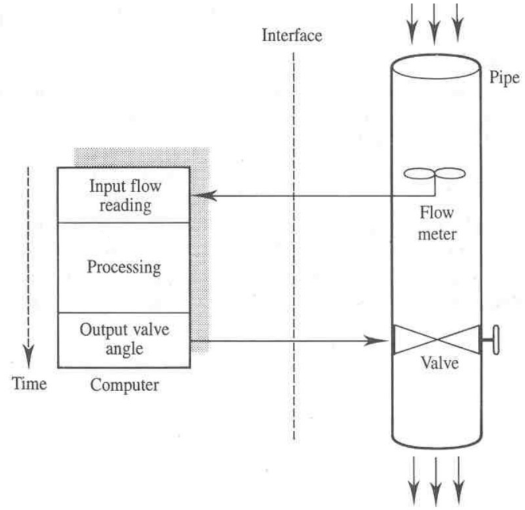
Many real-time systems combine "hard" and "soft" real-time tasks.

i.e. we optimize performance w.r.t. "soft" real-time tasks under the constraint that "hard" real-time tasks are finished before their deadlines

Examples of Real-Time Systems

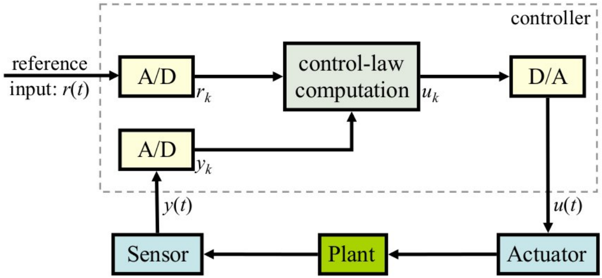
- Digital process control
  - anti-lock braking system
- Higher-level command and control
  - helicopter flight control
- Real-time databases
  - Stock trading systems

Digital Process Control



Computer controls the flow in the pipe in real-time

Digital Process Control



The controller (computer) controls the plant using the actuator (valve) based on sampled data from the sensor (flow meter)

- $y(t)$ – the measured state of the plant
- $r(t)$ – the desired state of the plant
- Calculate control output $u(t)$ as a function of $y(t), r(t)$
  e.g. $u_k = u_{k-2} + \alpha(r_k - y_k) + \beta(r_{k-1} - y_{k-1}) + \gamma(r_{k-2} - y_{k-2})$
  where α, β, γ are suitable constants

Digital Process Control

- Pseudo-code for the controller:

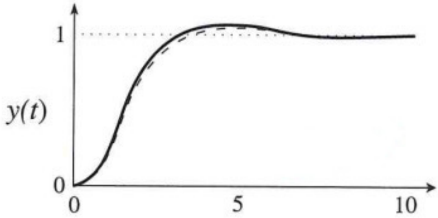
  set timer to interrupt periodically with period T
  foreach timer interrupt **do**
  | analogue-to-digital conversion of $y(t)$ to get y_k
  | compute control output u_k based on r_k and y_k
  | digital-to-analogue conversion of u_k to get $u(t)$
  end

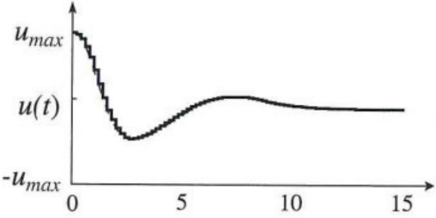
- Effective control of the plant depends on:
  - The correct reference input and control law computation
  - The accuracy of the sensor measurements
    - Resolution of the sampled data (i.e. bits per sample)
    - Frequency of interrupts (i.e. $1/T$)

- T is the *sampling period*
  - Small T better approximates the analogue behavior
  - Large T means less processor-time demand

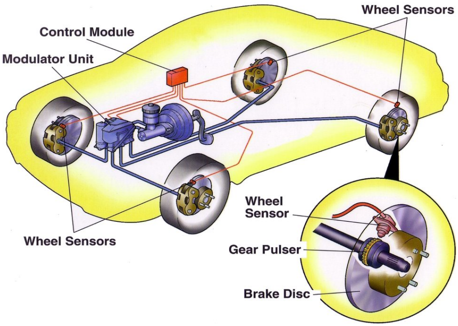
    ... but may result in unstable control

Example





Anti-Lock Braking System

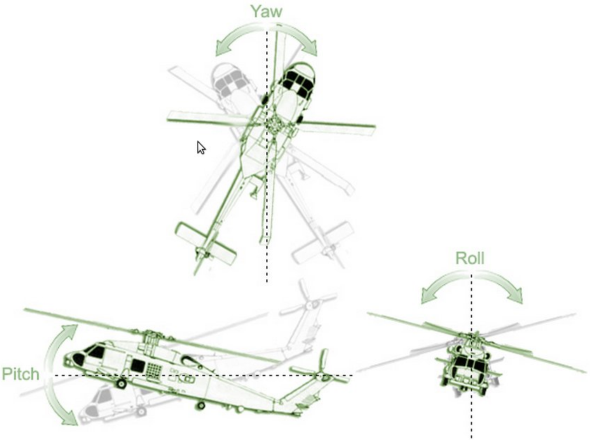


- The controller monitors the speed sensors in wheels

  Right before a wheel locks up, it experiences a rapid deceleration

- If a rapid deceleration of a wheel is observed, the controller alternately
  - reduces pressure on the corresponding brake until acceleration is observed
  - then applies brake until deceleration is observed

Multi-Rate DPC – Helicopter Flight Control



There are also three velocity components

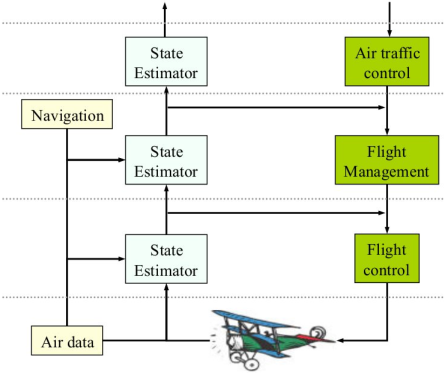
Two control loops: pilot's control (30Hz) and stabilization (90Hz)

Multi-Rate DPC – Helicopter Flight Control

Do the following in each 1/180-second cycle:

- Validate sensor data; in the presence of failures, reconfigure the system
- Do the following 30-Hz avionics tasks, each one every six cycles:
  - keyboard input and mode selection
  - data normalization and coordinate transformation
  - tracking reference update
- Do the following 30-Hz avionics tasks, each one every six cycles:
  - control laws of the outer pitch-control loop
  - control laws of the outer roll-control loop
  - control laws of the outer yaw- and collective-control loop
- Do each of the following 90-Hz computations once every two cycles, using outputs produced by 30-Hz computations and avionics tasks:
  - control laws of the inner pitch-control loop
  - control laws of the inner roll- and collective-control loop
- Compute the control laws of the inner yaw-control loop, using outputs produced by 90-Hz control-law computations as inputs
- Output commands
- Carry out built-in-test
- Wait until the beginning of the next cycle

Higher-Level Command and Control



Controllers organized into a hierarchy

- At the lowest level we place the digital control systems that operate on the physical environment
- Higher level controllers monitor the behavior of lower levels
- Time-scale and complexity of decision making increases as one goes up the hierarchy (from control to planning)

Real-Time Database System

- Databases that contain perishable data, i.e. relevance of data deteriorates with time

  Air traffic control, stock price quotation systems, tracking systems, etc.
- The temporal quality of data is quantified by *age of an image object*, i.e. the length of time since last update
- temporal consistency
  - absolute = max. age is bounded by a fixed threshold
  - relative = max. difference in ages is bounded by a threshold

    e.g. planning system correlating traffic density and flow of vehicles

Applications	Size	Ave. Resp. Time	Max Resp. Time	Abs. Cons.	Rel. Cons.
Air traffic control	20,000	0.50 ms	5.00 ms	3.00 sec.	6.00 sec.
Aircraft mission	3,000	0.05 ms	1.00 ms	0.05 sec.	0.20 sec.
Spacecraft control	5,000	0.05 ms	1.00 ms	0.20 sec.	1.00 sec.
Process control		0.80 ms	5.00 sec	1.00 sec.	2.00 sec

- Users of database compete for access – various models for trading consistency with time demands exist.

Stock-Trading System

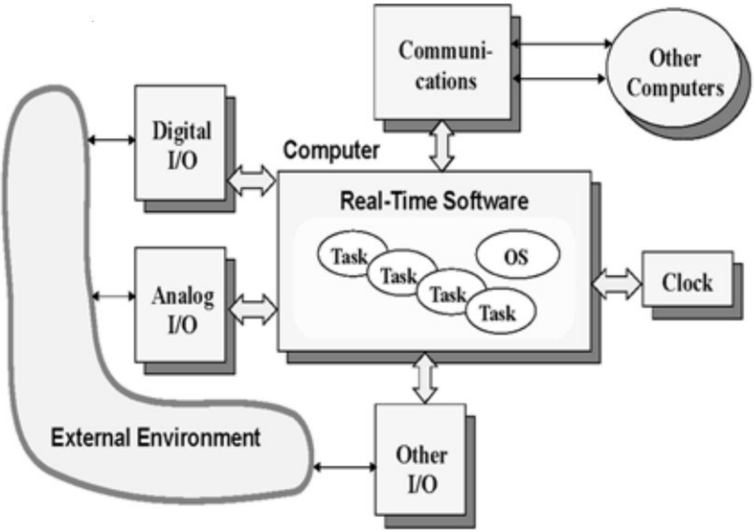
- A system for selling/buying stock at public prices
- Prices are volatile in their movement
- Stop orders:
  - set upper limit on prices for buying – buy for the best available price once the limit is reached

    e.g. stock currently trading at $30 should be bought when the price rises above $35
  - set lower limit on prices for selling – sell for the best available price once the limit is reached

    e.g. stock currently trading at $30 should be sold when the price sinks below $25
- Depending on the delay, the available price may be different from the limit

  successful stop orders depend on the timely delivery of stock trade data and the ability to trade on the changing prices in a timely manner

Structure of Real-Time (Embedded) Applications



Types of Real-Time Systems

- Purely cyclic
  - every task executes periodically; I/O operations are polled; demands in resources do not vary

  e.g. digital controllers
- Mostly cyclic
  - most tasks execute periodically; system also responds to external events (fault recovery and external commands) asynchronously

  e.g. avionics
- Asynchronous and somewhat predictable
  - durations between consecutive executions of a task as well as demands in resources may vary considerably. These variations have either bounded range, or known statistics.

  e.g. radar signal processing, tracking

Types of Real-Time Systems

- The type of application affects how we schedule tasks and prove correctness
- It is easier to reason about applications that are more cyclic, synchronous and predictable
  - Many real-time systems are designed in this manner
  - Safe, conservative, design approach, if it works

Real-Time Systems Failures

- AT&T *long* distance calls
- Therac-25 medical accelerator disaster
- Patriot missile mistiming

AT&T Long Distance Calls

114 computer-operated electronic switches scattered across USA
Handling up to 700,000 calls an hour

The problem:

- the switch in New York City neared its load limit
- entered a four-second maintenance reset
- sent “do not disturb” to neighbors
- after the reset, the switch began to distribute calls (quickly)

- then another switch received one of these calls from New York
- began to update its records that New York was back on line
- a second call from New York arrived less than 10 milliseconds after the first, i.e. while the first hadn’t yet been handled;
  this together with a SW bug caused maintenance reset

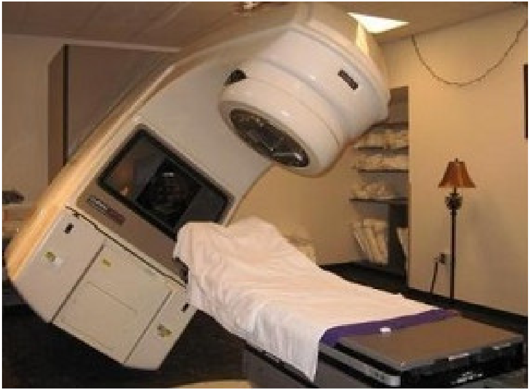
- the error was propagated further ....

The reason for failure: The system was unable to react to closely timed messages

Therac-25 medical accelerator disaster

Therac-25 = a machine for radiotheratpy

- between 1985 and 1987 (at least) six accidents involving enormous radiation overdoses to patients
- Half of these patients died due to the overdoses

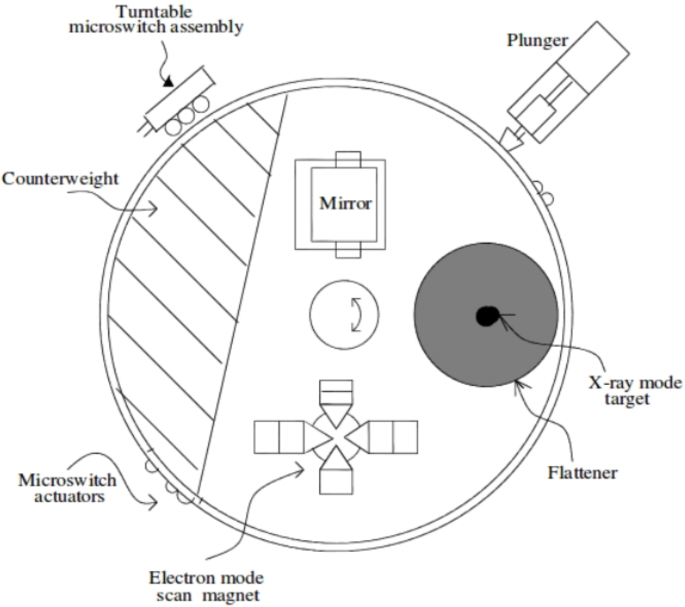


Therac-25 – the modes

1. electron mode
   - electron beam (low current)
   - various levels of energy (5 to 25-MeV)
   - scanning magnets used to spread the beam to a safe concentration
2. photon mode
   - only one level of energy (25-MeV), much larger electron-beam current
   - electron beam strikes a metal foil to produce X-rays (photons)
   - the X-ray beam is "flattened" by a device below the foil
3. light mode – just light beam used to illuminate the field on the surface of the patient's body that will be treated

All devices placed on a turntable, supposed to be rotated to the correct position before the beam is started up

Therac-25 – turntable



The Software

The software responsible for

- Operator
  - Monitoring input and editing changes from an operator
  - Updating the screen to show current status of machine
  - Printing in response to an operator commands
- Machine
  - monitoring the machine status
  - placement of turntable
  - strength and shape of beam
  - operation of bending and scanning magnets
  - setting the machine up for the specified treatment
  - turning the beam on
  - turning the beam off (after treatment, on operator command, or if a malfunction is detected)

Software running several safety critical tasks in parallel!

Insufficient hardware protection (as opposed to previous models)!!

Therac-25 – software

- The Therac-25 runs on a real-time operating system
- Four major components of software: stored data, a scheduler, a set of tasks, and interrupt services (e.g. the computer clock and handling of computer-hardware-generated errors)
- The software segregated the tasks above into
  - critical tasks: e.g. setup and operation of the beam
  - non-critical tasks: e.g. monitoring the keyboard
- The scheduler directs all non-interrupt events and orders simultaneous events
- Every 0.1 seconds tasks are initiated and critical tasks are executed first, with non-critical tasks taking up any remaining time

Communication between tasks based on shared variables (without proper atomic test-and-set instructions)

What happened?

There were several accidents due to various bugs in software

One of them proceeded as follows (much simplified):

- the operator entered parameters for X-rays treatment
- the machine started to set up for the treatment
- the operator changed the mode from X-rays to electron (within the interval from 1s to 8s from the end of the original editing)
- the patient received X-ray “treatment” with turntable in the electron position (i.e. unshielded)

The cause:

- The turntable and treatment parameters were set by *different* concurrent procedures HAND and DATENT, respectively.
- If the change in parameters came in the “right” time, only HAND reacted to the change.

Patriot missile mistiming

vs

Patriot missile mistiming

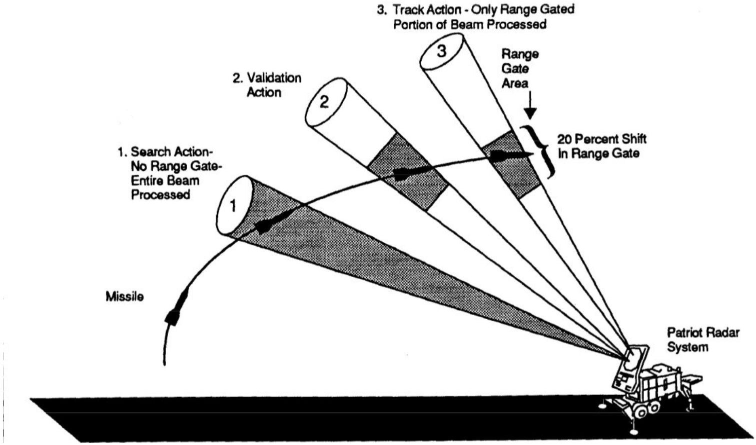
- Patriot – Air defense missile system
- Failed to intercept a scud missile on February 25, 1991 at Dhahran, Saudi Arabia

  (missile hit US army barracks, 28 persons killed)
- The problem was caused by incorrect measurement of time

Simplified principle of function:

- Patriot's radar detects an airborne object
- the object is identified as a scud missile (according to speed, size, etc.)
- the range gate computes an area in the air space where the system should next look for it
- finding the object in the calculated area confirms that it is a scud
- then the scud is intercepted

Patriot Missile Mistiming



Patriot Missile Mistiming

Prediction of the new area:

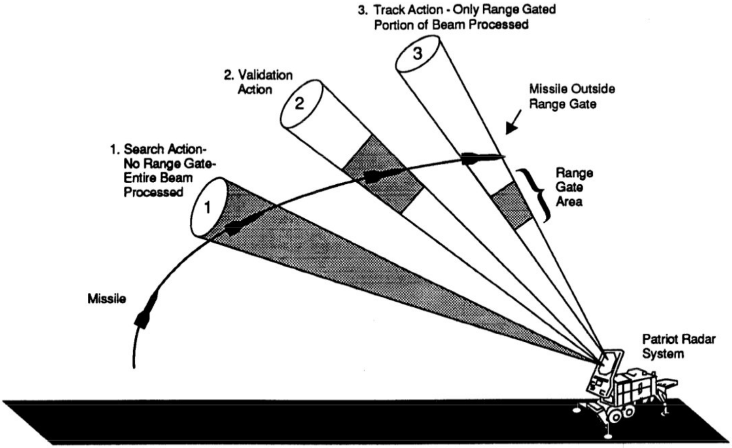
- a function of *velocity* and *time* of the last radar detection
- velocity represented as a real number
- **the current time kept by incrementing whole number counter counting tenths of seconds**
- computation in 24bit fixed floating point numbers

The time converted to 24bit real number and multiplied with 1/10 represented in 24bit (i.e. the real value of 1/10 was 0.099999905)

- the system was already running for 100 hours, i.e. the counter value was 360000, i.e. $360000 \cdot 0.099999905 = 35999.6568$
- the error was 0.3432 seconds, which means 687 m off MACH 5 scud missile
- the problem was not only in wrong conversion but in the fact that at some points correct conversion was used (after incomplete bug fix), so the errors did not cancel out

As a result, the tracking gate looked into wrong area

Patriot Missile Mistiming



(Rough) Course Outline

- Real-time scheduling
  - Time and priority driven
  - Resource control
  - Multi-processor (a bit)

- A little bit on programming real-time systems
  - Real-time operating systems
  - Real-time programming languages

Outline – Scheduling

The Scheduling problem:

Input:

- available processors, resources
- set of tasks/jobs

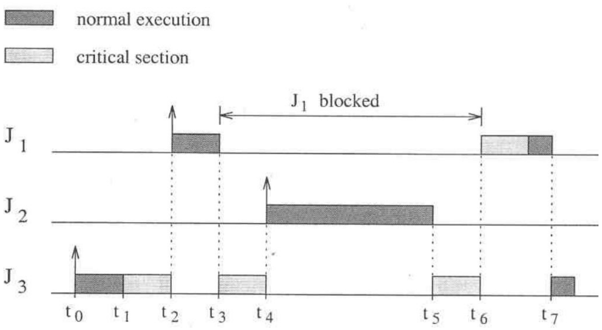
  with their requirements, deadlines, etc.

Question: How to assign processors and resources to tasks/jobs so that all requirements are met?

Example:

- 1 processor, one critical section shared by job 1 and job 3
- job 1: release time 1, computation time 4, deadline 8
- job 2: release time 1, computation time 2, deadline 5
- job 3: release time 0, computation time 3, deadline 4
- ...

Outline – Scheduling

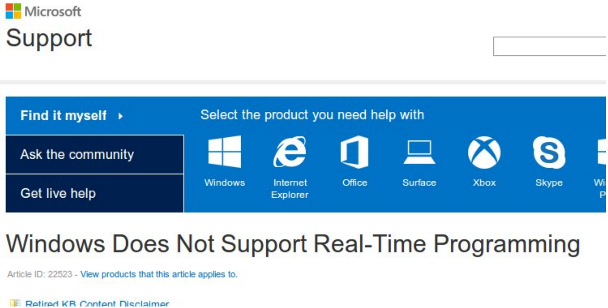


- We consider a formal model of systems with parallel jobs that possibly contend for shared resources

  consider periodic as well as aperiodic jobs
- Consider various algorithms that schedule jobs to meet their timing constraints

  offline and online algorithms, RM, EDF, etc.

