IA159 Formal Verification Methods

Shape Analysis via 3-Valued Logic

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Focus and sources

Focus

- shape analysis in general
- 3-valued logic approach
  - the logic and shape graphs
  - algorithm
  - TVLA and (semi)demo
- other approaches

Sources

- M. Sagiv, T. Reps, R. Wilhelm: *Parametric Shape Analysis via 3-Valued Logic*, ACM Trans. Program. Lang. Syst. 24(3), 2002.
- B. Jeannet, A. Loginov, T. Reps, M. Sagiv: *A Relational Approach to Interprocedural Shape Analysis*, SAS 2004.

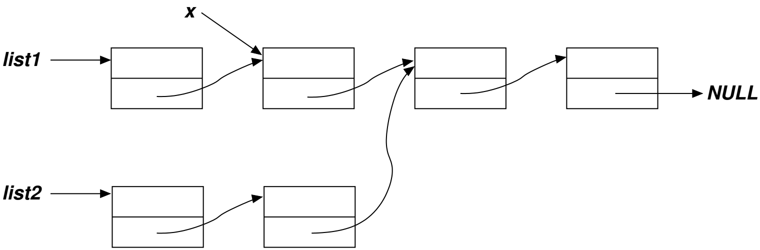
Goal

Shape analysis is a static analysis focused on program properties related to dynamically allocated memory. In particular, it aims to detect or verify the absence of heap-specific errors like

- null dereference
- memory leaking
- dangling pointer – a pointer to a deallocated memory
- violation of expected properties of dynamic datastructures (e.g. the datastructure is a cyclic list)
- ...

Basic idea

For each program location, we want to compute all reachable memory configurations.



Realistic approach

- The number of reachable memory configurations can be very large or even unbounded.
- We need to find finite representations of potentially infinite sets memory configurations.
- We compute over-approximations of sets of reachable memory configurations (an abstraction).
- The over-approximations are represented by finite shape graphs.
- Shape graphs can be represented using logics, graph structures, automata, ...

3-valued logic approach

Representing concrete memory configurations
with 2-valued logical structures

Logical representation of concrete configurations

- Configurations are represented by predicate logic formulas over the following core predicates:

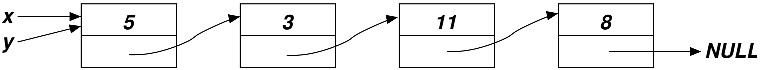
  unary predicate $x(v)$ for each pointer variable x
  binary predicate $n(v_1, v_2)$ for each stucture field n serving as a pointer
  binary predicate $eq(v_1, v_2)$

predicate	intended meaning
$x(v)$	variable x points to memory cell v
$n(v_1, v_2)$	field n of v_1 (i.e. $v_1.n$) points to v_2
$eq(v_1, v_2)$	v_1 and v_2 denote the same memory cell

- memory configurations correspond to interpretations
- allocated memory cells correspond to domain elements

Example

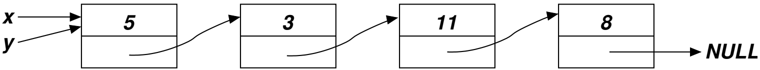
```
typedef struct node {
  struct node *n;
  int data;
} *List;
```



Logical representation

- domain $\{u_1, u_2, u_3, u_4\}$
- $x(u_1) = y(u_1) = 1$
- $n(u_1, u_2) = n(u_2, u_3) = n(u_3, u_4) = 1$
- $eq(u_1, u_1) = eq(u_2, u_2) = eq(u_3, u_3) = eq(u_4, u_4) = 1$
- values of all predicates on other arguments is 0

Example



Logical representation

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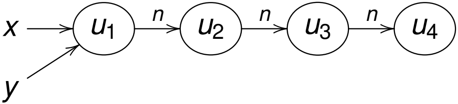
Visualisation of the logical representation



Notes

- storeless approach – it does not model precise location of allocated cells in the memory
- it cannot handle pointer arithmetics

