

IA159 Formal Verification Methods

Introduction

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Agenda

- basic information about the course
- quick overview of formal methods
- selected topics

What does “Formal Verification Methods” mean?

Formal methods are a collection of notations and techniques for describing and analyzing systems. Methods are **formal** in the sense that they are based on some mathematical theories, such as logic, automata or graph theory. [Pel01]

Verification is the process of applying a manual or an automatic technique that is supposed to establish whether the code either satisfies a given property or behaves in accordance with some higher-level description of it. [Pel01]

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In other words, methods that can find a bug or prove its absence.

Focus of the course

- The course is focused on theoretical and algorithmic bases of selected verification methods.
- The software engineering aspects of verification methods are beyond the scope of this course.

- Books (cover only some topics of the course):
 - *D. A. Peled: Software Reliability Methods, Springer, 2001.*
 - *E. M. Clarke, O. Grumberg, and D. A. Peled: Model Checking, MIT, 1999.*
 - *Ch. Baier and J.-P. Katoen: Principles of Model Checking, MIT, 2008.*
 - *D. S. Scott: The Seventeen Provers of the World, Springer, 2006.*
 - ...
- Other sources (mainly journal or conference papers) will be referred and available in Study materials in IS.

Mandatory prerequisites

- IV113 Introduction to Validation and Verification or
- IA169 System Verification and Assurance

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Other relevant courses

- IA006 Automata theory (aka FJA II)
- IA040 Modal and Temporal Logics for Processes
- IV101 Seminar on Verification
- IV022 Design and Verification of Algorithms

- There will be an **oral exam** at the end.
- No intrasemestral tests, no written exams, no mandatory homeworks.

Overview of verification methods

Basic verification methods

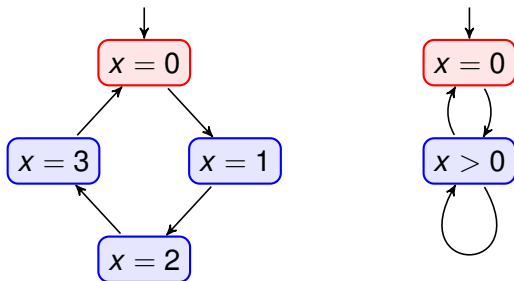
- testing
- deductive verification (with use of theorem provers)
- equivalence checking
- reachability analysis and model checking
- abstract interpretation and other static analyses
- symbolic execution

Other related techniques

- abstraction
- slicing
- SAT/SMT solving
- Craig interpolation

Abstraction

- reduces the size of systems to be analyzed
- can transform an infinite-state system into a finite one
- the set of system behaviours is usually increased (source of false alarms)



- reduces the size of systems on the source code level
- the reduced system preserves values of given variables at given control locations
- M. Weiser: *Program Slicing*, IEEE Transactions on Software Engineering 10(4), 1984.

Slicing: example

```
1: char *copy(char *dst, char *src, int n, int *L) {
2:     int i, len;
3:     len = 0;
4:     if (src != NULL && dst != NULL) {
5:         len = n;
6:         lock(L);
7:     }
8:     i = 0;
9:     while (i < len) {
10:        dst[i] = src[i];
11:        i++;
12:    }
13:    if (len > 0) {
14:        unlock(L);
15:    }
16:    return dst;
17: }
```

Assume that we are interested only in values of lock L at the end of line 16.

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SAT problem is to decide satisfiability of a given propositional logic formula.

Satisfiability Modulo Theories (SMT) problem is to decide satisfiability of a given first-order logic formula with respect to a given theory (e.g. theory of integers with addition and subtraction).

- crucial for symbolic execution, abstraction, deductive verification
- A. R. Bradley and Z. Manna: *The Calculus of Computation: Decision Procedures with Applications to Verification*, Springer, 2007.

Craig interpolation

- if $\varphi \implies \psi$ then there exists an **interpolant** ρ such that $\varphi \implies \rho \implies \psi$ and ρ uses only propositional variables occurring in both φ and ψ
- ρ overapproximates φ and it is usually smaller than φ
- crucial for ic3, Ultimate Automizer, and many methods/tools using abstraction refinement
- W. Craig: *Three uses of the Herbrand-Gentzen theorem in relating model theory and proof theory*, The Journal of Symbolic Logic 22(3), 1957.

- simple, feasible, very good cost/performance ratio
- very effective in early stages of debugging process
- applicable directly to real systems
- cannot guarantee that there are no errors
- **in practice**: standard technique for enhancing the quality of systems, wide tool support

Deductive verification

Deductive verification is a method for proving that, for any input values satisfying a given initial condition, a given program terminates and resulting variable values satisfy a given final assertion.