Outline – Programming



Basic information about RTOS and RT programming languages

- RTOS – overview
  - real-time in non-real-time operating systems
  - **implementation of theoretical concepts in freeRTOS**
- RT in programming languages – short overview

Real-Time Scheduling

Formal Model

[Some parts of this lecture are based on a real-time systems course of Colin Perkins

http://csperkins.org/teaching/rtes/index.html]

Real-Time Scheduling – Formal Model

- Introduce an abstract model of real-time systems
  - abstracts away unessential details
  - sets up consistent terminology
- Three components of the model
  - A workload model that describes applications supported by the system
    i.e. jobs, tasks, ...
  - A resource model that describes the system resources available to applications
    i.e. processors, passive resources, ...
  - Algorithms that define how the application uses the resources at all times
    i.e. scheduling and resource access protocols

Basic Notions

- A *job* is a unit of work that is scheduled and executed by a system

  compute a control law, transform sensor data, etc.

- A *task* is a set of related jobs which jointly provide some system function

  check temperature periodically, keep a steady flow of water

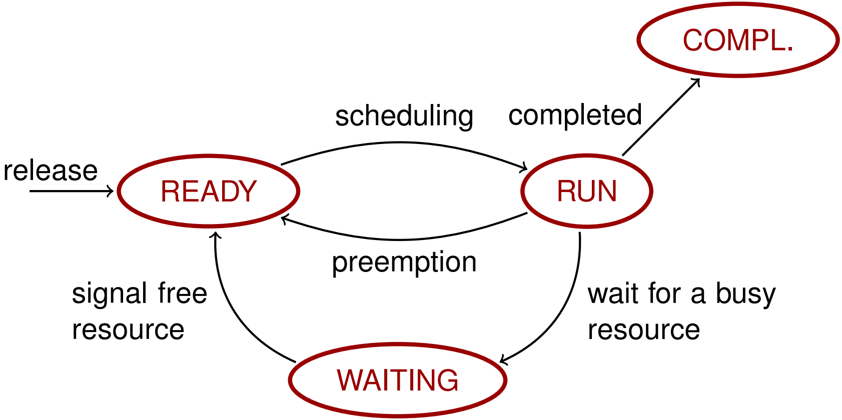
- A job executes on a *processor*

  CPU, transmission link in a network, database server, etc.

- A job may use some (shared) passive *resources*

  file, database lock, shared variable etc.

Life Cycle of a Job



Jobs – Parameters

We consider finite, or countably infinte number of jobs $J_1, J_2, \ldots$

Each job has several parameters.

There are four types of job parameters:

- temporal
  - release time, execution time, deadlines
- functional
  - Laxity type: hard and soft real-time
  - preemptability, (criticality)
- interconnection
  - precedence constraints
- resource
  - usage of processors and passive resources

Job Parameters – Execution Time

Execution time e_i **of a job** J_i – the amount of time required to complete the execution of J_i when it executes alone and has all necessary resources

- Value of e_i depends upon complexity of the job and speed of the processor on which it executes; may change for various reasons:
  - Conditional branches
  - Caches, pipelines, etc.
  - ...
- **Execution times fall into an interval** $[e_i^-, e_i^+]$; we assume that we know this interval (WCET analysis) but not necessarily e_i

We usually validate the system using only e_i^+ for each job i.e. assume $e_i = e_i^+$

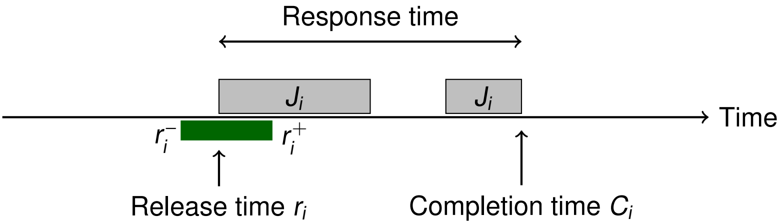
Job Parameters – Release and Response Time

Release time r_i – the instant in time when a job J_i becomes available for execution

- Release time may *jitter*, only an interval $[r_i^-, r_i^+]$ is known
- A job can be executed at any time at, or after, its release time, provided its processor and resource demands are met

Completion time C_i – the instant in time when a job completes its execution

Response time – the difference $C_i - r_i$ between the completion time and the release time

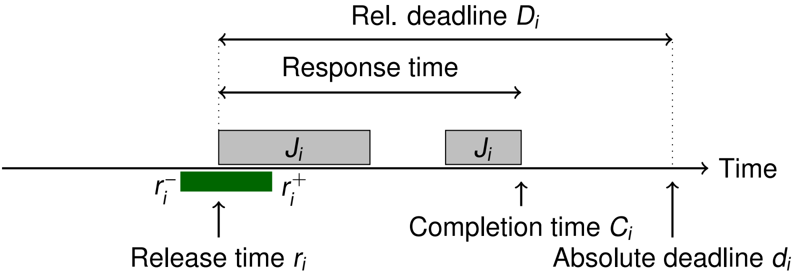


Job Parameters – Deadlines

Absolute deadline d_i – the instant in time by which a job must be completed

Relative deadline D_i – the maximum allowable response time i.e. $D_i = d_i - r_i$

Feasible interval is the interval $(r_i, d_i]$



A *timing constraint* of a job is specified using release time together with relative and absolute deadlines.

Laxity Type – Hard Real-Time

A **hard real-time constraint** specifies that a job should never miss its deadline.

Examples: Flight control, railway signaling, anti-lock brakes, etc.

Several more precise definitions occur in literature:

- A timing constraint is hard if the failure to meet it is considered a fatal error

  e.g. a bomb is dropped too late and hits civilians

- A timing constraint is hard if the usefulness of the results falls off abruptly (may even become negative) at the deadline

  Here the nature of abruptness allows to soften the constraint

Definition 5

A *timing constraint is hard* if the user requires *formal validation* that the job meets its timing constraint.

Laxity Type – Soft Real-Time

A **soft real-time constraint** specifies that a job could occasionally miss its deadline

Examples: stock trading, multimedia, etc.

Several more precise definitions occur in literature:

- A timing constraint is soft if the failure to meet it is undesirable but acceptable if the probability is low
- A timing constraint is soft if the usefulness of the results decreases at a slower rate with *tardiness* of the job

  e.g. the probability that a response time exceeds 50 ms is less than 0.2

Definition 6

A *timing constraint is soft* if either validation is not required, or only a demonstration that a *statistical constraint* is met suffices.

Jobs – Preemptability

Jobs may be interrupted by higher priority jobs

- A job is *preemptable* if its execution can be interrupted
- A job is *non-preemptable* if it must run to completion once started

  (Some preemptable jobs have periods during which they cannot be preempted)
- The *context switch time* is the time to switch between jobs

  (Most of the time we assume that this time is negligible)

Reasons for preemptability:

- Jobs may have different levels of criticality

  e.g. brakes vs radio tunning
- Priorities may make part of scheduling algorithm

  e.g. resource access control algorithms

Jobs – Precedence Constraints

Jobs may be constrained to execute in a particular order

- This is known as a *precedence constraint*
- A job J_i is a *predecessor* of another job J_k and J_k a *successor* of J_i (denoted by $J_i < J_k$) if J_k cannot begin execution until the execution of J_i completes
- J_i is an *immediate predecessor* of J_k if $J_i < J_k$ and there is no other job J_j such that $J_i < J_j < J_k$
- J_i and J_k are *independent* when neither $J_i < J_k$ nor $J_k < J_i$

A job with a precedence constraint becomes ready for execution when its release time has passed and when all predecessors have completed.

Example: authentication before retrieving an information, a signal processing task in radar surveillance system precedes a tracker task

Tasks – Modeling Reactive Systems

Reactive systems – run for unlimited amount of time

A system parameter: number of tasks

- may be known in advance (flight control)
- may change during computation (air traffic control)

We consider three types of tasks

- Periodic – jobs executed at regular intervals, hard deadlines
- Aperiodic – jobs executed in random intervals, soft deadlines
- Sporadic – jobs executed in random intervals, hard deadlines

... precise definitions later.

Processors

A processor, P, is an *active* component on which jobs are scheduled

The general case considered in literature:

m processors $P_1, \dots, P_m$, each P_i has its *type* and *speed*.

We mostly concentrate on *single processor* scheduling

- Efficient scheduling algorithms
- In a sense subsumes multiprocessor scheduling where tasks are assigned *statically* to individual processors

  i.e. all jobs of every task are assigned to a single processor

Multi-processor scheduling is a rich area of current research, we touch it only lightly (later).

Resources

A resource, R, is a *passive* entity upon which jobs may depend

In general, **we consider n resources $R_1, \ldots, R_n$ of distinct types**

Each R_i is used in a mutually exclusive manner

- A job that acquires a free resource locks the resource
- Jobs that need a busy resource have to wait until the resource is released
- Once released, the resource may be used by another job

  (i.e. it is not consumed)

(More generally, each resource may be used by k jobs concurrently, i.e., there are k units of the resource)

Resource requirements of a job specify

- which resources are used by the job
- the time interval(s) during which each resource is required

  (precise definitions later)

Scheduling

Schedule assigns, in every time instant, processors and resources to jobs.

More formally, a schedule is a function

$$\sigma : \{J_1, \ldots\} \times \mathbb{R}_0^+ \to \mathcal{P}(\{P_1, \ldots, P_m, R_1, \ldots, R_n\})$$

so that for every $t \in \mathbb{R}_0^+$ there are rational $0 \leq t_1 \leq t < t_2$ such that $\sigma(J_i, \cdot)$ is constant on $[t_1, t_2)$.

(We also assume that there is the least time quantum in which scheduler does not change its decisions, i.e. each of the intervals $[t_1, t_2)$ is larger than a fixed $\varepsilon > 0$.)

Valid and Feasible Schedule

A schedule is *valid* if it satisfies the following conditions:

- Every processor is assigned to at most one job at any time
- Every job is assigned to at most one processor at any time
- No job is scheduled before its release time
- The total amount of processor time assigned to a given job is equal to its actual execution time
- *All the precedence and resource usage constraints are satisfied*

A schedule is *feasible* if *all jobs with hard real-time constraints* complete before their deadlines

A set of jobs is *schedulable* if there is a feasible schedule for the set.

Scheduling – Algorithms

Scheduling algorithm computes a schedule for a set of jobs

A set of jobs is *schedulable according to a scheduling algorithm* if the algorithm produces a feasible schedule

Definition 7

A scheduling algorithm is optimal if it always produces a feasible schedule whenever such a schedule exists.

Real-Time Scheduling

Individual Jobs

Scheduling of Individual Jobs

We start with scheduling of finite sets of jobs $\{J_1, \ldots, J_m\}$ for execution on **single processor** systems.

Each J_i has a release time r_i, an execution time e_i and a relative deadline D_i.

We assume hard real-time constraints.

The question: Is there an optimal scheduling algorithm?

We proceed in the direction of growing generality:

1. No resources, independent, synchronized (i.e. $r_i = 0$ for all i)
2. No resources, independent but not synchronized
3. No resources but possibly dependent
4. The general case

No resources, Independent, Synchronized

	J_1	J_2	J_3	J_4	J_5
e_i	1	1	1	3	2
d_i	3	10	7	8	5

Is there a feasible schedule?

Note: Preemption does not help in synchronized case.

Theorem 8

If there are no resource contentions, then executing independent jobs in the order of non-decreasing deadline (EDD) produces a feasible schedule (if it exists).

Proof.

Let σ be a schedule. **Inversion** is a pair (J_a, J_b) such that J_a precedes J_b in σ but $d_b < d_a$.

Note that σ is EDD iff it does not contain any inversion.

Proof cont.

Assume $k > 0$ inversions in σ.

Let (J_a, J_b) be an inversion such that J_a is scheduled right before J_b.
There is always at least one such inversion (homework).

Let $t_a < t_b$ be the time instants when J_a, J_b start to be executed in σ.
Recall: C_a, C_b are completion times of J_a, J_b, and e_a, e_b are execution times.

Note that $C_a \leq d_a$ and that $C_b \leq d_b < d_a$.

Define a new schedule σ' in which:

- All jobs except J_a, J_b are scheduled as in σ,
- J_b starts at t_a,
- J_a starts at $t_a + e_b$.

Observe that σ' is still feasible:

- J_b is completed at $t_a + e_b < t_a + e_b + e_a = t_b + e_b = C_b \leq d_b$
- J_a is completed at $t_a + e_b + e_a = C_b \leq d_b < d_a$

Note that σ' has $k - 1$ inversions. By repeating the above procedure k times, we obtain an EDD schedule. □

No resources, Independent, Synchronized

Is there any simple schedulability test?

$\{J_1, \ldots, J_n\}$ where $d_1 \leq \cdots \leq d_n$ is schedulable iff
$\forall i \in \{1, \ldots, n\} \; : \; \sum_{k=1}^{i} e_k \leq d_i$

No resources, Independent (No Synchro)

	J_1	J_2	J_3
r_i	0	0	2
e_i	1	2	2
d_i	2	5	4

- find a (feasible) schedule (with and without preemption)
- determine response time of each job in your schedule

Preemption makes a difference.

No resources, Independent (No Synchro)

Earliest Deadline First (EDF) scheduling:

At any time instant, a job with the earliest absolute deadline is executed

Here EDF works in the preemptive case but not in the non-preemptive one.

	J_1	J_2
r_i	0	1
e_i	4	2
d_i	7	5

No Resources, Dependent (No Synchro)

Theorem 9

If there are no resource contentions, jobs are independent and preemption is allowed, the EDF algorithm finds a feasible schedule (if it exists).

Proof.

We show that any feasible schedule σ can be transformed in finitely many steps to EDF schedule which is feasible.

Let σ be a feasible schedule but not EDF. Assume, w.l.o.g., that for every $k \in \mathbb{N}$ at most one job is executed in the interval $[k, k+1)$ and that all release times and deadlines are in $\mathbb{N}$.

(Otherwise rescale by the least common multiple.)

No Resources, Dependent (No Synchro)

Proof cont.

We say that σ violates EDF at k if there are two jobs J_a and J_b that satisfy:

- J_a and J_b are ready for execution at k
- J_a is executed in $[k, k+1)$
- $d_b < d_a$

Let $k \in \mathbb{N}$ be the *least* time instant such that σ violates EDF at k as witnessed by jobs J_a and J_b.
Assume, w.l.o.g. that J_b has the minimum deadline among all jobs ready for execution at k.

There is $k < \ell < d_b$ such that J_b is executed in $[\ell, \ell+1)$.

Let us define a new schedule σ' which is the same as σ except:

- executes J_b in $[k, k+1)$
- executes J_a in $[\ell, \ell+1)$

Then σ' is feasible and does not violate EDF at any $k' \leq k$.

Finitely many steps transform any feasible schedule to EDF. □

No resources, Independent (No Synchro)

The **non-preemptive** case is NP-hard.

Heuristics are needed, such as the **Spring algorithm**, that usually work in much more general setting (with resources etc.)

Use the notion of *partial schedule* where only a subset of tasks has been scheduled.

Exhaustive search through partial schedules

- start with an empty schedule
- in every step either
  - add a job which maximizes a *heuristic function H* among jobs that have not yet been tried in this partial schedule
  - or backtrack if there is no such a job
- After failure, backtrack to previous partial schedule

Heuristic function identifies plausible jobs to be scheduled (earliest release, earliest deadline, etc.)

No resources, Dependent (No Synchro)

Theorem 10

Assume that there are no resource contentions and jobs are preemptable. There is a polynomial time algorithm which decides whether a feasible schedule exists and if yes, then computes one.

Idea: Reduce to independent jobs by changing release times and deadlines. Then use EDF.

Observe that if $J_i < J_k$ then replacing

- r_k with $\max\{r_k, r_i + e_i\}$
  (J_k cannot be scheduled for execution before $r_i + e_i$ because J_i cannot be finished before $r_i + e_i$)
- d_i with $\min\{d_i, d_k - e_k\}$
  (J_i must be finished before $d_k - e_k$ so that J_k can be finished before d_k)

does not change feasibility.

Replace systematically according to the precedence relation.

No Resources, Dependent (No Synchro)

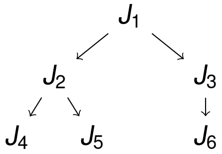
Define r_k^*, d_k^* systematically as follows:

- Pick J_k whose all predecessors have been processed and compute $r_k^* := \max\{r_k, \max_{J_i < J_k} r_i^* + e_i\}$. Repeat for all jobs.
- Pick J_k whose all successors have been processed and compute $d_k^* := \min\{d_k, \min_{J_k < J_i} d_i^* - e_i\}$. Repeat for all jobs.

Example:

	J_1	J_2	J_3	J_4	J_5	J_6
e_i	1	1	1	1	1	1
d_i	2	5	4	3	5	6

Dependencies:



Do you need the precedence constraints?

No Resources, Dependent (No Synchro)

Define r_k^*, d_k^* systematically as follows:

- Pick J_k whose all predecessors have been processed and compute $r_k^* := \max\{r_k, \max_{J_i < J_k} r_i^* + e_i\}$. Repeat for all jobs.
- Pick J_k whose all successors have been processed and compute $d_k^* := \min\{d_k, \min_{J_k < J_i} d_i^* - e_i\}$. Repeat for all jobs.

This gives a new set of jobs $J_1^*, \ldots, J_m^*$ where each J_k^* has the release time r_k^* and the absolute deadline d_k^*.

We impose **no precedence constraints** on $J_1^*, \ldots, J_m^*$.

Lemma 11

$\{J_1, \ldots, J_m\}$ is feasible iff $\{J_1^*, \ldots, J_m^*\}$ is feasible. If EDF schedule is feasible on $\{J_1^*, \ldots, J_m^*\}$, then the same schedule is feasible on $\{J_1, \ldots, J_m\}$.

The same schedule means that whenever J_i^* is scheduled at time t, then J_i is scheduled at time t.

No Resources, Dependent (No Synchro)

Recall: $r^*_k := \max\{r_k, \max_{J_i < J_k} r^*_i + e_i\}$ and
$d^*_k := \min\{d_k, \min_{J_k < J_i} d^*_i - e_i\}$

Proof of Lemma 11.

$\Rightarrow$: It is easy to show that in *no feasible schedule* on $\{J_1, \ldots, J_m\}$ any job J_k can be released before r^*_k and completed before d^*_k (otherwise, precedence constraints would be violated).

$\Leftarrow$: Assume that EDF σ is feasible on $\{J^*_1, \ldots, J^*_m\}$. Let us use σ on $\{J_1, \ldots, J_m\}$.

I.e. J_i is executed iff J^*_i is executed.

Timing constraints of $\{J_1, \ldots, J_m\}$ are satisfied since $r_k \leq r^*_k$ and $d_k \geq d^*_k$ for all k.

Precedence constraints: Assume that $J_s < J_t$. Then J^*_s executes completely before J^*_t since $r^*_s < r^*_s + e_s \leq r^*_t$ and $d^*_s \leq d^*_t - e_t < d^*_t$ and σ is EDF on $\{J^*_1 \ldots, J^*_m\}$.

Resources, Dependent, Not Synchronized

Even the preemptive case is NP-hard

- reduce the non-preemptive case without resources to the preemptive with resources
- Use a common resource R.
  - Whenever a job starts its execution it locks the resource R.
  - Whenever a job finishes its execution it releases the resourse R.

Could be solved using heuristics, e.g. the Spring algorithm.

Real-Time Scheduling

Scheduling of Reactive Systems

[Some parts of this lecture are based on a real-time systems course of Colin Perkins

http://csperkins.org/teaching/rtes/index.html]

Reminder of Basic Notions

- Jobs are executed on processors and need resources
- Parameters of jobs
  - temporal:
    - release time – r_i
    - execution time – e_i
    - absolute deadline – d_i
    - derived params: relative deadline (D_i), completion time, response time, ...
  - functional:
    - laxity type: hard vs soft
    - preemptability
  - interconnection
    - precedence constraints (independence)
  - resource
    - what resources and when are used by the job
- Tasks = sets of jobs

Reminder of Basic Notions

- Schedule assigns, in every time instant, processors and resources to jobs
- valid schedule = correct (common sense)
- Feasible schedule = valid and all hard real-time tasks meet deadlines
- Set of jobs is schedulable if there is a feasible schedule for it

- Scheduling algorithm computes a schedule for a set of jobs
- Scheduling algorithm is optimal if it always produces a feasible schedule whenever such a schedule exists, and if a cost function is given, minimizes the cost

Scheduling Reactive Systems

We have considered scheduling of individual jobs

From this point on we concentrate on reactive systems

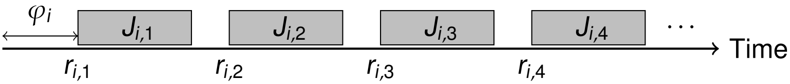
i.e. systems that run for unlimited amount of time

Recall that a task is a set of related jobs that jointly provide some system function.