- some interpretations do not represent any memory configuration, e.g. if $n(u, v) = n(u, w) = 1$ for some $v \neq w$
- these interpretations are eliminated by formulas called integrity constraints, e.g. $n(u, v) \wedge n(u, w) \implies eq(v, w)$

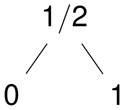
- the size of a configuration (and its logical representation) can be unbounded $\longrightarrow$ we use an abstraction to get a less precise, but bounded representation

3-valued logic approach

3-valued logical structures and shape graphs

3-valued logic

- uses 3 truth values: 0, 1, $1/2$ (indefinite value)
- new operation the least upper bound $\sqcup$
- operations $\wedge, \vee, \neg$ are extended



$\sqcup$	0	1	$1/2$
0	0	$1/2$	$1/2$
1	$1/2$	1	$1/2$
$1/2$	$1/2$	$1/2$	$1/2$

$\wedge$	0	1	$1/2$
0	0	0	0
1	0	1	$1/2$
$1/2$	0	$1/2$	$1/2$

$\vee$	0	1	$1/2$
0	0	1	$1/2$
1	1	1	1
$1/2$	$1/2$	1	$1/2$

$\neg$	
0	1
1	0
$1/2$	$1/2$

Abstraction of concrete configurations

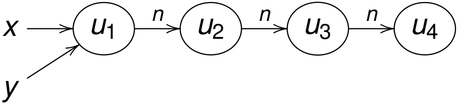
Abstraction

- we merge cells with identical values of all unary predicates
- values of unary predicates on merged cells keep unchanged (these are always 0 or 1)
- values of binary predicates on merged cells are defined as the least upper bound of the values on the original cells

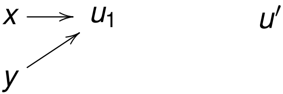
Example: if u_2, u_3, u_4 is merged into u' and u_1 is not, then

$$n(u_1, u') = n(u_1, u_2) \sqcup n(u_1, u_3) \sqcup n(u_1, u_4)$$

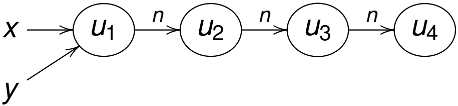
Example



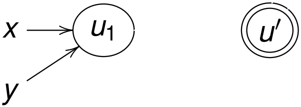
- let u_2, u_3, and u_4 be merged into u'



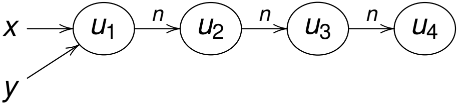
Example



- let u_2, u_3, and u_4 be merged into u'
- $eq(u', u') = eq(u_2, u_2) \sqcup eq(u_2, u_3) \sqcup \ldots \sqcup eq(u_4, u_4) = 1/2$
- cells with $eq(u, u) = 1/2$ are called summary nodes



Example



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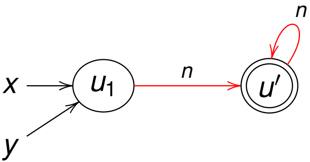
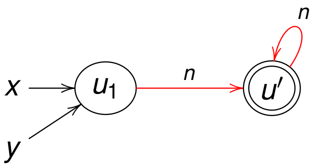
- cells with $eq(u, u) = 1/2$ are called summary nodes
- $n(u_1, u') = 1/2$ and $n(u', u') = 1/2$



Shape graph interpretation



This shape graph may represent:

- an acyclic list of 3+ elements pointed by x and y
- a cyclic list of 3+ elements pointed by x and y, with the first element not lying on the cycle
- besides of these, the configuration can contain other cyclic or acyclic lists not pointed by anything (i.e. garbage)

To refine the abstraction, we add instrumentation predicates.