If initial condition $x2 > 0$ holds, then the execution of

```
y1=0;  
y2=0;  
while (y2 < x2) {  
    y1 = y1 + x1;  
    y2++;  
}
```

always terminates and the resulting variable values satisfy final assertion

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always terminates and the resulting variable values satisfy final assertion $y1 = x1 * x2$.

Deductive verification

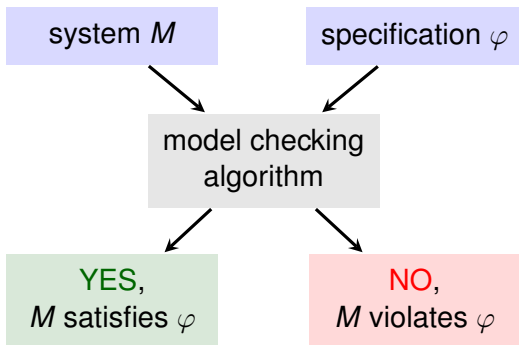
- applicable to models or small parts of real systems
- needs a huge effort of an expert on both deductive verification and systems under verification
- can guarantee that (a model of) a real system satisfies a given property
- **in practice:** used rarely (e.g. partial correctness of FPU in AMD processors)
- **tools:** Coq, ACL2, Dafny, ...

Equivalence checking decides whether two given systems are equivalent with respect to a given equivalence.

- applicable mainly to models of real systems
- needs a detailed formal specification of a system under verification (or another “second system”)
- there are no algorithms for reasonable equivalences and infinite-state systems
- **in practice**: some specific applications (e.g. equivalence of different levels of hardware design)

Reachability analysis and model checking

Reachability analysis decides whether any run of a given system can reach a given state. Model checking decides whether each run of a given system satisfies a given specification property (which is typically described by a temporal logic formula).

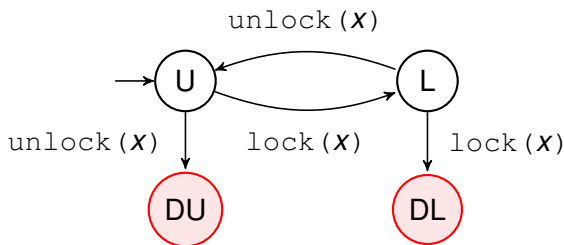


- needs formal description of the property to be checked
- fully automatic, but feasible mainly for relatively small finite-state systems
- successful verification of real systems may require provision of a suitable abstraction
- **in practice**: a standard technique for verification of simple hardware designs, used also for verification of small systems (e.g. communication protocols)
- **tools**: DIVINE, SPIN, NuSMV, . . .

Abstract interpretation and other static analyses

Abstract interpretation and other static analyses are typically used to overapproximate or underapproximate a set of reachable states of selected program variables in each program location. The analyzed code is not executed.

Consider the following states of a lock x :



U = unlocked
L = locked

error states: DU = double unlock
DL = double lock

Abstract interpretation and other static analyses

```
1: char *copy(char *dst, char *src, int n, int *L) {
2:     int i, len;                                U
3:     len = 0;                                    U
4:     if (src != NULL && dst != NULL) {          U
5:         len = n;                                U
6:         lock(L);                                L
7:     }                                           U,L
8:     i = 0;                                       U,L
9:     while (i < len) {                            U,L
10:        dst[i] = src[i];                          U,L
11:        i++;                                       U,L
12:    }                                           U,L
13:    if (len > 0) {                                U,L
14:        unlock(L);                                DU,U
15:    }                                           U,L
16:    return dst;                                    U,L
17: }
```

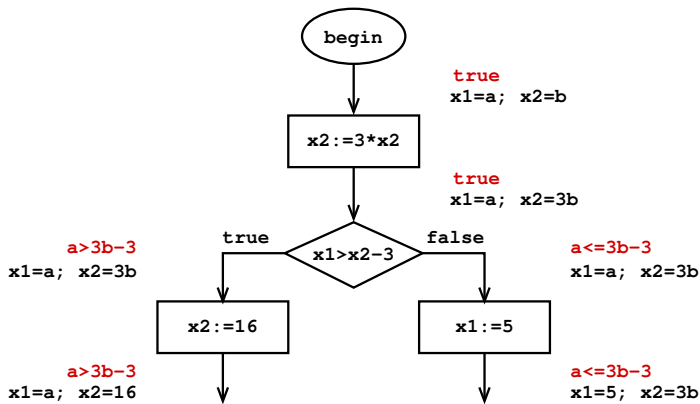
The indicated double unlock error is a false positive.

Abstract interpretation and other static analyses

- applicable directly to source code of real systems (or directly to executables)
- feasible
- can verify only a specific class of properties (including many interesting properties)
- may produce false alarms
- fully automatic
- **in practice:** some static analysis is performed by almost every compiler, there are many efficient tools able to work with big pieces of real software (e.g. Linux kernel)
- **tools:** Coverity, CodeSonar, Stansen, . . .