- We consider various types of tasks
  - Periodic
  - Aperiodic
  - Sporadic
- Differ in execution time patterns for jobs in the tasks
- Must be modeled differently
  - Differing scheduling algorithms
  - Differing impact on system performance
  - Differing constraints on scheduling

Periodic Tasks

- A set of jobs that are executed repeatedly at regular time intervals can be modeled as a *periodic task*



- Each periodic task T_i is a sequence of jobs $J_{i,1}, J_{i,2}, \ldots J_{i,n}, \ldots$
  - The *phase* φ_i of a task T_i is *the release time* $r_{i,1}$ *of the first job* $J_{i,1}$ in the task T_i ;
    tasks are *in phase* if their phases are equal
  - The *period* p_i of a task T_i is *the minimum length of all time intervals between release times of consecutive jobs* in T_i
  - The *execution time* e_i of a task T_i is *the maximum execution time of all jobs* in T_i
  - The *relative deadline* D_i is relative deadline of all jobs in T_i

(The period and execution time of every periodic task in the system are known with reasonable accuracy at all times)

Periodic Tasks – Notation

The 4-tuple $T_i = (\varphi_i, p_i, e_i, D_i)$ refers to a periodic task T_i with phase φ_i, period p_i, execution time e_i, and relative deadline D_i

For example: jobs of $T_1 = (1, 10, 3, 6)$ are

- released at times 1, 11, 21, ...,
- execute for 3 time units,
- have to be finished in 6 time units (the first by 7, the second by 17, ...)

Default phase of T_i is $\varphi_i = 0$ and default relative deadline is $d_i = p_i$

$T_2 = (10, 3, 6)$ satisfies $\varphi = 0$, $p_i = 10$, $e_i = 3$, $D_i = 6$, i.e. jobs of T_2 are

- released at times 0, 10, 20, ...,
- execute for 3 time units,
- have to be finished in 6 time units (the first by 6, the second by 16, ...)

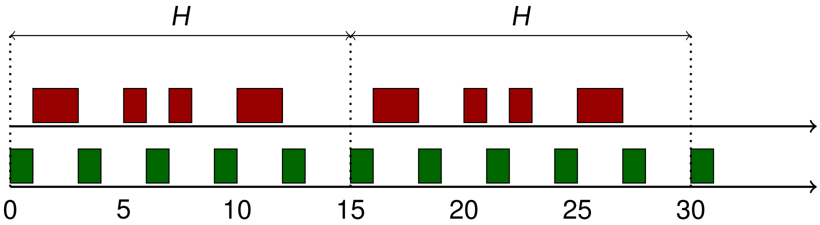
$T_3 = (10, 3)$ satisfies $\varphi = 0$, $p_i = 10$, $e_i = 3$, $D_i = 10$, i.e. jobs of T_3 are

- released at times 0, 10, 20, ...,
- execute for 3 time units,
- have to be finished in 10 time units (the first by 10, the second by 20, ...)

Periodic Tasks – Hyperperiod

The *hyper-period* H of a set of periodic tasks is the least common multiple of their periods

If tasks are in phase, then H is the time instant after which the pattern of job release/execution times starts to repeat



Aperiodic and Sporadic Tasks

- Many real-time systems are required to respond to external events
- The tasks resulting from such events are *sporadic* and *aperiodic* tasks
  - Sporadic tasks – hard deadlines of jobs
    e.g. autopilot on/off in aircraft
  - Aperiodic tasks – soft deadlines of jobs
    e.g. sensitivity adjustment of radar surveilance system
- Inter-arrival times between consecutive jobs are identically and independently distributed according to a probability distribution $A(x)$
- Execution times of jobs are identically and independently distributed according to a probability distribution $B(x)$
- In the case of sporadic tasks, the usual goal is to decide, whether a newly released job can be feasibly scheduled with the remaining jobs in the system
- In the case of aperiodic tasks, the usual goal is to minimize the average response time

Scheduling – Classification of Algorithms

- Off-line vs Online
  - Off-line – sched. algorithm is executed on the whole task set before activation
  - Online – schedule is updated at runtime every time a new task enters the system
- Optimal vs Heuristic
  - Optimal – algorithm computes a feasible schedule and minimizes cost of soft real-time jobs
  - Heuristic – algorithm is guided by heuristic function; tends towards optimal schedule, may not give one

The main division is on

- Clock-Driven
- Priority-Driven

Scheduling – Clock-Driven

- Decisions about what jobs execute when are made at specific time instants
  - these instants are chosen before the system begins execution
  - Usually regularly spaced, implemented using a periodic timer interrupt
  - Scheduler awakes after each interrupt, schedules jobs to execute for the next period, then blocks itself until the next interrupt

    E.g. the helicopter example with the interrupt every 1/180 th of a second
- Typically in clock-driven systems:
  - All parameters of the real-time jobs are fixed and known
  - A schedule of the jobs is computed off-line and is stored for use at runtime; thus scheduling overhead at run-time can be minimized
  - Simple and straight-forward, not flexible

Scheduling – Priority-Driven

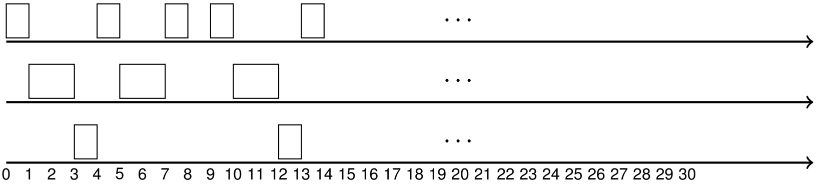
- Assign priorities to jobs, based on some algorithm
- Make scheduling decisions based on the priorities, when events such as releases and job completions occur
  - Priority scheduling algorithms are *event-driven*
  - Jobs are placed in one or more queues; at each event, the ready job with the highest priority is executed

  (The assignment of jobs to priority queues, along with rules such as whether preemption is allowed, completely defines a priority-driven alg.)
- Priority-driven algs. make *locally optimal* scheduling decisions
  - Locally optimal scheduling is often *not* globally optimal
  - Priority-driven algorithms *never* intentionally leave idle processors
- Typically in priority-driven systems:
  - Some parameters do not have to be fixed or known
  - A schedule is computed online; usually results in larger scheduling overhead as opposed to clock-driven scheduling
  - Flexible – easy to add/remove tasks or modify parameters

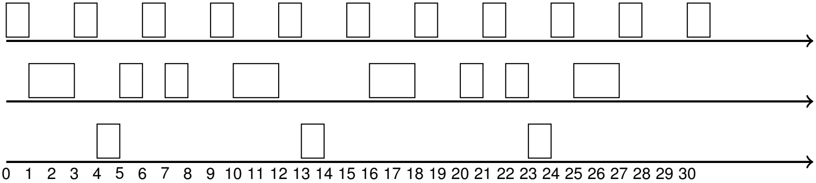
Clock-Driven & Priority-Driven Example

	T_1	T_2	T_3
p_i	3	5	10
e_i	1	2	1

Clock-Driven:



Priority-driven: $T_1 \succ T_2 \succ T_3$



Real-Time Scheduling

Scheduling of Reactive Systems

Clock-Driven Scheduling

Current Assumptions

- Fixed number, n, of periodic tasks $T_1, \dots, T_n$
- Parameters of periodic tasks are known a priori
  - Execution time $e_{i,k}$ of each job $J_{i,k}$ in a task T_i is fixed
  - For a job $J_{i,k}$ in a task T_i we have
    - $r_{i,1} = \varphi_i = 0$ (i.e., synchronized)
    - $r_{i,k} = r_{i,k-1} + p_i$
- We allow aperiodic jobs
  - assume that the system maintains a single queue for aperiodic jobs
  - Whenever the processor is available for aperiodic jobs, the job at the head of this queue is executed
- We treat sporadic jobs later

Static, Clock-Driven Scheduler

- Construct a *static schedule* offline
  - The schedule specifies exactly when each job executes
  - The amount of time allocated to every job is equal to its execution time
  - The schedule repeats each hyperperiod

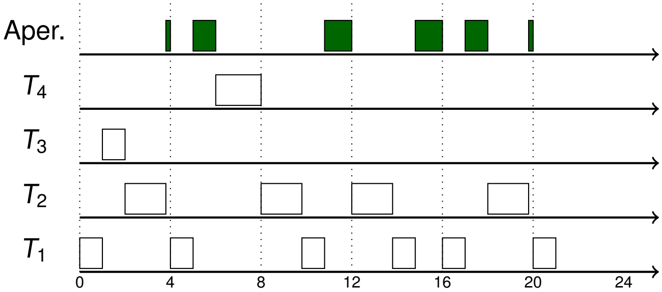
    i.e. it suffices to compute the schedule up to hyperperiod
- Can use complex algorithms offline
  - Runtime of the scheduling algorithm is not relevant
  - Can compute a schedule that optimizes some characteristics of the system

    e.g. a schedule where the idle periods are nearly periodic (useful to accommodate aperiodic jobs)

Example

$T_1 = (4, 1)$, $T_2 = (5, 1.8)$, $T_3 = (20, 1)$, $T_4 = (20, 2)$

Hyperperiod $H = 20$



Implementation of Static Scheduler

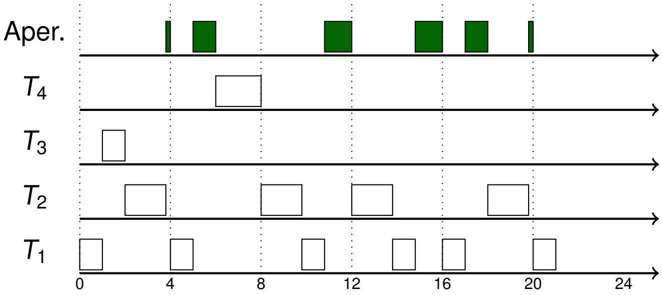
- Store pre-computed schedule as a table
  - Each entry $(t_k, T(t_k))$ gives
    - a decision time t_k
    - scheduling decision $T(t_k)$ which is either a task to be executed, or idle (denoted by I)
- The system creates all tasks that are to be executed:
  - Allocates memory for the code and data
  - Brings the code into memory
- Scheduler sets the hardware timer to interrupt at the first decision time $t_0 = 0$
- On receipt of an interrupt at t_k:
  - Scheduler sets the timer interrupt to t_{k+1}
  - If previous task overrunning, handle failure
  - If $T(t_k) = I$ and aperiodic job waiting, start executing it
  - Otherwise, start executing the next job in $T(t_k)$

k	t_k	$T(t_k)$
0	0.0	T_1
1	1.0	T_3
2	2.0	T_2
3	3.8	I
4	4.0	T_1
5	5.0	I
6	6.0	T_4
7	8.0	T_2
8	9.8	T_1
9	10.8	I
10	12.0	T_2
11	13.8	T_1
12	14.8	I
13	17.0	T_1
14	17.0	I
15	18.0	T_2
16	19.8	I

Example

$T_1 = (4, 1)$, $T_2 = (5, 1.8)$, $T_3 = (20, 1)$, $T_4 = (20, 2)$

Hyperperiod $H = 20$



t_k	0.0	1.0	2.0	3.8	4.0	5.0	6.0	$\cdots$
$T(t_k)$	T_1	T_3	T_2	I	T_1	I	T_4	$\cdots$

Frame Based Scheduling

- Arbitrary table-driven cyclic schedules flexible, but inefficient
  - Relies on accurate timer interrupts, based on execution times of tasks
  - High scheduling overhead
- Easier to implement if a structure is imposed
  - Make scheduling decisions at periodic intervals *(frames)* of length f
  - Execute a fixed list of jobs within each frame; **no preemption within frames**
- Gives two benefits:
  - Scheduler can easily check for overruns and missed deadlines at the end of each frame.
  - Can use a periodic clock interrupt, rather than programmable timer.

Frame Based Scheduling – Cyclic Executive

- Modify previous table-driven scheduler to be frame based
- Table that drives the scheduler has F entries, where $F = H/f$
  - The k-th entry $L(k)$ lists the names of the jobs that are to be scheduled in frame k ($L(k)$ is called *scheduling block*)
  - Each job is implemented by a procedure
- Cyclic executive executed by the clock interrupt that signals the start of a frame:
  - If an aperiodic job is executing, preempts it; if a periodic overruns, handles the overrun
  - Determines the appropriate scheduling block for this frame
  - Executes the jobs in the scheduling block
  - Executes jobs from the head of the aperiodic job queue for the remainder of the frame
- Less overhead than pure table driven cyclic scheduler, since only interrupted on frame boundaries, rather than on each job

Frame Based Scheduling – Frame Size

How to choose the frame length?
(Assume that periods are in $\mathbb{N}$ and choose frame sizes in $\mathbb{N}$.)

1. Necessary condition for avoiding preemption of jobs is

$$f \geq \max_i e_i$$

   (i.e. we want each job to have a chance to finish within a frame)

2. To minimize the number of entries in the cyclic schedule, the hyper-period should be an integer multiple of the frame size, i.e.

$$\exists i : p_i \mod f = 0$$

3. To allow scheduler to check that jobs complete by their deadline, at least one frame should lie between release time of a job and its deadline, which is equivalent to

$$\forall i : 2 * f - gcd(p_i, f) \leq D_i$$

All three constraints should be satisfied.

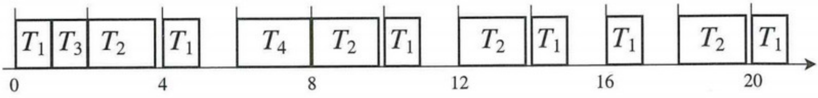
Frame Based Scheduling – Frame Size – Example

1. $f \geq \max_i e_i$
2. $\exists i : p_i \mod f = 0$
3. $\forall i : 2 * f - gcd(p_i, f) \leq D_i$

Example 12

$T_1 = (4, 1.0)$, $T_2 = (5, 1.8)$, $T_3 = (20, 1.0)$, $T_4 = (20, 2.0)$
Then $f \in \mathbb{N}$ satisfies 1.–3. iff $f = 2$.

With $f = 2$ is schedulable:



Frame Based Scheduling – Job Slices

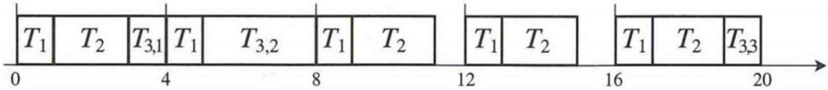
- Sometimes a system cannot meet all three frame size constraints simultaneously (and even if it meets the constraints, no non-preemptive schedule is feasible)
- Can be solved by partitioning a job with large execution time into slices with shorter execution times

  This, in effect, allows preemption of the large job

- Consider $T_1 = (4, 1)$, $T_2 = (5, 2, 7)$, $T_3 = (20, 5)$
- Cannot satisfy constraints: 1. $\Rightarrow f \geq 5$ but 3. $\Rightarrow f \leq 4$
- Solve by splitting T_3 into $T_{3,1} = (20, 1)$, $T_{3,2} = (20, 3)$, and $T_{3,3} = (20, 1)$

  (Other splits exist)

- Result can be scheduled with $f = 4$



Building a Structured Cyclic Schedule

To construct a schedule, we have to make three kinds of design decisions (that cannot be taken independently):

- Choose a frame size based on constraints
- Partition jobs into slices
- Place slices into frames

There are efficient algorithms for solving these problems based e.g. on a reduction to the network flow problem.

Scheduling Aperiodic Jobs

So far, aperiodic jobs scheduled in the background after all jobs with hard deadlines

This may unnecessarily delay aperiodic jobs

Note: There is no advantage in completing periodic jobs early

Ideally, finish periodic jobs by their respective deadlines.

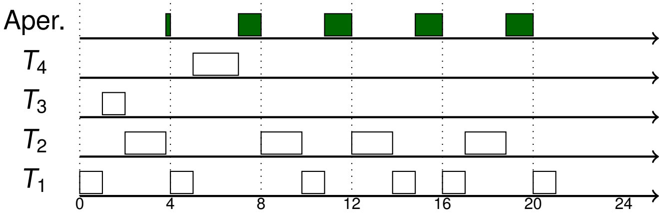
Slack Stealing:

- Slack time in a frame = the time left in the frame after all (remaining) slices execute
- Schedule aperiodic jobs ahead of periodic in the slack time of periodic jobs
  - The cyclic executive keeps track of the slack time left in each frame as the aperiodic jobs execute, preempts them with periodic jobs when there is no more slack
  - As long as there is slack remaining in a frame and the aperiodic jobs queue is non-empty, the executive executes aperiodic jobs, otherwise executes periodic
- Reduces resp. time for aper. jobs, but requires accurate timers

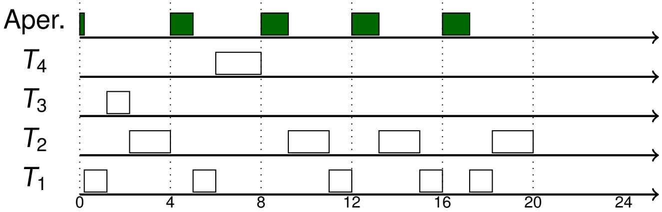
Example

Assume that the aperiodic queue is never empty.

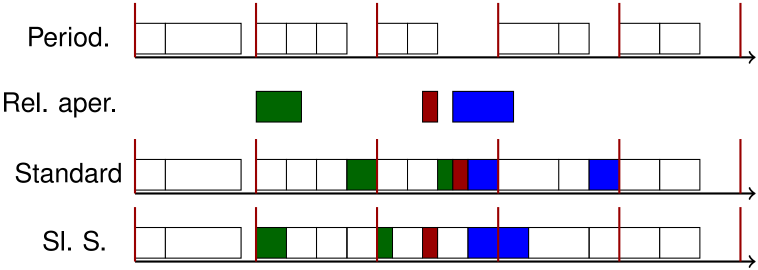
Aperiodic at the ends of frames:



Slack stealing:



Slack Stealing – cont.



Frame Based Scheduling – Sporadic Jobs

Let us allow sporadic jobs

i.e. hard real-time jobs whose release and exec. times are not known a priori

The scheduler determines whether to accept a sporadic job when it arrives (and its parameters become known)

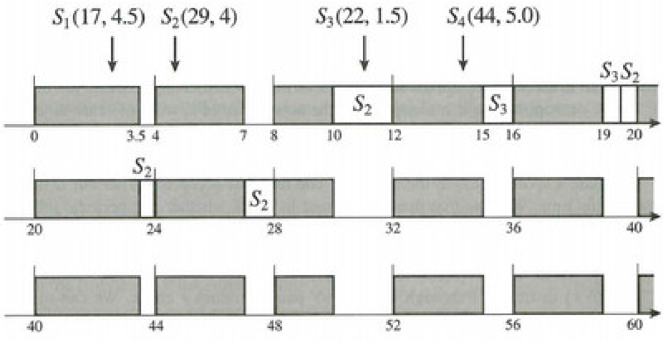
- Perform *acceptance test* to check whether the new sporadic job can be feasibly scheduled with all the jobs (periodic and sporadic) in the system at that time

  Acceptance check done at the beginning of the next frame; has to keep execution times of the parts of sporadic jobs that have already executed
- If there is sufficient slack time in the frames before the new job's deadline, the new sporadic job is accepted; otherwise, rejected
- Among themselves, sporadic jobs scheduled according to EDF

  This is optimal for sporadic jobs

Note: rejection is often better than missing deadline

e.g. a robotic arm taking defective parts off a conveyor belt: if the arm cannot meet deadline, the belt may be slowed down or stopped



- $S_1(17, 4.5)$ released at 3 with abs. deadline 17 and execution time 4.5; acceptance test at 4; must be scheduled in frames $2, 3, 4$; total slack in these frames is 4, i.e. rejected
- $S_2(29, 4)$ released at 5 with abs. deadline 29 and exec. time 4; acc. test at 8; total slack in frames 3-7 is 5.5, i.e. accepted
- $S_3(22, 1.5)$ released at 11 with abs. deadline 22 and exec. time 1.5; acc. test at 12;

  2 units of slack in frames $4, 5$ as S_3 will be executed *ahead of the remaining parts of S_2* by EDF – check whether there will be enough slack for the remaining parts of S_2, accepted
- $S_4(44, 5.0)$ is rejected (only 4.5 slack left)

Handling Overruns

Overruns may happen due to failures

e.g. unexpectedly large data over which the system operates, hardware failures, etc.

Ways to handle overruns:

- Abort the overrun job at the beginning of the next frame; log the failure; recover later

  e.g. control law computation of a robust digital controller
- Preempt the overrun job and finish it as an aperiodic job

  use this when aborting job would cause "costly" inconsistencies
- Let the overrun job finish – start of the next frame and the execution jobs scheduled for this frame are delayed

  This may cause other jobs to be delayed

  depends on application

Clock-drive Scheduling: Conclusions

Advantages:

- Conceptual simplicity
  - Complex dependencies, communication delays, and resource contention among jobs can be considered when constructing the static schedule
  - Entire schedule in a static table
  - No concurrency control or synchronization needed
- Easy to validate, test and certify

Disadvantages:

- Inflexible
  - If any parameter changes, the schedule must be usually recomputed

    Best suited for systems which are rarely modified (e.g. controllers)
  - Parameters of the jobs must be fixed

    As opposed to most priority-driven schedulers

Real-Time Scheduling

Scheduling of Reactive Systems

Priority-Driven Scheduling

Current Assumptions

- Single processor
- Fixed number, n, of *independent periodic* tasks

  i.e. there is no dependency relation among jobs

  - Jobs can be preempted at any time and never suspend themselves
  - No aperiodic and sporadic jobs
  - No resource contentions

Moreover, unless otherwise stated, we assume that

- **Scheduling decisions take place precisely at**
  - release of a job
  - completion of a job

  (and nowhere else)
- Context switch overhead is negligibly small

  i.e. assumed to be zero
- There is an unlimited number of priority levels

Fixed-Priority vs Dynamic-Priority Algorithms

A priority-driven scheduler is on-line
i.e. it does not precompute a schedule of the tasks

- It assigns priorities to jobs after they are released and places the jobs in a ready job queue in the priority order

  with the highest priority jobs at the head of the queue

- At each scheduling decision time, the scheduler updates the ready job queue and then schedules and executes the job at the head of the queue

  i.e. one of the jobs with the highest priority

Fixed-priority = *all jobs in a task* are assigned the same priority

Dynamic-priority = jobs in a task may be assigned different priorities

Note: In our case, a priority assigned to a job does not change. There are *job-level dynamic priority* algorithms that vary priorities of individual jobs – we won't consider such algorithms.

Fixed-priority Algorithms – Rate Monotonic

Best known fixed-priority algorithm is *rate monotonic (RM)* scheduling that assigns priorities to tasks based on their periods

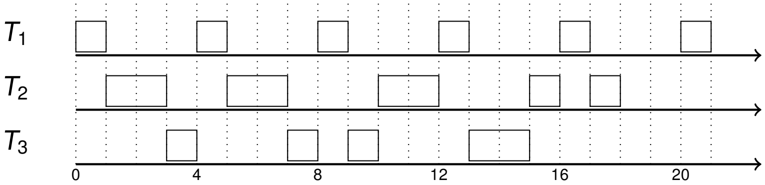
- The shorter the period, the higher the priority
- The *rate* is the inverse of the period, so jobs with higher rate have higher priority

RM is very widely studied and used

Example 13

$T_1 = (4, 1)$, $T_2 = (5, 2)$, $T_3 = (20, 5)$
with rates 1/4, 1/5, 1/20, respectively

The priorities: $T_1 \succ T_2 \succ T_3$



Fixed-priority Algorithms – Deadline Monotonic

The *deadline monotonic (DM)* algorithm assigns priorities to tasks based on their *relative deadlines*

- the shorter the deadline, the higher the priority

Observation: When relative deadline of every task matches its period, then RM and DM give the same results

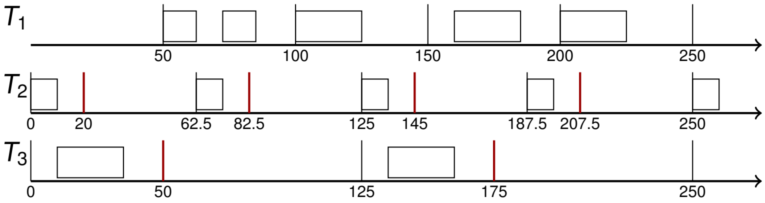
Proposition 1

When the relative deadlines are arbitrary DM can sometimes produce a feasible schedule in cases where RM cannot.

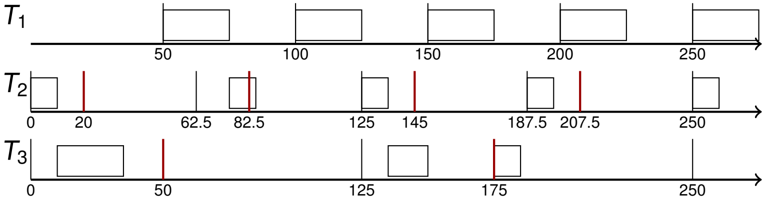
Rate Monotonic vs Deadline Monotonic

$T_1 = (50, 50, 25, 100)$, $T_2 = (0, 62.5, 10, 20)$, $T_3 = (0, 125, 25, 50)$

DM is optimal (with priorities $T_2 \succ T_3 \succ T_1$):



RM is not optimal (with priorities $T_1 \succ T_2 \succ T_3$):



Dynamic-priority Algorithms

Best known is *earliest deadline first (EDF)* that assigns priorities based on *current* (absolute) deadlines

- At the time of a scheduling decision, the job queue is ordered by earliest deadline

Another one is the *least slack time (LST)*

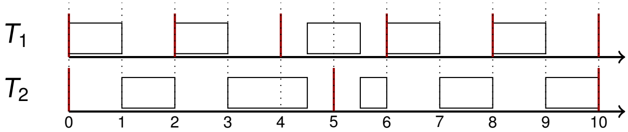
- The job queue is ordered by least slack time

Recall that the *slack time* of a job J_i at time t is equal to $d_i - t - x$ where x is the remaining computation time of J_i at time t

We focus on EDF here.

EDF – Example

$T_1 = (2, 1)$ and $T_2 = (5, 2.5)$



Note that the processor is 100% "utilized", not surprising :-)

Summary of Priority-Driven Algorithms

We consider:
Dynamic-priority:

- **EDF** = at the time of a scheduling decision, the job queue is ordered by the earliest deadline

Fixed-priority:

- **RM** = assigns priorities to tasks based on their periods
- **DM** = assigns priorities to tasks based on their relative deadlines

(In all cases, ties are broken arbitrarily.)

We consider the following questions:

- Are the algorithms optimal?
- How to efficiently (or even online) test for schedulability?

To measure abilities of scheduling algorithms and to get fast online tests of schedulability we use a notion of **utilization**

Utilization

- *Utilization u_i of a periodic task T_i* with period p_i and execution time e_i is defined by $u_i := e_i/p_i$

  u_i is the fraction of time a periodic task with period p_i and execution time e_i keeps a processor busy
- *Total utilization $U^{\mathcal{T}}$ of a set of tasks $\mathcal{T} = \{T_1, \ldots, T_n\}$* is defined as the sum of utilizations of all tasks of $\mathcal{T}$, i.e. by

$$U^{\mathcal{T}} := \sum_{i=1}^{n} u_i$$

- U is a *schedulable utilization* of an algorithm ALG if all sets of tasks $\mathcal{T}$ satisfying $U^{\mathcal{T}} \leq U$ are schedulable by ALG.

  Maximum schedulable utilization U_{ALG} of an algorithm ALG is the *supremum of schedulable utilizations of ALG*.
  - If $U^{\mathcal{T}} < U_{ALG}$, then $\mathcal{T}$ is schedulable by ALG.
  - If $U > U_{ALG}$, then there is $\mathcal{T}$ with $U^{\mathcal{T}} \leq U$ that is not schedulable by ALG.

Utilization – Example

- $T_1 = (2, 1)$ then $u_1 = \frac{1}{2}$
- $T_1 = (11, 5, 2, 4)$ then $u_1 = \frac{2}{5}$
  (i.e., the phase and deadline do not play any role)
- $\mathcal{T} = \{T_1, T_2, T_3\}$ where $T_1 = (2, 1), T_2 = (6, 1), T_3 = (8, 3)$
  then

$$U^{\mathcal{T}} = \frac{1}{2} + \frac{1}{6} + \frac{3}{8} = \frac{25}{24}$$

Real-Time Scheduling

Priority-Driven Scheduling

Dynamic-Priority

Optimality of EDF

Theorem 14

Let $\mathcal{T} = \{T_1, \ldots, T_n\}$ be a set of independent, preemptable periodic tasks with $D_i \geq p_i$ for $i = 1, \ldots, n$. The following statements are equivalent:

1. *$\mathcal{T}$ can be feasibly scheduled on one processor*
2. *$U^{\mathcal{T}} \leq 1$*
3. *$\mathcal{T}$ is schedulable using EDF*

(i.e., in particular, $U_{EDF} = 1$)

Proof.

1.⇒2. We prove that $U^{\mathcal{T}} > 1$ implies that $\mathcal{T}$ is not schedulable

2.⇒3. Next slides and whiteboard ...

3.⇒1. Trivial

□

Proof of 1.⇒2.

Assume that $U^{\mathcal{T}} = \sum_{i=1}^{N} \frac{e_i}{p_i} > 1$.

Consider a time instant $t > \max_i \varphi_i$
(i.e. a time when all tasks are already "running")

Observe that the number of jobs of T_i that are released in the time interval $[0, t]$ is $\left\lceil \frac{t-\varphi_i}{p_i} \right\rceil$. Thus a single processor needs $\sum_{i=1}^{n} \left\lceil \frac{t-\varphi_i}{p_i} \right\rceil \cdot e_i$ time units to finish all jobs *released before or at t*.

However,

$$\sum_{i=1}^{n} \left\lceil \frac{t-\varphi_i}{p_i} \right\rceil \cdot e_i \geq \sum_{i=1}^{n} (t-\varphi_i) \cdot \frac{e_i}{p_i} = \sum_{i=1}^{n} t u_i - \varphi_i u_i = \sum_{i=1}^{n} t u_i - \sum_{i=1}^{n} \varphi_i u_i = t \cdot U^{\mathcal{T}} - \sum_{i=1}^{n} \varphi_i u_i$$

Here $\sum_{i=1}^{n} \varphi_i u_i$ does not depend on t.

Note that $\lim_{t\to\infty} \left(t \cdot U^{\mathcal{T}} - \sum_{i=1}^{n} \varphi_i u_i\right) - t = \infty$. So there exists t such that $t \cdot U^{\mathcal{T}} - \sum_{i=1}^{n} \varphi_i u_i > t + \max_i D_i$.

So in order to complete all jobs released before time t we need more time than $t + \max_i D_i$. However, the latest deadline of a job released before t is $t + \max_i D_i$. So at least one job misses its deadline.

Proof of 2.⇒3. – Simplified

Let us start with a proof of a special case (see the assumptions A1 and A2 below). Then a complete proof will be presented.

We prove ¬3.⇒ ¬2. assuming that $D_i = p_i$ for $i = 1, \ldots, n$.
(Note that the general case immediately follows.)

Assume that $\mathcal{T}$ is not schedulable according to EDF.
(Our goal is to show that $U^{\mathcal{T}} > 1$.)

This means that there must be at least one job that misses its deadline when EDF is used.

Simplifying assumptions:

A1 Suppose that all tasks are in phase, i.e. the phase $\varphi_\ell = 0$ for every task T_ℓ.

A2 Suppose that *the first job* $J_{i,1}$ of a task T_i misses its deadline.

By A1, $J_{i,1}$ is released at 0 and misses its deadline at p_i. Assume w.l.o.g. that this is the first time when a job misses its deadline.
(To simplify even further, you may (privately) assume that no other job has its deadline at p_i.)

Proof of 2.⇒3. – Simplified

Let G be the set of all jobs that are released in $[0, p_i]$ and have their deadlines in $[0, p_i]$.

Crucial observations:

- G contains $J_{i,1}$ and all jobs that preempt $J_{i,1}$.

  We are using EDF, so if a job preempts $J_{i,1}$, then its deadline must be in $[0, p_i]$.

- During $[0, p_i]$, the processor is never idle and executes *only* jobs of G.

  The processor is not idle because $J_{i,1}$ is ready for computation throughout $[0, p_i]$. Jobs that do not belong to G are *not* executed as $J_{i,1}$ is not completed in $[0, p_i]$ and only jobs of G can preempt $J_{i,1}$.

Denote by E_G the total execution time of G, that is, the sum of execution times of all jobs in G.

Corollary of the crucial observation: $E_G > p_i$ because otherwise $J_{i,1}$ (and all jobs that preempt it) would complete by p_i.

Let us compute E_G.

Proof of 2.⇒3. – Simplified

Since we assume $\varphi_\ell = 0$ for every T_ℓ, the first job of T_ℓ is released at 0, and thus $\left\lfloor \frac{p_i}{p_\ell} \right\rfloor$ jobs of T_ℓ belong to G.

E.g., if $p_\ell = 2$ and $p_i = 5$ then three jobs of T_ℓ are released in $[0, 5]$ (at times 0, 2, 4) but only $2 = \left\lfloor \frac{5}{2} \right\rfloor = \left\lfloor \frac{p_i}{p_\ell} \right\rfloor$ of them have their deadlines in $[0, p_i]$.

Thus the total execution time E_G of all jobs in G is

$$E_G = \sum_{\ell=1}^{n} \left\lfloor \frac{p_i}{p_\ell} \right\rfloor e_\ell$$

But then

$$p_i < E_G = \sum_{\ell=1}^{n} \left\lfloor \frac{p_i}{p_\ell} \right\rfloor e_\ell \leq \sum_{\ell=1}^{n} \frac{p_i}{p_\ell} e_\ell \leq p_i \sum_{\ell=1}^{n} u_\ell \leq p_i \cdot U^{\mathcal{T}}$$

which implies that $U^{\mathcal{T}} > 1$.

Proof of 2.⇒3. – Complete

Now let us drop the simplifying assumptions A1 and A2 !

Notation: Given a set of tasks $\mathcal{L}$, we denote by $\bigcup \mathcal{L}$ the set of all jobs of the tasks in $\mathcal{L}$.

We prove ¬3.⇒ ¬2. assuming that $D_i = p_i$ for $i = 1, \ldots, n$ (note that the general case immediately follows).

Assume that $\mathcal{T}$ is not schedulable by EDF. We show that $U^{\mathcal{T}} > 1$.

Suppose that a job $J_{i,k}$ of T_i misses its deadline at time $t = r_{i,k} + p_i$. *Assume that this is the earliest deadline miss.*

Let $\mathcal{T}'$ be the set of all tasks whose jobs have deadlines (and thus also release times) in $[r_{i,k}, t]$

(i.e., a task belongs to $\mathcal{T}'$ iff at least one job of the task is released in $[r_{i,k}, t]$).

Let t_{-} be the end of the *latest* interval before t in which either jobs of $\bigcup(\mathcal{T} \setminus \mathcal{T}')$ are executed, or the processor is idle.

Then $r_{i,k} \geq t_{-}$ since all jobs of $\bigcup(\mathcal{T} \setminus \mathcal{T}')$ waiting for execution during $[r_{i,k}, t]$ have deadlines later than t (thus have lower priorities than $J_{i,k}$).

Proof of 2.⇒3. – Complete (cont.)

It follows that

- no job of $\bigcup(\mathcal{T} \setminus \mathcal{T}')$ is executed in $[t_-, t]$,

  (by definition of t_-)
- the processor is fully utilized in $[t_-, t]$.

  (by definition of t_-)
- *all jobs* (that all must belong to $\bigcup \mathcal{T}'$) *executed in $[t_-, t]$ are released in $[t_-, t]$ and have their deadlines in $[t_-, t]$* since
  - no job of $\bigcup \mathcal{T}'$ executes just before t_-
  - all jobs of $\bigcup \mathcal{T}'$ released in $[t_-, r_{i,k}]$ have deadlines in $[r_{i,k}, t]$, i.e. before t,
  - jobs of $\bigcup \mathcal{T}'$ released in $[r_{i,k}, t]$ with deadlines after t are not executed in $[r_{i,k}, t]$ as they have lower priorities than $J_{i,k}$.

Let G be the set of all jobs that are released in $[t_-, t]$ and have their deadlines in $[t_-, t]$.

Note that $J_{i,k} \in G$ since $r_{i,k} \geq t_-$.

Denote by E_G the sum of all execution times of all jobs in G (the total execution time of G).

Proof of 2.⇒3. – Complete (cont.)

Now $E_G > t - t_-$ because otherwise $J_{i,k}$ would complete in $[t_-, t]$.

How to compute E_G?

For $T_\ell \in \mathcal{T}'$, denote by R_ℓ the earliest release time of a job in T_ℓ during the interval $[t_-, t]$.

For every $T_\ell \in \mathcal{T}'$, exactly $\left\lfloor \frac{t-R_\ell}{p_\ell} \right\rfloor$ jobs of T_ℓ belong to G. (For every $T_\ell \in \mathcal{T} \setminus \mathcal{T}'$, exactly 0 jobs belong to G.)

Thus

$$E_G = \sum_{T_\ell \in \mathcal{T}'} \left\lfloor \frac{t - R_\ell}{p_\ell} \right\rfloor e_\ell$$

As argued above:

$$t - t_- < E_G = \sum_{T_\ell \in \mathcal{T}'} \left\lfloor \frac{t - R_\ell}{p_\ell} \right\rfloor e_\ell \leq \sum_{T_\ell \in \mathcal{T}'} \frac{t - t_-}{p_\ell} e_\ell \leq (t - t_-) \sum_{T_\ell \in \mathcal{T}'} u_\ell \leq (t - t_-) U^{\mathcal{T}}$$

which implies that $U^{\mathcal{T}} > 1$.

Density and EDF

What about tasks with $D_i < p_i$?

Density of a task T_i with period p_i, execution time e_i and relative deadline D_i is defined by

$$e_i / \min(D_i, p_i)$$

Total density $\Delta^{\mathcal{T}}$ of a set of tasks $\mathcal{T}$ is the sum of densities of tasks in $\mathcal{T}$

Note that if $D_i < p_i$ for some i, then $\Delta^{\mathcal{T}} > U^{\mathcal{T}}$

Theorem 15

A set $\mathcal{T}$ of independent, preemptable, periodic tasks can be feasibly scheduled on one processor if $\Delta^{\mathcal{T}} \leq 1$.

Note that this is NOT a necessary condition! (Example whiteb.)

Schedulability Test For EDF

The problem: Given a set of independent, preemptable, periodic tasks $\mathcal{T} = \{T_1, \ldots, T_n\}$ where each T_i has a period p_i, execution time e_i, and relative deadline D_i, decide whether $\mathcal{T}$ is schedulable by EDF.

Solution using utilization and density:

If $p_i \leq D_i$ for each i, then it suffices to decide whether $U^{\mathcal{T}} \leq 1$.

Otherwise, decide whether $\Delta^{\mathcal{T}} \leq 1$:

- If yes, then $\mathcal{T}$ is schedulable with EDF
- If not, then $\mathcal{T}$ does not have to be schedulable

Note that

- Phases of tasks do not have to be specified
- Parameters may vary: increasing periods or deadlines, or decreasing execution times does not prevent schedulability

Schedulability Test for EDF – Example

Consider a digital robot controller

- A control-law computation
  - takes no more than 8 ms
  - the sampling rate: 100 Hz, i.e. computes every 10 ms

  Feasible? Trivially yes ....
- Add Built-In Self-Test (BIST)
  - maximum execution time 50 ms
  - want a minimal period that is feasible (max one second)

  With 250 ms still feasible ....
- Add a telemetry task
  - maximum execution time 15 ms
  - want to minimize the deadline on telemetry
    period may be large

  Reducing BIST to once a second, deadline on telemetry may be set to 100 ms ....

Real-Time Scheduling

Priority-Driven Scheduling

Fixed-Priority

Fixed-Priority Algorithms

Recall that we consider a set of n tasks $\mathcal{T} = \{T_1, \ldots, T_n\}$

Any fixed-priority algorithm schedules tasks of $\mathcal{T}$ according to fixed (distinct) priorities *assigned to tasks.*

We write $T_i \sqsupset T_j$ whenever T_i has a higher priority than T_j.

To simplify our reasoning, assume that

all tasks are in phase, i.e. $\varphi_k = 0$ **for all** T_k**.**

We will remove this assumption at the end.

Fixed-Priority Algorithms – Reminder

Recall that Fixed-Priority Algorithms do not have to be optimal.

Consider $\mathcal{T} = \{T_1, T_2\}$ where $T_1 = (2, 1)$ and $T_2 = (5, 2.5)$

$U^{\mathcal{T}} = 1$ and thus $\mathcal{T}$ is schedulable by EDF

If $T_1 \sqsupset T_2$, then $J_{2,1}$ misses its deadline

If $T_2 \sqsupset T_1$, then $J_{1,1}$ misses its deadline

We consider the following algorithms:

- **RM** = assigns priorities to tasks based on their periods

  the priority is inversely proportional to the period p_i
- **DM** = assigns priorities to tasks based on their relative deadlines

  the priority is inversely proportional to the relative deadline D_i

(In all cases, ties are broken arbitrarily.)

We consider the following questions:

- Are the algorithms optimal?
- How to efficiently (or even online) test for schedulability?

Maximum Response Time

Which job of a task T_i has the maximum response time?

As all tasks are in phase, the first job of T_i is released together with (first) jobs of all tasks that have higher priority than T_i.

This means, that $J_{i,1}$ is the most preempted of jobs in T_i.

It follows, that $J_{i,1}$ has the maximum response time.
Note that this relies heavily on the assumption that tasks are in phase!