Symbolic execution

Symbolic execution executes the code on abstract symbols instead of input values.



Symbolic execution

- can be seen as exhaustive testing
- applicable directly to source code of real systems (or directly to executables)
- fully automatic
- does not report false alarms
- feasible, but the computation usually did not finish due to large or even infinite number of execution paths
- **in practice**: several successful applications, but computational cost of pure symbolic execution is too high
- **tools**: Klee, . . .

Combined methods

- popular combinations:
 - model checking + abstraction + counter-example guided abstraction refinement (CEGAR)
 - abstract interpretation + CEGAR
 - testing + model checking
 - testing + symbolic execution + Craig interpolation
- the aim is to develop methods which are automatic (as much as possible) and applicable directly to sources or binaries of real systems
- may be incomplete and/or produce some false alarms
- **in practice**: already has some specific applications in verification (e.g. verification of Windows drivers by **Static Driver Verifier**, **CPAchecker**, **Ultimate Automizer**) and many applications in test-generation and bug-finding (e.g. **SAGE**, **PEX**, **CBMC**)
- **the most promising approaches usually combine several basic techniques**

Verification of infinite-state systems

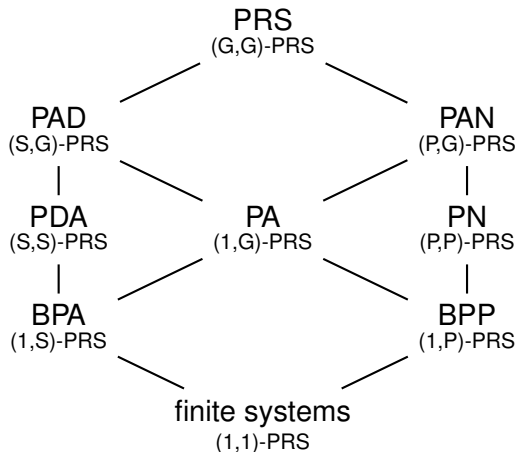
Finite vs. infinite-state systems

```
y1=0;  
y2=0;  
while (y2 < x2) {  
    y1 = y1 + x1;  
    y2++;  
}
```

- verification of algorithm vs. verification of programs
- all verification problems are decidable for finite systems
- for infinite-state systems, decidability depends on the problem and type of the system
- explicit and symbolic (BDD-based) model checking applicable only to finite systems

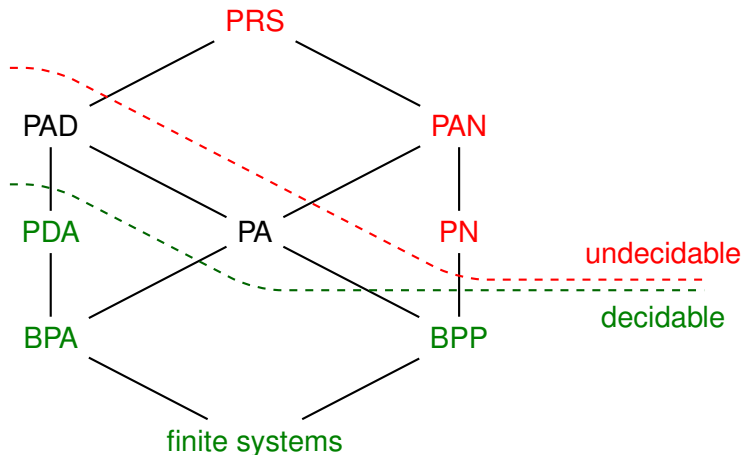
PRS-hierarchy of infinite-state systems

The hierarchy compares expressive power of many classes of infinite-state systems including **BPA**, **BPP**, **PA**, **Petri nets (PN)**, and **pushdown processes (PDA)**. systems.



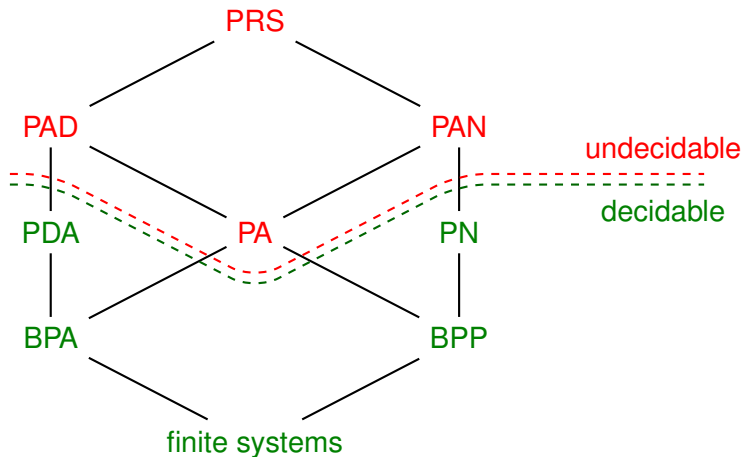
Decidability of equivalence checking

The decidability boundary of **strong bisimulation** in the PRS-hierarchy.