Thus in order to decide whether $\mathcal{T}$ is schedulable, it suffices to test for schedulability of the first jobs of all tasks.

Optimality of RM for Simply Periodic Tasks

Definition 16

A set $\{T_1, \ldots, T_n\}$ is **simply periodic** if for every pair T_i, T_ℓ satisfying $p_i > p_\ell$ we have that p_i is an integer multiple of p_ℓ

Example 17

The helicopter control system from the first lecture.

Theorem 18

A set $\mathcal{T}$ of n simply periodic, independent, preemptable tasks with $D_i = p_i$ is schedulable on one processor according to RM **iff** $U^{\mathcal{T}} \leq 1$.

i.e. on simply periodic tasks RM is as good as EDF

Note: Theorem 18 is true in general, no "in phase" assumption is needed.

Proof of Theorem 18

By Theorem 14, every schedulable set $\mathcal{T}$ satisfies $U^{\mathcal{T}} \leq 1$.

We prove that if $\mathcal{T}$ is not schedulable *according to RM*, then $U^{\mathcal{T}} > 1$.

Assume that a job $J_{i,1}$ of T_i misses its deadline at $D_i = p_i$. W.l.o.g., we assume that $T_1 \sqsupset \cdots \sqsupset T_n$ according to RM.

Let us compute the total execution time of $J_{i,1}$ and all jobs that preempt it:

$$E = e_i + \sum_{\ell=i+1}^{n} \left\lceil \frac{p_i}{p_\ell} \right\rceil e_\ell = \sum_{\ell=i}^{n} \frac{p_i}{p_\ell} e_\ell = p_i \sum_{\ell=i}^{n} u_\ell \leq p_i \sum_{\ell=1}^{n} u_\ell = p_i U^{\mathcal{T}}$$

Now $E > p_i$ because otherwise $J_{i,1}$ meets its deadline. Thus

$$p_i < E \leq p_i U^{\mathcal{T}}$$

and we obtain $U^{\mathcal{T}} > 1$.

Optimality of DM (RM) among Fixed-Priority Algs.

Theorem 19
A set of independent, preemptable periodic tasks with $D_i \leq p_i$ that are in phase (i.e., $\varphi_i = 0$ for all $i = 1, \ldots, n$) can be feasibly scheduled on one processor according to DM if it can be feasibly scheduled by some fixed-priority algorithm.

Proof.
Assume a fixed-priority feasible schedule with $T_1 \sqsupset \cdots \sqsupset T_n$.

Consider the least i such that the relative deadline D_i of T_i is larger than the relative deadline D_{i+1} of T_{i+1}.

Swap the priorities of T_i and T_{i+1}.

The resulting schedule is still feasible.

DM is obtained by using finitely many swaps. □

Note: If the assumptions of the above theorem hold and all relative deadlines are equal to periods, then RM is optimal among all fixed-priority algorithms.

Fixed-Priority Algorithms: Schedulability

We consider two schedulability tests:

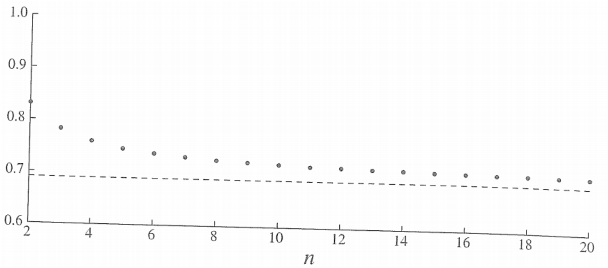
- Schedulable utilization U_{RM} of the RM algorithm.
- Time-demand analysis based on response times.

Schedulable Utilization for RM

Theorem 20

Let us fix $n \in \mathbb{N}$ and consider only independent, preemptable periodic tasks with $D_i = p_i$.

- *If $\mathcal{T}$ is a set of n tasks satisfying $U^{\mathcal{T}} \leq n(2^{1/n} - 1)$, then $U^{\mathcal{T}}$ is schedulable according to the RM algorithm.*
- *For every $U > n(2^{1/n} - 1)$ there is a set $\mathcal{T}$ of n tasks satisfying $U^{\mathcal{T}} \leq U$ that is not schedulable by RM.*



Note: Theorem 20 holds in general, no "in phase" assumption is needed.

Schedulable Utilization for RM

It follows that the maximum schedulable utilization U_{RM} over independent, preemptable periodic tasks satisfies

$$U_{RM} = \inf_n n(2^{1/n} - 1) = \lim_{n \to \infty} n(2^{1/n} - 1) = \ln 2 \approx 0.693$$

Note that $U^{\mathcal{T}} \leq n(2^{1/n} - 1)$ is a sufficient but not necessary condition for schedulability of $\mathcal{T}$ using the RM algorithm (an example will be given later)

We say that a set of tasks $\mathcal{T}$ is *RM-schedulable* if it is schedulable according to RM.

We say that $\mathcal{T}$ is *RM-infeasible* if it is not RM-schedulable.

Proof – Special Case

To simplify, we restrict to two tasks and always assume $p_2 \leq 2p_1$.
(the latter condition is w.l.o.g., proof omitted)

Outline: Given p_1, p_2, e_1, denote by max_e_2 the maximum execution time so that $\mathcal{T} = \{(p_1, e_1), (p_2, max\_e_2)\}$ is RM-schedulable.

We define $U_{e_1}^{p_1,p_2}$ to be $U^{\mathcal{T}}$ where $\mathcal{T} = \{(p_1, e_1), (p_2, max\_e_2)\}$.
We say that $\mathcal{T}$ fully utilizes the processor, any increase in an execution time causes RM-infeasibility.

Now **we find the (global) minimum** $minU$ **of** $U_{e_1}^{p_1,p_2}$**.**
Note that this suffices to obtain the desired result:

- Given a set of tasks $\mathcal{T} = \{(p_1, e_1), (p_2, e_2)\}$ satisfying $U^{\mathcal{T}} \leq minU$ we get $U^{\mathcal{T}} \leq minU \leq U_{e_1}^{p_1,p_2}$, and thus the execution time e_2 cannot be larger than max_e_2. Thus, $\mathcal{T}$ is RM-schedulable.
- Given $U > minU$, there must be p_1, p_2, e_1 satisfying $minU \leq U_{e_1}^{p_1,p_2} < U$ where $U_{e_1}^{p_1,p_2} = U^{\mathcal{T}}$ for a set of tasks $\mathcal{T} = \{(p_1, e_1), (p_2, max\_e_2)\}$.
  However, now increasing e_1 by a sufficiently small $\varepsilon > 0$ makes the set RM-infeasible without making utilization larger than U.

Proof – Special Case (Cont.)

Consider two cases depending on e_1:

1. $e_1 < p_2 - p_1$:

Maximum RM-feasible max_e_2 (with p_1, p_2, e_1 fixed) is $p_2 - 2e_1$. Which gives the utilization

$$U_{e_1}^{p_1,p_2} = \frac{e_1}{p_1} + \frac{max\_e_2}{p_2} = \frac{e_1}{p_1} + \frac{p_2 - 2e_1}{p_2} = \frac{e_1}{p_1} + \frac{p_2}{p_2} - \frac{2e_1}{p_2} = 1 + \frac{e_1}{p_2}\left(\frac{p_2}{p_1} - 2\right)$$

As $\frac{p_2}{p_1} - 2 \leq 0$, the utilization $U_{e_1}^{p_1,p_2}$ is minimized by maximizing e_1.

2. $e_1 \geq p_2 - p_1$:

Maximum RM-feasible max_e_2 (with p_1, p_2, e_1 fixed) is $p_1 - e_1$. Which gives the utilization

$$U_{e_1}^{p_1,p_2} = \frac{e_1}{p_1} + \frac{max\_e_2}{p_2} = \frac{e_1}{p_1} + \frac{p_1 - e_1}{p_2} = \frac{e_1}{p_1} + \frac{p_1}{p_2} - \frac{e_1}{p_2} = \frac{p_1}{p_2} + \frac{e_1}{p_2}\left(\frac{p_2}{p_1} - 1\right)$$

As $\frac{p_2}{p_1} - 1 \geq 0$, the utilization $U_{e_1}^{p_1,p_2}$ is minimized by minimizing e_1.

The minimum of $U_{e_1}^{p_1,p_2}$ is attained at $e_1 = p_2 - p_1$.
(Both expressions defining $U_{e_1}^{p_1,p_2}$ give the same value for $e_1 = p_2 - p_1$.)

Proof – Special Case (Cont.)

Substitute $e_1 = p_2 - p_1$ into the expression for $U_{e_1}^{p_1,p_2}$:

$$U_{p_2-p_1}^{p_1,p_2} = \frac{p_1}{p_2} + \frac{p_2 - p_1}{p_2}\left(\frac{p_2}{p_1} - 1\right) = \frac{p_1}{p_2} + \left(1 - \frac{p_1}{p_2}\right)\left(\frac{p_2}{p_1} - 1\right)$$

$$= \frac{p_1}{p_2} + \frac{p_1}{p_2}\left(\frac{p_2}{p_1} - 1\right)\left(\frac{p_2}{p_1} - 1\right) = \frac{p_1}{p_2}\left(1 + \left(\frac{p_2}{p_1} - 1\right)^2\right)$$

Denoting $G = \frac{p_2}{p_1} - 1$ we obtain

$$U_{p_2-p_1}^{p_1,p_2} = \frac{p_1}{p_2}(1 + G^2) = \frac{1 + G^2}{p_2/p_1} = \frac{1 + G^2}{1 + G}$$

Differentiating w.r.t. G we get

$$\frac{G^2 + 2G - 1}{(1 + G)^2}$$

which attains minimum at $G = -1 \pm \sqrt{2}$. Here only $G = -1 + \sqrt{2} > 0$ is acceptable since the other root is negative.

Proof – Special Case (Cont.)

Thus the minimum value of $U_{e_1}^{p_1,p_2}$ is

$$\frac{1+(\sqrt{2}-1)^2}{1+(\sqrt{2}-1)} = \frac{4-2\sqrt{2}}{\sqrt{2}} = 2(\sqrt{2}-1)$$

It is attained at periods satisfying

$$G = \frac{p_2}{p_1} - 1 = \sqrt{2} - 1 \qquad \text{i.e. satisfying } p_2 = \sqrt{2}p_1.$$

The execution time e_1 which at full utilization of the processor (due to max_e_2) gives the minimum utilization is

$$e_1 = p_2 - p_1 = (\sqrt{2}-1)p_1$$

and the corresponding $max\_e_2 = p_1 - e_1 = p_1 - (p_2 - p_1) = 2p_1 - p_2$.

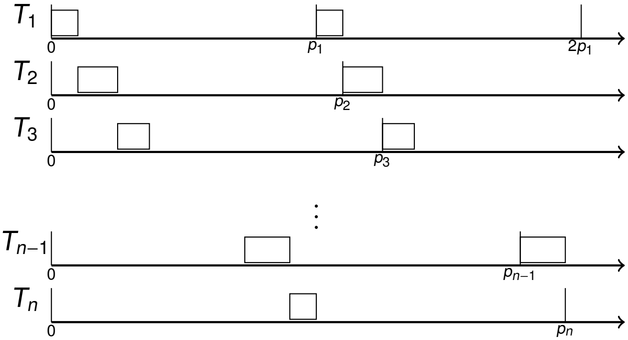
Scaling to $p_1 = 1$, we obtain a completely determined example

$$p_1 = 1 \quad p_2 = \sqrt{2} \approx 1.41 \quad e_1 = \sqrt{2}-1 \approx 0.41 \quad max\_e_2 = 2-\sqrt{2} \approx 0.59$$

that fully utilizes the processor (no execution time can be increased) but has the minimum utilization $2(\sqrt{2}-1)$.

Proof Idea of Theorem 20

Fix periods $p_1 < \cdots < p_n$ so that (w.l.o.g.) $p_n \leq 2p_1$. Then the following set of tasks has the smallest utilization among all task sets that fully utilize the processor (i.e., any increase in any execution time makes the set unschedulable).



$$e_k = p_{k+1} - p_k \qquad \text{for } k = 1, \ldots, n-1$$

$$e_n = p_n - 2\sum_{k=1}^{n-1} e_k = 2p_1 - p_n$$

Time-Demand Analysis

Consider a set of n tasks $\mathcal{T} = \{T_1, \ldots, T_n\}$.
Recall that we consider only independent, preemptable, in phase (i.e. $\varphi_i = 0$ for all i) tasks without resource contentions.

Assume that $D_i \leq p_i$ for every i, and consider an arbitrary fixed-priority algorithm. W.l.o.g. assume $T_1 \sqsupset \cdots \sqsupset T_n$.

Idea: For every task T_i and every time instant $t \geq 0$, compute the total execution time $w_i(t)$ (the time demand) of the first job $J_{i,1}$ and of all higher-priority jobs released up to time t.

If $w_i(t) \leq t$ for some time $t \leq D_i$, then $J_{i,1}$ is schedulable, and hence all jobs of T_i are schedulable.

Time-Demand Analysis

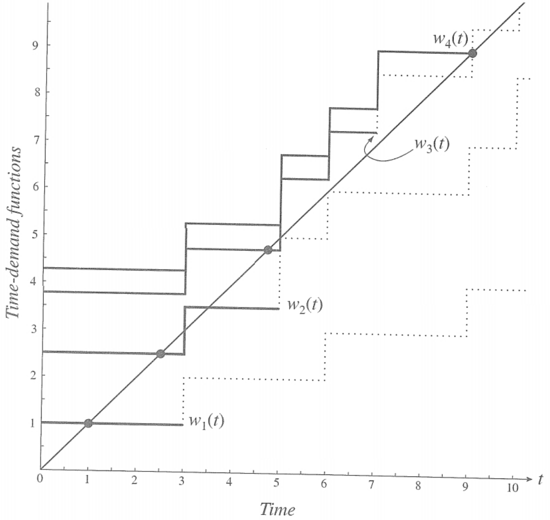
- Consider one task T_i at a time, starting with highest priority and working to lowest priority.
- Focus on the first job $J_{i,1}$ of T_i.

  If $J_{i,1}$ makes it, all jobs of T_i will make it due to $\varphi_i = 0$.
- At time t for $t \geq 0$, the processor time demand $w_i(t)$ for this job and all higher-priority jobs released in $[0, t]$ is bounded by

$$w_i(t) = e_i + \sum_{\ell=1}^{i-1} \left\lceil \frac{t}{p_\ell} \right\rceil e_\ell \qquad \text{for } 0 < t \leq p_i$$

  (Note that the smallest t for which $w_i(t) \leq t$ is the response time of $J_{i,1}$, and hence the maximum response time of jobs in T_i).
- If $w_i(t) \leq t$ for some $t \leq D_i$, the job $J_{i,1}$ meets its deadline D_i.
- If $w_i(t) > t$ for all $0 < t \leq D_i$, then the first job of the task cannot complete by its deadline.

Time-Demand Analysis – Example



Example: $T_1 = (3, 1)$, $T_2 = (5, 1.5)$, $T_3 = (7, 1.25)$, $T_4 = (9, 0.5)$

This is schedulable by RM even though
$U^{\{T_1,\dots,T_4\}} = 0.85 > 0.757 = U_{RM}(4)$

Time-Demand Analysis

- The time-demand function $w_i(t)$ is a staircase function
  - Steps in the time-demand for a task occur at multiples of the period for higher-priority tasks
  - The value of $w_i(t) - t$ linearly decreases from a step until the next step
- If our interest is the schedulability of a task, it suffices to check if $w_i(t) \leq t$ at the time instants when a higher-priority job is released and at D_i
- Our schedulability test becomes:
  - Compute $w_i(t)$
  - Check whether $w_i(t) \leq t$ for some t equal either to D_i, or to $j \cdot p_k$ where $k = 1, 2, \ldots, i$ and $j = 1, 2, \ldots, \lfloor D_i / p_k \rfloor$

Time-Demand Analysis – Comments

- Time-demand analysis schedulability test is more complex than the schedulable utilization test but more general:
  - Works for *any* fixed-priority scheduling algorithm, provided the tasks have short response time ($D_i \leq p_i$)
    Can be extended to tasks with arbitrary deadlines
- Still more efficient than exhaustive simulation.
- Assuming that the tasks are in phase the time demand analysis is complete.

We have considered the time demand analysis for tasks in phase. In particular, we used the fact that the first job has the maximum response time.

This is not true if the jobs are not in phase, we need to identify the so called *critical instant*, the time instant in which the system is most loaded, and has its worst response time.

Critical Instant – Formally

Definition 21
A **critical instant** t_{crit} of a task T_i is a time instant in which a job $J_{i,k}$ in T_i is released so that $J_{i,k}$ either does not meet its deadline, or has the maximum response time of all jobs in T_i.

Theorem 22
In a fixed-priority system where every job completes before the next job in the same task is released, a critical instant of a task T_i occurs when one of its jobs $J_{i,k}$ is released at the same time with a job from every higher-priority task.

Note that the situation described in the theorem does not have to occur if tasks are not in phase!

Critical Instant and Schedulability Tests

We use critical instants to get upper bounds on schedulability as follows:

- Set phases of all tasks to zero, which gives a new set of tasks $\mathcal{T}' = \{T'_1, \ldots, T'_n\}$

  By Theorem 22, the response time of the first job $J'_{i,1}$ of T'_1 in $\mathcal{T}'$ is at least as large as the response time of every job of T_i in $\mathcal{T}$.

- Decide schedulability of $\mathcal{T}'$, e.g. using the timed-demand analysis.
  - If $\mathcal{T}'$ if schedulable, then also $\mathcal{T}$ is schedulable.
  - If $\mathcal{T}'$ is not schedulable, then $\mathcal{T}$ does not have to be schedulable.

    But may be schedulable, which make the time-demand analysis incomplete in general for tasks not in phase.

Dynamic vs Fixed Priority

- EDF
  - pros:
    - optimal
    - very simple and complete test for schedulability
  - cons:
    - difficult to predict which job misses its deadline
    - strictly following EDF in case of overloads assigns higher priority to jobs that missed their deadlines
    - larger scheduling overhead
- DM (RM)
  - pros:
    - easier to predict which job misses its deadline (in particular, tasks are not blocked by lower priority tasks)
    - easy implementation with little scheduling overhead
    - (optimal in some cases often occurring in practice)
  - cons:
    - not optimal
    - incomplete and more involved tests for schedulability

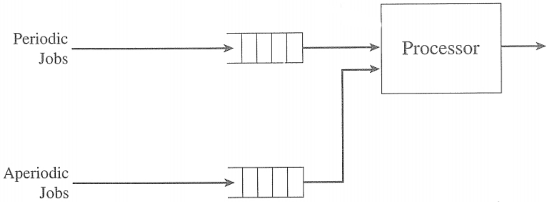
Real-Time Scheduling

Priority-Driven Scheduling

Aperiodic Tasks

Current Assumptions

- Single processor
- Fixed number, *n*, of *independent periodic* tasks
  - Jobs can be preempted at any time and never suspend themselves
  - No resource contentions
- Aperiodic jobs exist
  - They are independent of each other, and of the periodic tasks
  - They can be preempted at any time
- There are no sporadic jobs (for now)
- Jobs are scheduled using a priority driven algorithm



Scheduling Aperiodic Jobs

Consider:

- A set $\mathcal{T} = \{T_1, \ldots, T_n\}$ of periodic tasks
- An aperiodic task A

Recall that:

- A schedule is feasible if all jobs with hard real-time constraints complete before their deadlines

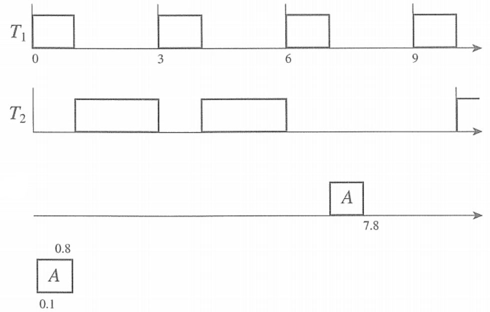
  $\Rightarrow$ This includes all periodic jobs
- A scheduling algorithm is optimal if it always produces a feasible schedule whenever such a schedule exists, and if a cost function is given, minimizes the cost

We assume that the periodic tasks are scheduled using a priority-driven algorithm

Background Scheduling of Aperiodic Jobs

- Aperiodic jobs are scheduled and executed only at times when there are no periodic jobs ready for execution
- Advantages
  - Clearly produces feasible schedules
  - Extremely simple to implement
- Disadvantages
  - Not optimal since the execution of aperiodic jobs may be unnecessarily delayed

Example: $T_1 = (3, 1)$, $T_2 = (10, 4)$

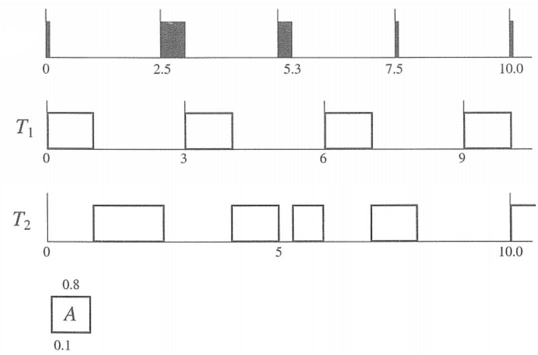


Polled Execution of Aperiodic Jobs

- We may use a *polling server*
  - A periodic job (p_s, e_s) scheduled according to the periodic algorithm, generally as the highest priority job
  - When executed, it examines the aperiodic job queue
    - If an aperiodic job is in the queue, it is executed for up to e_s time units
    - If the aperiodic queue is empty, the polling server self-suspends, giving up its execution slot
    - The server does not wake-up once it has self-suspended, aperiodic jobs which become active during a period are not considered for execution until the next period begins
- Simple to prove correctness, performance less than ideal – executes aperiodic jobs in particular timeslots

Polled Execution of Aperiodic Jobs

Example: $T_1 = (3, 1)$, $T_2 = (10, 4)$, $poller = (2.5, 0.5)$



Can we do better?

Yes, polling server is a special case of *periodic-server* for aperiodic jobs.

Periodic Severs – Terminology

periodic server = a task that behaves much like a periodic task, but is created for the purpose of executing aperiodic jobs

- A periodic server, $T_S = (p_S, e_S)$
  - p_S is a period of the server
  - e_S is the (maximal) *budget* of the server
- The budget can be *consumed* and *replenished*; the budget is *exhausted* when it reaches 0

  (Periodic servers differ in how they consume and replenish the budget)
- A periodic server is
  - *backlogged* whenever the aperiodic job queue is non-empty
  - *idle* if the queue is empty
  - *eligible* if it is backlogged and the budget is not exhausted
- When a periodic server is eligible, it is scheduled as any other periodic task with parameters (p_S, e_S)

Periodic Severs

Each periodic server is thus specified by

- *consumption rules*: How the budget is consumed
- *replenishment rules*: When and how the budget is replenished

Polling server

- *consumption rules*:
  - Whenever the server executes, the budget is consumed at the rate one per unit time.
  - Whenever the server becomes idle, the budget gets immediately exhausted
- *replenishment rule*: At each time instant $k \cdot p_S$ replenish the budget to e_S

Periodic Severs

Deferrable sever

- *Consumption rules*:
  - The budget is consumed at the rate of one per unit time whenever the server executes
  - Unused budget is retained throughout the period, to be used whenever there are aperiodic jobs to execute
    (i.e. instead of discarding the budget if no aperiodic job to execute at start of period, keep in the hope a job arrives)
- *Replenishment rule*:
  - The budget is set to e_S at multiples of the period
    - i.e. time instants $k \cdot p_S$ for $k = 0, 1, 2, \ldots$

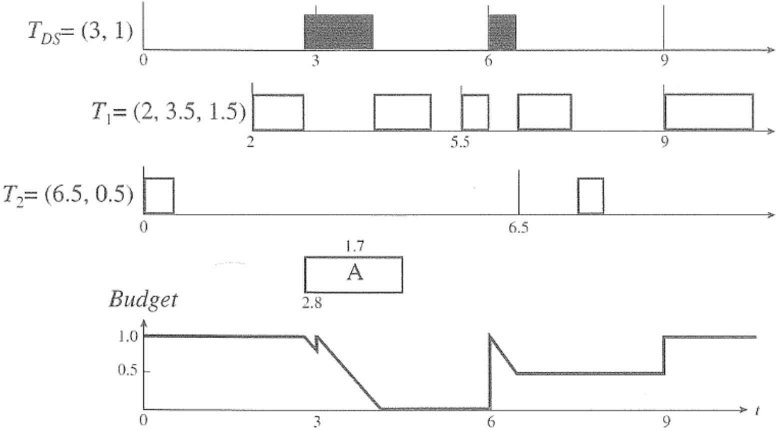
    (Note that the server is not able tu cumulate the budget over periods)

We consider both

- Fixed-priority scheduling
- Dynamic-priority scheduling (EDF)

Deferrable Server – RM

Here the tasks are scheduled using RM.

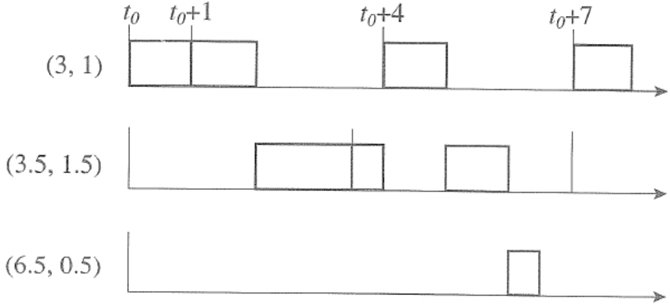


Is it possible to increase the budget of the server to 1.5 ?

Deferrable Server – RM

Consider $T_1 = (3.5, 1.5)$, $T_2 = (6.5, 0.5)$ and $T_{DS} = (3, 1)$

A **critical instant** for $T_1 = (3.5, 1.5)$ looks as follows:



i.e. increasing the budget above 1 may cause T_1 to miss its deadline

Deferrable Server – Critical Instant

Lemma 23

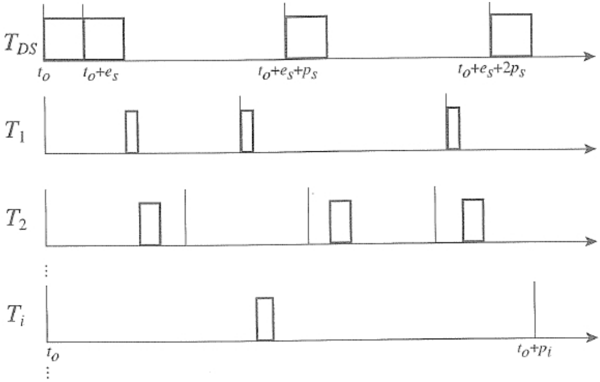
Assume a fixed-priority scheduling algorithm. Assume that $D_i \leq p_i$ and that the deferrable server (p_S, e_S) has the highest priority among all tasks. Then a critical instant of every periodic task T_i occurs at a time t_0 when all of the following are true:

- *One of its jobs $J_{i,c}$ is released at t_0*
- *A job in every higher-priority periodic task is released at t_0*
- *The budget of the server is e_S at t_0, one or more aperiodic jobs are released at t_0, and they keep the server backlogged hereafter*
- *The next replenishment time of the server is $t_0 + e_S$*

Deferrable Server – Critical Instant

Assume $T_{DS} \sqsupset T_1 \sqsupset T_2 \sqsupset \cdots \sqsupset T_n$

(i.e. T_1 has the highest pririty and T_n lowest)



Deferrable Server – Time Demand Analysis

Assume that the deferrable server has the highest priority

- The definition of critical instant is identical to that for the periodic tasks without the deferrable server + the worst-case requirements for the server
- Thus the expression for the time-demand function becomes

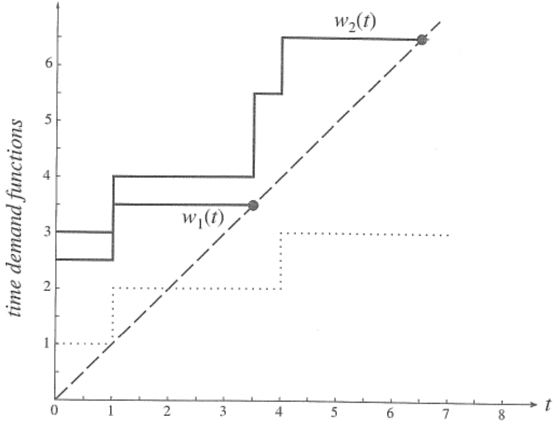
$$w_i(t) = e_i + \sum_{k=1}^{i-1} \left\lceil \frac{t}{p_k} \right\rceil e_k + e_S + \left\lceil \frac{t - e_S}{p_S} \right\rceil e_S \qquad \text{for } 0 < t \leq p_i$$

- To determine whether the task T_i is schedulable, we simply check whether $w_i(t) \leq t$ for some $t \leq D_i$

  Note that this is a *sufficient condition*, not necessary.
- Check whether $w_i(t) \leq t$ for some t equal either
  - to D_i, or
  - to $j \cdot p_k$ where $k = 1, 2, \ldots, i$ and $j = 1, 2, \ldots, \lfloor D_i / p_k \rfloor$, or
  - to $e_S, e_S + p_S, e_S + 2p_S, \ldots, e_S + \lfloor (D_i - e_i)/p_S \rfloor p_S$

Deferrable Server – Time Demand Analysis

$T_{DS} = (3, 1.0)$, $T_1 = (3.5, 1.5)$, $T_2 = (6.5, 0.5)$



Deferrable Server – Schedulable Utilization

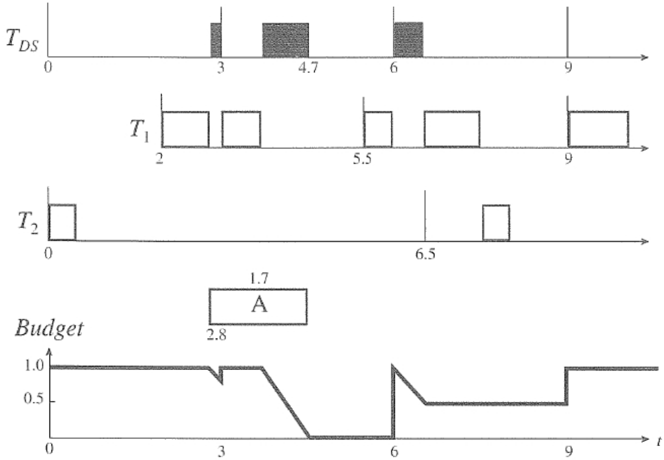
- No maximum schedulable utilization is known in general
- A special case:
  - A set T of n independent, preemptable periodic tasks whose periods satisfy $p_S < p_1 < \cdots < p_n < 2p_S$ and $p_n > p_S + e_S$ and whose relative deadlines are equal to their respective periods, can be scheduled according to RM with a deferrable server provided that

$$U^T \leq U_{RM/DS}(n) := (n-1)\left[\left(\frac{u_S + 2}{u_S + 1}\right)^{\frac{1}{n-1}} - 1\right]$$

where $u_S = e_S/p_S$

Deferrable Server – EDF

Here the tasks are scheduled using EDF.
$T_{DS} = (3, 1)$, $T_1 = (2, 3.5, 1.5)$, $T_2 = (6.5, 0.5)$



Deferrable Server – EDF – Schedulability

Theorem 24

A set of n independent, preemptable, periodic tasks satisfying $p_i \leq D_i$ for all $1 \leq i \leq n$ is schedulable with a deferrable server with period p_S, execution budget e_S and utilization $u_S = e_S/p_S$ according to the EDF algorithm if:

$$\sum_{k=1}^{n} u_k + u_S\left(1 + \frac{p_S - e_S}{\min_i D_i}\right) \leq 1$$

Sporadic Server – Motivation

- Problem with polling server: $T_{PS} = (p_S, e_S)$ executes aperiodic tasks at the multiples of p_S
- Problem with deferrable server: $T_{DS} = (p_S, e_S)$ may delay lower priority jobs longer than periodic task (p_S, e_S)
  Therefore special version of time-demand analysis and utilization bounds were needed.
- **Sporadic server** $T_{SS} = (e_S, p_S)$
  - may execute jobs "in the middle" of its period
  - *never* delays periodic tasks for longer time than the periodic task (p_S, e_S)
    Thus can be tested for schedulability as an ordinary periodic task.

Originally proposed by Sprunt, Sha, Lehoczky in 1989

original version contains a bug which allows longer delay of lower priority jobs

Part of POSIX standard

also incorrect as observed and (probably) corrected by Stanovich in 2010

Very Simple Sporadic Server

For simplicity, we consider only fixed priority scheduling, i.e. assume $T_1 \sqsupset T_2 \sqsupset \cdots \sqsupset T_n$ and consider a sporadic server $T_{SS} = (p_S, e_S)$ with the *highest priority*

Notation:

- t_r = the *latest* replenishment time
- t_f = first instant after t_r at which server begins to execute
- n_r = a variable representing the *next* replenishment

- *Consumption rule*: The budget is consumed (at the rate of one per unit time) whenever the current time t satisfies $t \geq t_f$
- *Replenishment rules*: At the beginning, $t_r = n_r = 0$
  - Whenever the current time is equal to n_r, the budget is set to e_S and t_r is set to the current time
  - At the first instant t_f after t_r at which the server starts executing, n_r is set to $t_f + p_S$

(Note that such server resembles a periodic task with the highest priority whose jobs are released at times t_f and execution times are at most e_S)

Very Simple Sporadic/Background Server

New notation:

- t_r = the *latest* replenishment time
- t_f = first instant after t_r at which server begins to execute and *at least one task of $\mathcal{T}$ is not idle*
- n_r = a variable representing the *next* replenishment

- *Consumption rule*: The budget is consumed (at the rate of one per unit time) whenever the current time t satisfies $t \geq t_f$ and at least one task of $\mathcal{T}$ is not idle
- *Replenishment rules*: At the beginning, $t_r = n_r = 0$
  - Whenever the current time is equal to n_r, the budget is set to e_S and t_r is set to the current time
  - At the beginning of an idle interval of $\mathcal{T}$, the budget is set to e_S and n_r is set to the end of this interval
  - At the first instant t_f after t_r at which the server starts executing and $\mathcal{T}$ is not idle, n_r is set to $t_f + p_S$

This combines the very simple sporadic server with background scheduling.

Very Simple Sporadic Server

Correctness (informally):

Assuming that $\mathcal{T}$ never idles, the sporadic server resembles a periodic task with the highest priority whose jobs are released at times t_f and execution times are at most e_S

Whenever $\mathcal{T}$ idles, the sporadic server executes in the background, i.e. does not block any periodic task, hence does not consume the budget

Whenever an idle interval of $\mathcal{T}$ ends, we may treat this situation as a restart of the system with possibly different phases of tasks (so that it is safe to have the budget equal to e_S)

Note that in both versions of the sporadic server, e_S units of execution time are available for aper. jobs every p_S units of time

This means that if the server is always backlogged, then it executes for e_S time units every p_S units of time

Real-Time Scheduling

Priority-Driven Scheduling

Sporadic Tasks

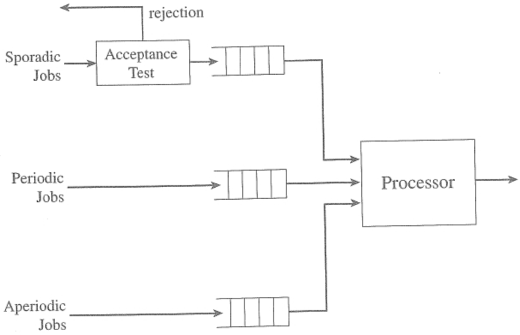
Current Assumptions

- Single processor
- Fixed number, n, of *independent periodic* tasks, $T_1, \ldots, T_n$ where $T_i = (\varphi_i, p_i, e_i, D_i)$
  - Jobs can be preempted at any time and never suspend themselves
  - No resource contentions
- Sporadic tasks
  - Independent of the periodic tasks
  - Jobs can be preempted at any time
- Aperiodic tasks

  For simplicity scheduled in the background – i.e. we may ignore them
- Jobs are scheduled using a priority driven algorithm

A sporadic job = a job of a sporadic task

Our situation



- Based on the execution time and deadline of each newly arrived sporadic job, decide whether to accept or reject the job
- Accepting the job implies that the job will complete within its deadline, without causing any periodic job or previously accepted sporadic job to miss its deadline
- Do not accept a sporadic job if cannot guarantee it will meet its deadline

Scheduling Sporadic Jobs – Correctness and Optimality

- A *correct* schedule is one where all periodic tasks, and all sporadic jobs that have been accepted, meet their deadlines
- A scheduling algorithm supporting sporadic jobs is a *correct* algorithm if it only produces correct schedules for the system
- A sporadic job scheduling algorithm is *optimal* if it accepts a new sporadic job, and schedules that job to complete by its deadline, iff the new job can be correctly scheduled to complete in time

Model for Scheduling Sporadic Jobs with EDF

- Assume that all jobs in the system are scheduled by EDF
- if more sporadic jobs are released at the same time their acceptance test is done in the EDF order
- Definitions:
  - Sporadic jobs are denoted by $S(r, d, e)$ where r is the release time, d the (absolute) deadline, and e is the maximum execution time
  - The **density** of $S(r, d, e)$ is defined by $e/(d - r)$
  - The **total density** of a set of sporadic jobs is the sum of densities of these jobs
  - The sporadic job $S(r, d, e)$ is *active at time t* iff $t \in (r, d]$

Note that each job of a periodic task (φ, p, e, D) can be seen as a sporadic job; to simplify, we **assume that always** $D \leq p$.
This in turn means that there is always at most one job of a given task active at a given time instant.
For every job of this task released at r with abs. deadline d, we obtain the density $e/(d - r) = e/D$

Schedulability of Sporadic Jobs with EDF

Theorem 25

A set of independent preemptable sporadic jobs is schedulable according to EDF if at every time instant t the total density of all *jobs active at time t is at most one.*

Proof.

By contradiction, suppose that a job misses its deadline at t, no deadlines missed before t

Let t_{-1} be the supremum of time instants before t when either the system idles, or a job with a deadline after t executes

Suppose that jobs $J_1, \ldots, J_k$ execute in $[t_{-1}, t]$ and that they are ordered w.r.t. increasing deadline (J_k misses its deadline at t)

Let L be the number of releases and completions in $[t_{-1}, t]$, denote by t_i the i-th time instant when i-th such event occurs (then $t_{-1} = t_1$, we denote by t_{L+1} the time instant t)

Denote by X_i the set of all jobs that are active during the interval $(t_i, t_{i+1}]$ and let Δ_i be their total density

The rest on whiteboard .... □

Sporadic Jobs with EDF – Example

Note that the above theorem includes both the periodic as well as sporadic jobs

This test is sufficient but not necessary

Example 26

Three sporadic jobs: $S_1(0, 2, 1)$, $S_2(0.5, 2.5, 1)$, $S_3(1, 3, 1)$

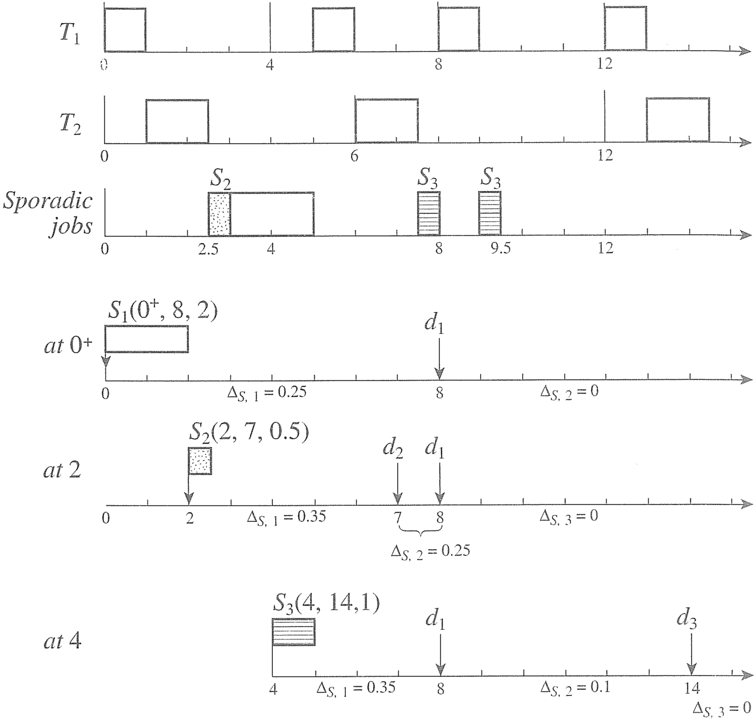
Total density at time 1.5 is 1.5

Yet, the jobs are schedulable by EDF

Admission Control for Sporadic Jobs with EDF

Let Δ be the total density of *periodic tasks*.
Assume that a new sporadic job $S(t, d, e)$ is released at time t.

- At time t there are n active sporadic jobs in the system
- The EDF scheduler maintains a list of the jobs, in non-decreasing order of their deadlines
  - The deadlines partition the time from t to ∞ into $n + 1$ discrete intervals $I_1, I_2, \ldots, I_{n+1}$
    - I_1 begins at t and ends at the earliest sporadic job deadline
    - For each $1 \leq k \leq n$, each I_{k+1} begins when the interval I_k ends, and ends at the next deadline in the list (or ∞ for I_{n+1})
  - The scheduler maintains the total density $\Delta_{S,k}$ of sporadic jobs active in each interval I_k
- Let I_ℓ be the interval containing the deadline d of the new sporadic job $S(t, d, e)$
  - The scheduler accepts the job if $e/(d - t) + \Delta_{S,k} \leq 1 - \Delta$ for all $k = 1, 2, \ldots, \ell$
  - i.e. accept if the new sporadic job can be added, without increasing density of any intervals past 1



T_1
0
4
8
12
T_2
0
6
12
S_2
S_3
S_3
Sporadic jobs
0
2.5
4
8
9.5
12
$S_1(0^+, 8, 2)$
d_1
at 0^+
0
$\Delta_{S,1} = 0.25$
8
$\Delta_{S,2} = 0$
$S_2(2, 7, 0.5)$
d_2
d_1
at 2
0
2
$\Delta_{S,1} = 0.35$
7
8
$\Delta_{S,3} = 0$
$\Delta_{S,2} = 0.25$
$S_3(4, 14, 1)$
d_1
d_3
at 4
4
$\Delta_{S,1} = 0.35$
8
$\Delta_{S,2} = 0.1$
14
$\Delta_{S,3} = 0$

Admission Control for Sporadic Jobs with EDF

This acceptance test is not optimal: a sporadic job may be rejected even though it could be scheduled.