Decidability of model checking

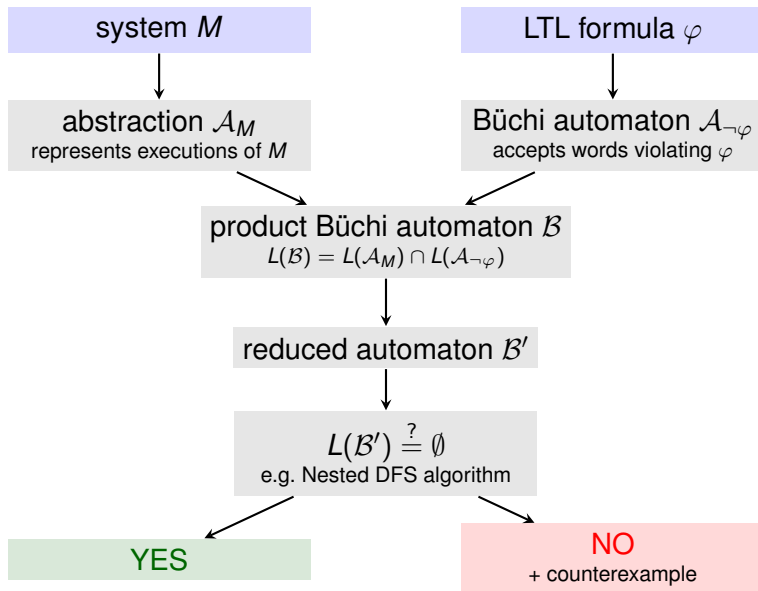
The decidability boundary of the **action-based LTL model checking** in the PRS-hierarchy.



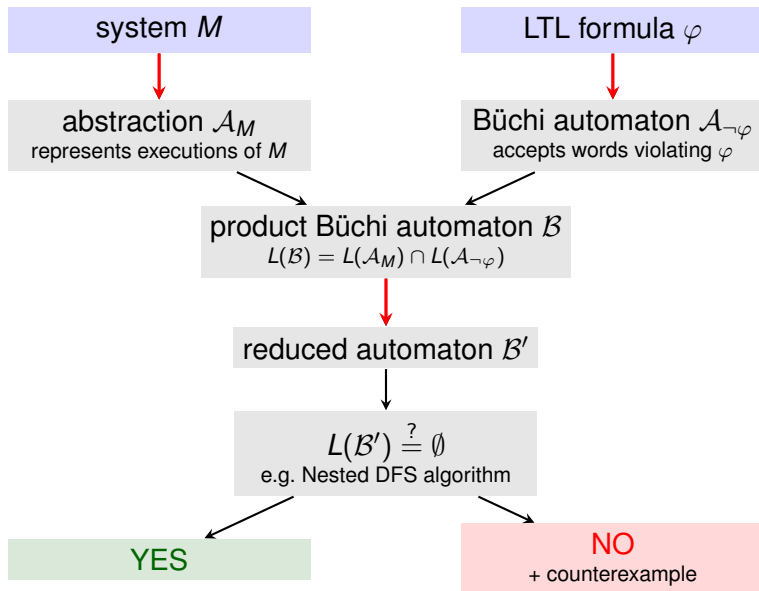
Actual topics of the course

- deductive verification
 - theorem prover ACL2 + Demo
- reachability analysis & verification of infinite-state systems
 - reachability analysis of pushdown systems
- LTL model checking
 - translation of LTL to Büchi automata (via alternating aut.)
 - partial order reduction
 - abstraction and CEGAR
- static analysis
 - abstract interpretation
 - shape analysis (abs. int. of dynamic memory operations)
- Ultimate Automizer: verification via automata, symbolic execution, and interpolation

Automata-based LTL model checking of finite systems



Automata-based LTL model checking of finite systems



An extra piece of motivation

- Formal verification is used in **Microsoft, Intel, AMD**,...
- Formal verification is usually a supplementary method, the main methods are testing or simulation.
- **In development of execution cluster of Core i7, formal verification has been used as a primary validation vehicle (simulation has been dropped)**
- only 3 bugs escaped to silicon (2 other bugs were detected during the pre-silicon stage by full chip testing)
- this number is usually about 40
- the previous minimum is 11
- More information in Kaivola et al: *Replacing Testing with Formal Verification in Intel Core i7 Processor execution Engine Validation*, CAV 2009, LNCS 5643, Springer, 2009.

Theorem prover ACL2

<http://www.cs.utexas.edu/users/moore/acl2/>

- How it works?
- What is it good for?
- Including a **live show!**