- The test is based on the density and hence is sufficient but not necessary.
- It is possible to derive a – much more complex – expression for schedulability which takes into account slack time, and is optimal. Unclear if the complexity is worthwhile.

Sporadic Jobs with EDF

- One way to schedule sporadic jobs in a **fixed-priority** system is to use a sporadic server to execute them
- Because the server (p_S, e_S) has e_S units of processor time every p_S units of time, the scheduler can compute the least amount of time available to every sporadic job in the system
  - Assume that sporadic jobs are ordered among themselves according to EDF
  - When first sporadic job $S_1(t, d_{S,1}, e_{S,1})$ arrives, there is at least

    $$\lfloor (d_{S,1} - t)/p_S \rfloor e_S$$

    units of processor time available to the server before the deadline of the job
  - Therefore it accepts S_1 if the slack of the job

    $$\sigma_{S,1}(t) = \lfloor (d_{S,1} - t)/p_S \rfloor e_S - e_{S,1} \geq 0$$

Sporadic Jobs with EDF

- To decide if a new job $S_i(t, d_{S,i}, e_{S,i})$ is acceptable when there are n sporadic jobs in the system, the scheduler first computes the slack $\sigma_{S,i}(t)$ of S_i:

$$\sigma_{S,i}(t) = \lfloor (d_{S,i} - t)/p_S \rfloor e_S - e_{S,i} - \sum_{d_{S,k} < d_{S,i}} (e_{S,k} - \xi_{S,k})$$

  where $\xi_{S,k}$ is the execution time of the completed part of the existing job S_k

  Note that the sum is taken over sporadic jobs with earlier deadline as S_i since sporadic jobs are ordered according to EDF
- The job cannot be accepted if $\sigma_{S,i}(t) < 0$
- If $\sigma_{S,i}(t) \geq 0$, the scheduler checks if any existing sporadic job S_k with deadline equal to, or after $d_{S,i}$ may be adversely affected by the acceptance of S_i, i.e. check if $\sigma_{S,k}(t) \geq e_{S,i}$

Real-Time Scheduling

Resource Access Control

[Some parts of this lecture are based on a real-time systems course of Colin Perkins

http://csperkins.org/teaching/rtes/index.html]

Current Assumptions

- Single processor
- Individual jobs
  (that possibly belong to periodic/aperiodic/sporadic tasks)
  - Jobs can be preempted at any time and never suspend themselves
- Jobs are scheduled using a priority-driven algorithm
  i.e., jobs are assigned priorities, scheduler executes jobs according to these priorities
- n resources $R_1, \ldots, R_n$ of distinct types
  - used in non-preemptable and mutually exclusive manner; *serially reusable*

Motivation & Notation

Resources may represent:

- Hardware devices such as sensors and actuators
- Disk or memory capacity, buffer space
- Software resources: locks, queues, mutexes etc.

Assume a lock-based concurrency control mechanism

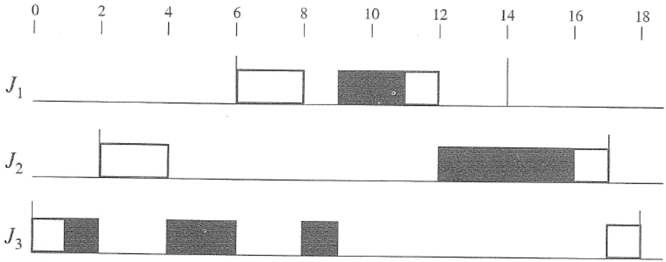
- A job wanting to use a resource R_k executes $L(R_k)$ to lock the resource R_k
- When the job is finished with the resource R_k, unlocks this resource by executing $U(R_k)$
- If lock request fails, the requesting job is **blocked** and has to wait, when the requested resource becomes available, it is unblocked

  In particular, a job holding a lock cannot be preempted by a higher priority job needing that lock

The segment of a job that begins at a lock and ends at a matching unlock is a *critical section* (CS)

- CS must be properly nested if a job needs multiple resources

Example



J_1, J_2, J_3 scheduled according to EDF.

- At 0, J_3 is ready and executes
- At 1, J_3 executes $L(R)$ and is granted R
- J_2 is released at 2, preempts J_3 and begins to execute
- At 4, J_2 executes $L(R)$, becomes blocked, J_3 executes
- At 6, J_1 becomes ready, preempts J_3 and begins to execute
- At 8, J_1 executes $L(R)$, becomes blocked, and J_3 executes
- At 9, J_3 executes $U(R)$ and both J_1 and J_2 are unblocked. J_1 has higher priority than J_2 and executes
- At 11, J_1 executes $U(R)$ and continues executing

Priority Inversion

Definition 27
Priority inversion occurs when

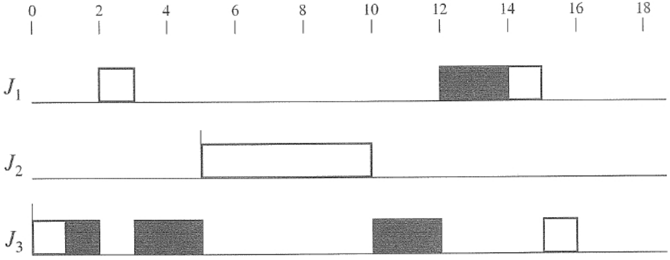
- a high priority job
- is blocked by a low priority job
- which is subsequently preempted by a medium priority job

Then effectively the medium priority job executes with higher priority than the high priority job even though they do not contend for resources

There may be arbitrarily many medium priority jobs that preempt the low priority job $\Rightarrow$ uncontrolled priority inversion

Priority Inversion – Example

Uncontrolled priority inversion:



High priority job (J_1) can be blocked by low priority job (J_3) for unknown amount of time depending on middle priority jobs (J_2)

Deadlock

Definition 28 (suitable for resource access control)

A deadlock occurs when there is a set of jobs $\mathcal{D}$ such that each job of $\mathcal{D}$ is waiting for a resource previously allocated by another job of $\mathcal{D}$.

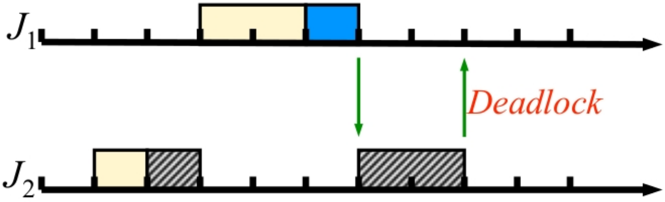
Deadlocks can be

- *detected:* regularly check for deadlock, e.g. search for cycles in a resource allocation graph regularly
- *avoided:* postpone unsafe requests for resources even though they are available (banker's algorithm, priority-ceiling protocol)
- *prevented:* many methods invalidating sufficient conditions for deadlock (e.g., impose locking order on resources)

See your operating systems course for more information ....

Deadlock – Example

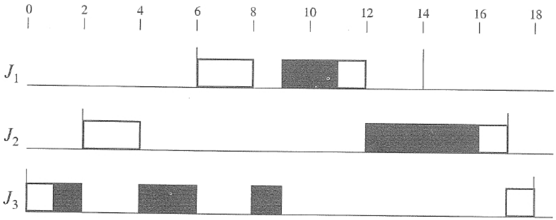
Deadlock can result from piecemeal acquisition of resources: classic example of two jobs J_1 and J_2 both needing both resources R and R'



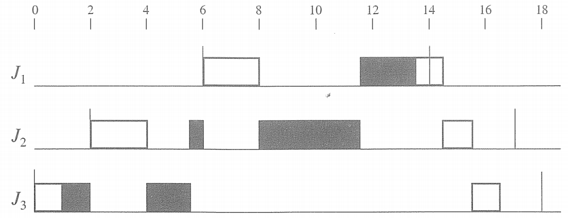
- J_2 locks R' and J_1 locks R
- J_1 tries to get R' and is blocked
- J_2 tries to get R and is blocked

Timing Anomalies due to Resources

Previous example, the critical section of J_3 has length 4



... the critical section of J_3 shortened to 2.5



... but response of J_1 becomes longer!

Controlling Timing Anomalies

Contention for resources causes timing anomalies, priority inversion and deadlock

Several protocols exist to control the anomalies

- Non-preemptive CS
- Priority inheritance protocol
- Priority ceiling protocol
- ....

Terminology:

- A job J_h is *blocked* by a job J_k when
  - the priority of J_k is lower than the priority of J_h and
  - J_k holds a resource R and
  - J_h executes $L(R)$.

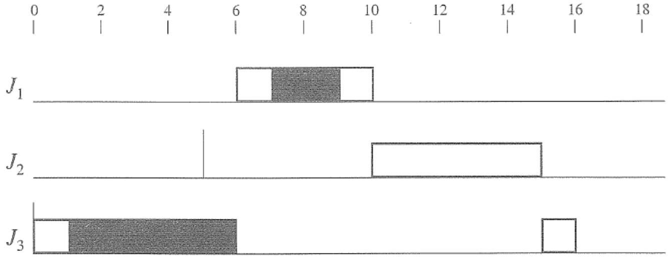
  In such situation we sometimes say that J_h is blocked by the corresponding critical section of J_k.

Non-preemptive Critical Sections

The protocol: when a job locks a resource, it is scheduled with priority higher than all other jobs (i.e., is non-preemptive)

Example 29

Jobs J_1, J_2, J_3 with release times $2, 5, 0$, resp., and with execution times $4, 5, 7$, resp.



Non-preemptive Critical Sections – Features

- no deadlock as no job holding a resource is ever preempted
- no priority inversion:
  - A job J_h can be blocked (by a lower priority job) *only at release time*.

    (Indeed, if J_h is not blocked at the release time r_h, it means that no lower priority job holds any resource at r_h. However, no lower priority job can be executed before completion of J_h, and thus no lower priority job may block J_h.)
  - If J_h is blocked at release time, then once the blocking critical section completes, no lower priority job can block J_h.
  - It follows that any job can be blocked only once, at release time, blocking time is bounded by duration of one critical section of a lower priority job.

Advantage: very simple; easy to implement both in fixed and dynamic priority; no prior knowledge of resource demands of jobs needed

Disadvantage: every job can be blocked by every lower-priority job with a critical section, even if there is no resource conflict

Priority-Inheritance Protocol

Idea: adjust the scheduling priorities of jobs during resource access, to reduce the duration of timing anomalies

(As opposed to non-preemptive CS protocol, this time the priority is not always increased to maximum)

Notation:

- *assigned priority* = priority assigned to a job according to a standard scheduling algorithm
- At any time t, each ready job J_k is scheduled and executes at its *current priority* $\pi_k(t)$ which may differ from its assigned priority and may vary with time
  - The current priority $\pi_k(t)$ of a job J_k may be raised to the higher priority $\pi_h(t)$ of another job J_h
  - In such a situation, the lower-priority job J_k is said to *inherit* the priority of the higher-priority job J_h, and J_k executes at its inherited priority $\pi_h(t)$

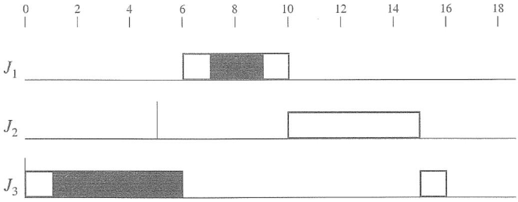
Priority-Inheritance Protocol

- **Scheduling rules**:
  - Jobs are scheduled in a preemptable priority-driven manner *according to their current priorities*
  - At release time, the current priority of a job is equal to its assigned priority
  - The current priority remains equal to the assigned priority, except when the priority-inheritance rule is invoked
- **Priority-inheritance rule**:
  - When a job J_h becomes blocked on a resource R, the job J_k which blocks J_h inherits the current priority $\pi_h(t)$ of J_h;
  - J_k executes at its inherited priority until it releases R; at that time, the priority of J_k is *set to the highest priority of all jobs still blocked by J_k after releasing R.* (the resulting priority may still be an inherited priority)
- **Resource allocation**: When a job J requests a resource R at t:
  - If R is free, R is allocated to J until J releases it
  - If R is not free, the request is denied and J is blocked

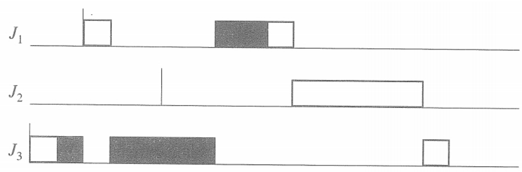
(Note that J is only denied R if the resource is held by another job.)

Priority-Inheritance Simple Example

non-preemptive CS:

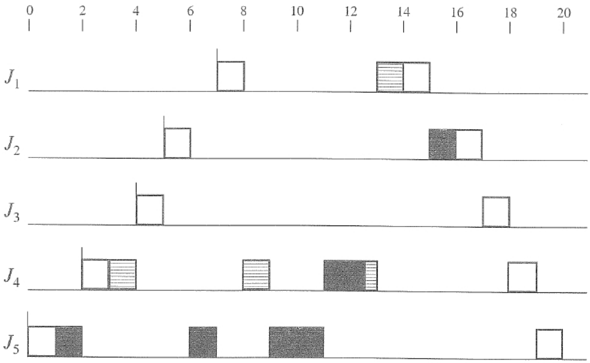


priority-inheritance:



- At 3, J_1 is blocked by J_3, J_3 inherits priority of J_1
- At 5, J_2 is released but cannot preempt J_3 since the inherited priority of J_3 is higher than the (assigned) priority of J_2

Priority-Inheritance Example



- At 0, J_5 starts executing at priority 5, at 1 it executes $L(\mathit{Black})$
- At 2, J_4 preempts J_5 and executes
- At 3, J_4 executes $L(\mathit{Shaded})$, J_4 continues to execute
- At 4, J_3 preempts J_4; at 5, J_2 preempts J_3
- At 6, J_2 executes $L(\mathit{Black})$ and is blocked by J_5. Thus J_5 inherits the priority 2 of J_2 and executes
- At 8, J executes $L(\mathit{Shaded})$ and is blocked by J. Thus J inherits the

Properties of Priority-Inheritance Protocol

- Simple to implement, does not require prior knowledge of resource requirements
- Jobs exhibit two types of "blocking"
  - (Direct) blocking due to resource locks
    i.e., a job J_ℓ locks a resource R, J_h executes $L(R)$ is directly blocked by J_ℓ on R
  - Priority-inheritance "blocking"
    i.e., a job J_h is preempted by a lower-priority job that inherited a higher priority
- Jobs may exhibit transitive blocking
  In the previous example, at 9, J_5 blocks J_4 and J_4 blocks J_1, hence J_5 inherits the priority of J_1
- Deadlock is *not* prevented
  In the previous example, let J_5 request *shaded* at 6.5, then J_4 and J_5 become deadlocked
- Can reduce blocking time (see next slide) compared to non-preemptable CS but does not guarantee to minimize blocking

Priority-Inheritance – Blocking Time (Optional)

$z_{\ell,k}$ = the k-th critical section of J_ℓ

A job J_h is blocked by $z_{\ell,k}$ if J_h has higher assigned priority than J_ℓ but has to wait for J_ℓ to exit $z_{\ell,k}$ in order to continue

$\beta^*_{h,\ell}$ = the set of all maximal critical sections $z_{\ell,k}$ that *may* block J_h, i.e., which correspond to resources that are (potentially) used by jobs with priorities equal or higher than J_h.

(recall that CS are properly nested, maximal CS which may block J_h is the one which is not contained within any other CS which may block J_h)

Theorem 30
Let J_h be a job and let $J_{h+1}, \ldots, J_{h+m}$ be jobs with lower priority than J_h. Then J_h can be blocked for at most the duration of one critical section in each *of $\beta^*_{h,\ell}$ where $\ell \in \{h+1, \ldots, h+m\}$.*

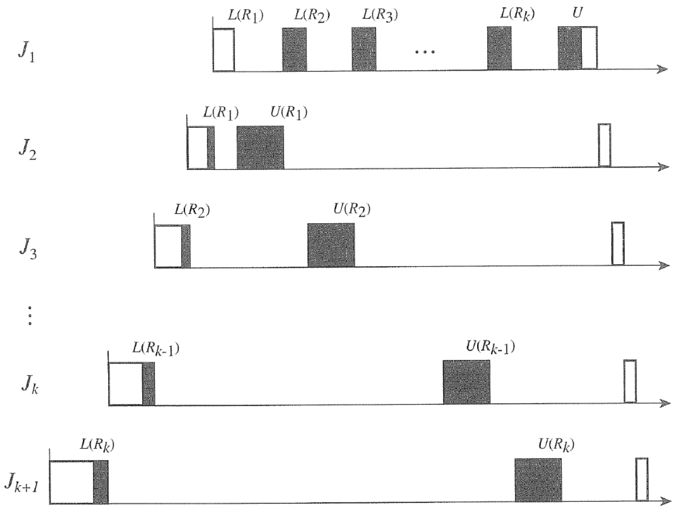
The theorem is a direct consequence of the next lemma.

Lemma 31

J_h can be blocked by J_ℓ only if J_ℓ is executing within a critical section $z_{\ell,k}$ of $\beta^*_{h,\ell}$ when J_h is released

- Assume that J_h is released at t and J_ℓ is in no CS of $\beta^*_{h,\ell}$ at t. We show that J_ℓ never executes between t and completion of J_h:
  - If J_ℓ is not in any CS at t, then its current priority at t is equal to its assigned priority and cannot increase. Thus, J_ℓ has to wait for completion of J_h as the current priority of J_h is always higher than the assigned priority of J_ℓ.
  - If J_ℓ is still in a CS at t, then this CS does not belong to $\beta^*_{h,\ell}$ and thus cannot block J_h before completion and cannot execute before completion of J_h.
- Assume that J_ℓ leaves $z_{\ell,k} \in \beta^*_{h,\ell}$ at time t. We show that J_ℓ never executes between t and completion of J_h:
  - If J_ℓ is not in any CS at t, then, as above, J_ℓ never executes before completion of J_h and cannot block J_h.
  - If J_ℓ is still in a CS at t, then this CS does not belong to $\beta^*_{h,\ell}$ because otherwise $z_{\ell,k}$ would not be maximal. Thus J_ℓ cannot block J_h, and thus J_ℓ is never executed before completion of J_h.

Priority-Inheritance – The Worst Case



J_1 is blocked for the total duration of all critical sections in all lower priority jobs.

Priority-Ceiling Protocol

The goal: to furhter reduce blocking times due to resource contention and to prevent deadlock

- in its basic form priority-ceiling protocol works under the assumption that the priorities of jobs and resources required by all jobs are known apriori

  can be extended to dynamic priority (job-level fixed priority), see later

Notation:

- The *priority ceiling* of any resource R_k is the highest priority of all the jobs that require R_k and is denoted by $\Pi(R_k)$
- At any time t, the current priority ceiling $\Pi(t)$ of the system is equal to the highest priority ceiling of the resources that are in use at the time
- If all resources are free, $\Pi(t)$ is equal to Ω, a newly introduced priority level that is lower than the lowest priority level of all jobs

Priority-Ceiling Protocol

The scheduling and priority-inheritance rules are the same as for priority-inheritance protocol

- **Scheduling rules**:
  - Jobs are scheduled in a preemptable priority-driven manner *according to their current priorities*
  - At release time, the current priority of a job is equal to its assigned priority
  - The current priority remains equal to the assigned priority, except when the priority-inheritance rule is invoked
- **Priority-inheritance rule**:
  - When job J_h becomes blocked on a resource R, the job J_k which blocks J_h inherits the current priority $\pi_h(t)$ of J_h;
  - J_k executes at its inherited priority until it releases R; at that time, the priority of J_k is *set to the highest priority of all jobs still blocked by J_k after releasing R.* (which may still be an inherited priority)

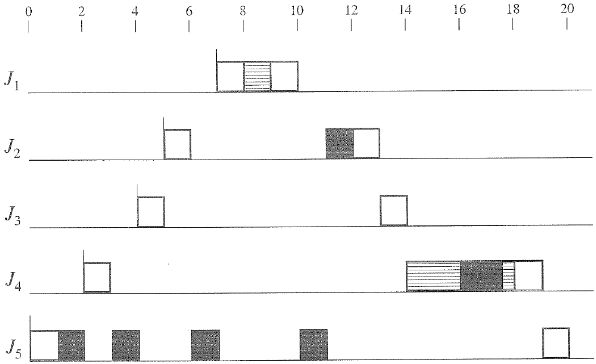
Priority-Ceiling Protocol

Resource allocation rules:

- When a job J requests a resource R held by another job, the request fails and the requesting job blocks
- When a job J requests a resource R at time t, and that resource is free:
  - If J's priority $\pi(t)$ is *strictly higher* than current priority ceiling $\Pi(t)$, R is allocated to J
  - If J's priority $\pi(t)$ is not higher than $\Pi(t)$, R is allocated to J only if J is the job holding the resource(s) whose priority ceiling is equal to $\Pi(t)$, otherwise J is blocked
    (Note that only one job may hold the resources whose priority ceiling is equal to $\Pi(t)$)

Note that unlike priority-inheritance protocol, the priority-ceiling protocol can deny access to an available resource.

Priority-Ceiling Protocol



- At 1, $\Pi(t) = \Omega$, J_5 executes $L(\mathit{Black})$, continues executing
- At 3, $\Pi(t) = 2$, J_4 executes $L(\mathit{Shaded})$; because the ceiling of the system $\Pi(t)$ is higher than the current priority of J_4, job J_4 is blocked, J_5 inherits J_4's priority and executes at priority 4
- At 4, J_3 preempts J_5; at 5, J_2 preempts J_3. At 6, J_2 requests *Black* and is directly blocked by J_5. Consequently, J_5 inherits priority 2 and executes until preempted by J_1

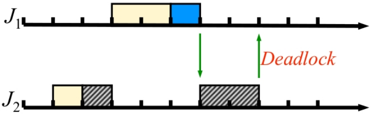
Priority-Ceiling Protocol

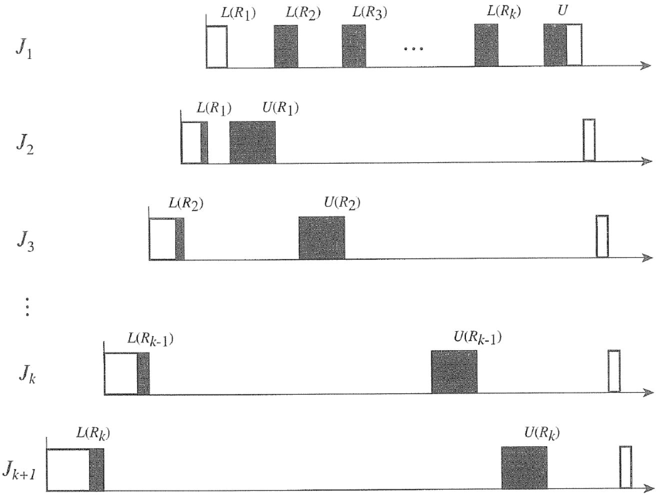
Theorem 32

Assume a system of preemptable jobs with fixed assigned priorities. Then

- *deadlock may never occur,*
- *a job can be blocked for at most the duration of one critical section.*

These situations cannot occur with priority ceiling protocol:





Differences between the priority-inheritance and priority-ceiling

- Priority-inheritance is greedy, while priority ceiling is not

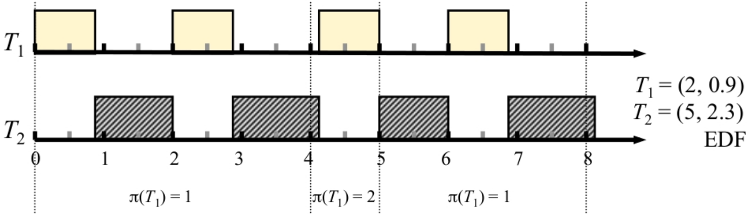
  The priority-ceiling protocol may withhold access to a free resource, i.e., a job can be prevented from execution by a lower-priority job which does not hold the requested resource – *avoidance "blocking"*
- The priority ceiling protocol forces a fixed order onto resource accesses thus eliminating deadlock

Resources in Dynamic Priority Systems

The priority ceiling protocol assumes fixed and known priorities

In a dynamic priority system, the priorities of the periodic tasks change over time, while the set of resources is required by each task remains constant

- As a consequence, the priority ceiling of each resource changes over time



What happens if T_1 uses resource X, but T_2 does not?

- Priority ceiling of X is 1 for $0 \leq t \leq 4$, becomes 2 for $4 \leq t \leq 5$, etc. even though the set of resources is required by the tasks remains unchanged

Resources in Dynamic Priority Systems

- If a system is job-level fixed priority, but task-level dynamic priority, a priority ceiling protocol can still be applied
  - Each job in a task has a fixed priority once it is scheduled, but may be scheduled at different priority to other jobs in the task (e.g. EDF)
  - Update the priority ceilings of all resources each time a new job is introduced; use until updated on next job release
- Has been proven to prevent deadlocks and no job is ever blocked for longer than the length of one critical section
  - But: very inefficient, since priority ceilings updated frequently
  - May be better to use priority inheritance, accept longer blocking

Schedulability Tests with Resources

How to adjust schedulability tests?

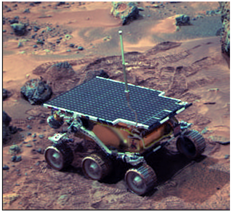
Add the blocking times to execution times of jobs; then run the test as normal

The blocking time b_i of a job J_i can be determined for all three protocols:

- non-preemptable CS $\Rightarrow$ b_i is bounded by the maximum length of a critical section in lower priority jobs
- priority-inheritance $\Rightarrow$ b_i is bounded by the total length of the m longest critical sections where m is the number of jobs that may block J_i

  (For a more precise formulation see Theorem 2.)
- priority-ceiling $\Rightarrow$ b_i is bounded by the maximum length of a critical section

Mars Pathfinder vs Priority Inversion



- Mars Pathfinder = a US spacecraft that landed on Mars in July 4th, 1997.
- Consisted of a lander and a lightweight wheeled robotic Mars rover called Sojourner
  - What Happened:
    - Few days in to the mission, not long after Pathfinder started gathering meteorological data, it began experiencing total system resets, each resulting in losses of data.
    - Apparently a software problem caused these resets.
  - The system:
    - Pathfinder used the well-known real-time embedded systems kernel VxWorks by Wind River.
    - VxWorks uses preemptive priority-based scheduling, in this case a deadline monotonic algorithm.
    - Pathfinder contained an "information bus" (a shared memory) used for communication, synchronized by locks.

Mars Pathfinder – The Problem

- Problematic tasks:
  - A bus management task ran frequently with high priority to move data in/out of the bus. If the bus has been locked, then this thread itself had to wait.
  - A meteorological data gathering task ran as an infrequent, low priority thread, and used the bus to publish its data.
  - The bus was also used by a communication task that ran with medium priority.
- Occasionally the communication task (medium priority) was invoked at the precise time when the bus management task (high priority) was blocked by the meteorological data gathering task (low priority) – priority inversion!
- The bus management task was blocked for considerable amount of time by the communication task, which caused a watchdog timer to go off, notice that the bus management task has not been executed for some time, which typically means that something had gone drastically wrong, and initiate a total system reset.

Mars Pathfinder – Solution

- JPL (Jet Propulsion Laboratory) engineers spent hours and hours running the system on a spacecraft replica.
- Early in the morning, after all but one engineer had gone home, the engineer finally reproduced a system reset on the replica.

Solution: Turn the priority inheritance on!

This was done online using a C language interpreter which allowed to execute C functions on-the-fly.

A short code changed a mutex initialization parameter from FALSE to TRUE.

Real-Time Scheduling

Multiprocessor Real-Time Systems

Multiprocessor Real-time Systems

- Many embedded systems are composed of many processors (control systems in cars, aircraft, industrial systems etc.)
- Today most processors in computers have multiple cores

  The main reason is that increasing frequency of a single processor is no more feasible (mostly due to power consumption problems, growing leakage currents, memory problems etc.)

Applications must be developed specifically for multiprocessor systems.

Multiprocessor Frustration

In case of real-time systems, multiple processors bring serious difficulties concerning correctness, predictability and efficiency.

The “root of all evil” in global scheduling: (Liu, 1969)

Few of the results obtained for a single processor generalize directly to the multiple processor case; bringing in additional processors adds a new dimension to the scheduling problem. The simple fact that a task can use only one processor even when several processors are free at the same time adds a surprising amount of difficulty to the scheduling of multiple processors.

The Model

- A *job* is a unit of work that is scheduled and executed by a system

  (Characterised by the release time r_i, execution time e_i and deadline d_i)
- A *task* is a set of related jobs which jointly provide some system function
- Jobs execute on *processors*

In this lecture we consider m processors

- Jobs may use some (shared) passive *resources*

Schedule

Schedule assigns, in every time instant, processors and resources to jobs.

A schedule is *feasible* if *all jobs with hard real-time constraints* complete before their deadlines.

A set of jobs is *schedulable* if there is a feasible schedule for the set.

A scheduling algorithm is optimal if it always produces a feasible schedule whenever such a schedule exists.
(and if a cost function is given, minimizes the cost)

We also consider *online* scheduling algorithms that do not use any knowlede about jobs that will be released in the future but are given a complete information about jobs that have been released.
(e.g. EDF is online)

Multiprocessor Taxonomy

- Identical processors: All processors identical, have the same computing power
- Uniform processors: Each processor is characterized by its own computing capacity κ, completes κt units of execution after t time units
- Unrelated processors: There is an execution rate ρ_{ij} associated with each job-processor pair (J_i, P_j) so that J_i completes $\rho_{ij} t$ units of execution by executing on P_j for t time units

In addition, cost of communication can be included etc.

Assumptions – Priority Driven Scheduling

Throughout this lecture we assume:

- Unless otherwise stated, consider *m identical* processors
- Jobs can be preempted at any time and never suspend themselves
- Context switch overhead is negligibly small

  i.e. assumed to be zero
- There is an unlimited number of priority levels
- For simplicity, we assume *independent* jobs that do not contend for resources

Unless otherwise stated, we assume that scheduling decisions take place only when a job is released, or completed.

Multiprocessor Scheduling Taxonomy

Multiprocessor scheduling attempts to solve two problems:

- the *allocation problem*, i.e., on which processor a given job executes
- the *priority problem*, i.e., when and in what order the jobs execute

Issues

What results from single processor scheduling remain valid in multiprocessor setting?

- Are there simple optimal scheduling algorithms?
- Are there optimal *online* scheduling algorithms
  (i.e. those that do not know what jobs come in future)
- Are there efficient tests for schedulability?

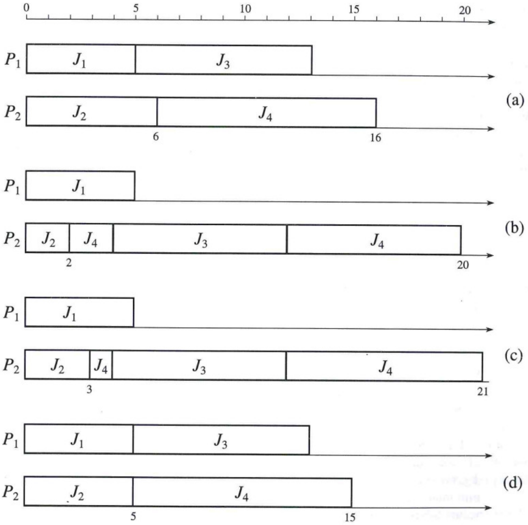
In this lecture we consider:

- Individual jobs
- Periodic tasks

Start with n individual jobs $\{J_1, \ldots, J_n\}$

Individual Jobs – Timing Anomalies

Priority order: $J_1 \sqsupset \cdots \sqsupset J_4$



Individual Jobs – EDF

EDF on m identical processors: At any time instant, jobs with the earliest absolute deadlines are executed on available processors.
(Recall: no job can be executed on more than one processor at a given time!)

Is this optimal? NO!

Example:
J_1, J_2, J_3 where

- $r_i = 0$ for $i \in \{1, 2, 3\}$
- $e_1 = e_2 = 1$ and $e_3 = 5$
- $d_1 = 1$, $d_2 = 2$, $d_3 = 5$

2 processors.

Individual Jobs – Speedup Helps(?)

Theorem 33
If a set of jobs is feasible on m identical processors, then the same set of jobs will be scheduled to meet all deadlines by EDF on identical processors in which the individual processors are $(2 - \frac{1}{m})$ *times as fast as in the original system.*

The result is tight for EDF (assuming dynamic job priority):

Theorem 34
There are sets of jobs that can be feasibly scheduled on m identical processors but EDF cannot schedule them on m processors that are only $(2 - \frac{1}{m} - \varepsilon)$ *faster for every* $\varepsilon > 0$.

... there are also general lower bounds for online algorithms:

Theorem 35
There are sets of jobs that can be feasibly scheduled on m (here m is even) identical processors but **no online** *algorithm can schedule them on m processors that are only* $(1 + \varepsilon)$ *faster for every* $\varepsilon < \frac{1}{5}$.

[Optimal Time-Critical Scheduling Via Resource Augmentation, Phillips et al, STOC 1997]

Reactive Systems

Consider fixed number, n, of *independent periodic* tasks
$\mathcal{T} = \{T_1, \ldots, T_n\}$
i.e. there is no dependency relation among jobs

- Unless otherwise stated, assume no phase and deadlines equal to periods
- Ignore aperiodic tasks
- No sporadic tasks unless otherwise stated

Utilization u_i of a periodic task T_i with period p_i and execution time e_i is defined by $u_i := e_i/p_i$
u_i is the fraction of time a periodic task with period p_i and execution time e_i keeps a processor busy

Total utilization $U^{\mathcal{T}}$ of a set of tasks $\mathcal{T} = \{T_1, \ldots, T_n\}$ is defined as the sum of utilizations of all tasks of $\mathcal{T}$, i.e. by $U^{\mathcal{T}} := \sum_{i=1}^{n} u_i$

Given a scheduling algorithm *ALG*, the schedulable utilization U_{ALG} of *ALG* is the maximum number U such that for all $\mathcal{T}$: $U_{\mathcal{T}} \leq U$ implies $\mathcal{T}$ is schedulable by *ALG*.

Multiprocessor Scheduling Taxonomy

Allocation (migration type)

- No migration: each **task** is allocated to a processor
- (Task-level migration: **jobs** of a task may execute on different processors; however, each job is assigned to a single processor)
- Job-level migration: A single job can migrate and execute on different processors

  (however, parallel execution of one job is not permitted and migration takes place only when the job is rescheduled)

Priority type

- Fixed task-level priority (e.g. EDF)
- Fixed job-level priority (e.g. RM)
- (Dynamic job-level priority)

Partitioned scheduling = No migration

Global scheduling = job-level migration

Fundamental Limit – Fixed Job-Level Priority

Consider m processors and $m+1$ tasks $\mathcal{T} = \{T_1, \ldots, T_{m+1}\}$, each $T_i = (L, 2L-1)$.

Then
$U_{\mathcal{T}} = \sum_{i=1}^{m+1} L/(2L-1) = (m+1)\,(L/(2L-1)) = (m+1)/2 \cdot L/(L-1)$
For very large L, this number is close to $(m+1)/2$.

The set $\mathcal{T}$ is not schedulable using any *fixed job-level* priority algorithm.

In other words, the schedulable utilization of fixed job-level priority algorithms is at most $(m+1)/2$, i.e., half of the processors capacity.

There are variants of EDF achieving this bound (see later slides).

Partitioned vs Global Scheduling

Most algorithms up to the end of 1990s based on *partitioned scheduling*

- no migration

From the end of 1990s, many results concerning *global scheduling*

- job-level migration

The task-level migration has not been much studied, so it is not covered in this lecture.

We consider fixed job-level priority (e.g. EDF) and fixed task-level priority (e.g. RM).

As before, we ignore dynamic job-level priority.

Partitioned Scheduling & Fixed Job-Level Priority

The algorithm proceeds in two phases:

1. Allocate tasks to processors, i.e., partition the set of tasks into m possibly empty *modules* $M_1, \ldots, M_m$
2. Schedule tasks of each M_i on the processor i according to a given single processor algorithm

The quality of task assignment is determined by the number of assigned processors

- Use EDF to schedule modules
- Suffices to test whether the total utilization of each module is ≤ 1

  (or, possibly, $\leq \hat{U}$ where $\hat{U} < 1$ in order to accomodate aperiodic jobs ...)

Finding an optimal schedule is equivalent to a simple *uniform-size bin-packing problem* (and hence is NP-complete)

Similarly, we may use RM for fixed task-level priorities (total utilization in modules $\leq \log 2$, etc.)

Global Scheduling

- All ready jobs are kept in a global queue
- When selected for execution, a job can be assigned to any processor
- When preempted, a job goes to the global queue (i.e., forgets on which processor it executed)

Global Scheduling – Fixed Job-Level Priority

Dhall's effect:

- Consider $m > 1$ processors
- Let $\varepsilon > 0$
- Consider a set of tasks $\mathcal{T} = \{T_1, \ldots, T_m, T_{m+1}\}$ such that
  - $T_i = (2\varepsilon, 1)$ for $1 \leq i \leq m$
  - $T_{m+1} = (1, 1 + \varepsilon)$
- $\mathcal{T}$ is schedulable
- Stadnard EDF and RM schedules are not feasible (whiteb.)

However,

$$U_{\mathcal{T}} = m\frac{2\varepsilon}{1} + \frac{1}{1+\varepsilon}$$

which means that for small ε the utilization $U_{\mathcal{T}}$ is close to 1
(i.e., $U_{\mathcal{T}}/m$ is very small for $m >> 0$ processors)

How to avoid Dhall's effect?

- Note that RM and EDF only account for task periods and ignore the execution time!
- (Partial) Solution: Dhall's effect can be avoided by giving high priority to tasks with high utilization

  Then in the previous example, T_{m+1} is executed whenever it comes and the other tasks are assigned to the remaining processors – produces a feasible schedule

Global Scheduling – Fixed Job-Level Priority

Apparently there is a problem with long jobs due to Dhall's effect.

There is an improved version of EDF called EDF-US(1/2) which

- assigns the highest priority to tasks with $u_i \geq 1/2$
- assigns priorities to the rest according to deadlines

which reaches the generic schedulable utilization bound $(m + 1)/2$.

Partitioned vs Global

Advantages of the global scheduling:

- Load is automatically balanced
- Better average response time (follows from queueing theory)

Disadvantages of the global scheduling:

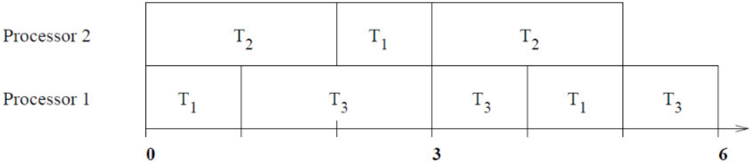
- Problems caused by migration (e.g. increased cache misses)
- Schedulability tests more difficult (active area of research)

Is either of the approaches better from the schedulability standpoint?

Global Beats Partitioned

There are sets of tasks schedulable only with global scheduler:

- $\mathcal{T} = \{T_1, T_2, T_3\}$ where $T_1 = (1,2)$, $T_2 = (2,3)$, $T_3 = (2,3)$, can be scheduled using a global scheduler:

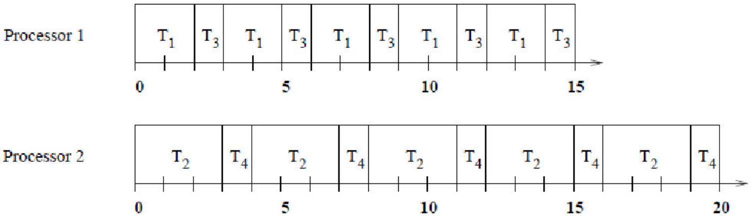


- No feasible partitioned schedule exists, always at least one processor gets tasks with total utilization higher than 1.

Partitioned Beats Global

There are task sets that can be scheduled only with partitioned scheduler (assuming fixed task-level priority assignment):

- $\mathcal{T} = \{T_1, \ldots, T_4\}$ where $T_1 = (2, 3)$, $T_2 = (3, 4)$, $T_3 = (5, 15)$, $T_4 = (5, 20)$, can be scheduled using a fixed task-level priority partitioned schedule:



- Global scheduling (fixed job-level priority): There are 9 jobs released in the interval $[0, 12)$. Any of the 9! possible priority assignments leads to a deadline miss.

Optimal Algorithm?

There IS an optimal algorithm in the case of job-level migration & dynamic job-level priority. However, the algorithm is *time driven*.

The *priority fair* (PFair) algorithm is optimal for periodic systems with deadlines equal to periods

Idea (of PFair): In any interval $(0, t]$ jobs of a task T_i with utilization u_i execute for amount of time W so that $u_i t - 1 < W < u_i t + 1$
(Here every parameter is assumed to be a natural number)

This is achieved by cutting time into small quanta and scheduling jobs in these quanta so that the execution times are always (more or less) in proportion.

There are other optimal algorithms, all of them suffer from a large number of preemptions/migrations.

No optimal algorithms are known for more general settings: deadlines bounded by periods, arbitrary deadlines.

Recall, that no optimal *on-line* scheduling possible