Instrumentation predicates

Instrumentation predicates

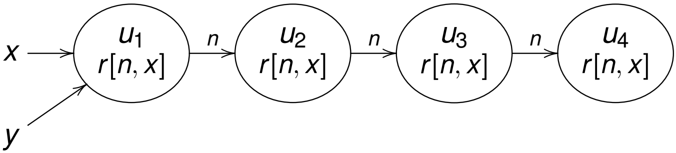
- are defined by first-order formulas over core predicates
- may also use transitive (or reflexive and transitive) closures of binary predicates

Typical instrumentation predicates for linked lists

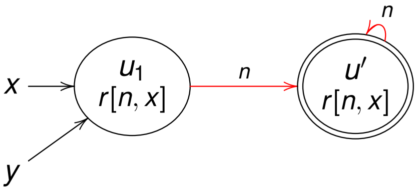
predicate	meaning	definition
$t[n](v_1, v_2)$	v_2 is reachable from v_1 via *n*-fields	$n^*(v_1, v_2)$
$r[n, x](v)$	v is reachable from variable x via *n*-fields	$\exists v_1 . x(v_1) \land t[n](v_1, v)$
$c[n](v)$	v lies on an cycle of *n*-fields	$\exists v_1 . n(v, v_1) \land t[n](v_1, v)$

Example

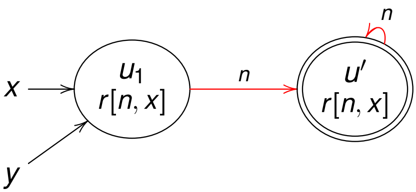
- we add instrumentation predicates $r[n, x]$ a $c[n]$



- there are more unary predicates determining cell merging



Example



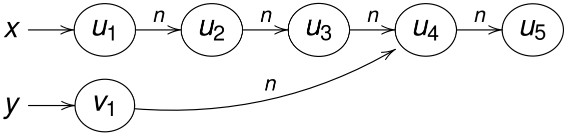
Now it represents exactly all acyclis lists of 3+ elements:

- all nodes satisfy $r[n, x]$, hence they are reachable from x (i.e. there is no garbage)
- $c[n]$ does not hold in any node, hence the list is acyclic

The choice of instrumentation predicates is crucial for obtaining some useful output.

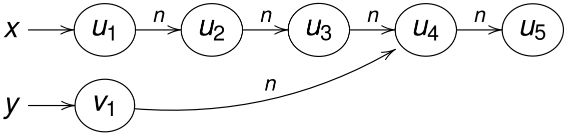
Example

Compute the shape graph given by core predicates and instrumentation predicates $r[n, x], r[n, y]$:

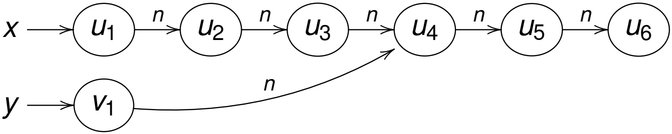


Example

Compute the shape graph given by core predicates and instrumentation predicates $r[n, x], r[n, y]$:

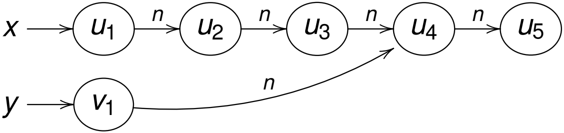


Decide whether the shape graph represents also the configuration below.



Example

Compute the shape graph given by core predicates and instrumentation predicates $r[n,x], r[n,y]$:



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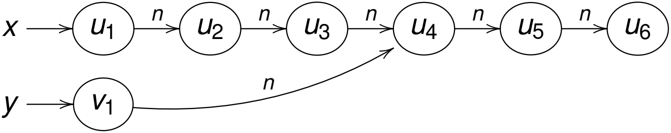
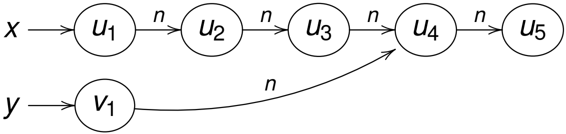


Suggest an instrumentation predicate that would make shape graphs for the two configurations different.

Example

Compute the shape graph given by core predicates and instrumentation predicates $r[n, x], r[n, y]$:



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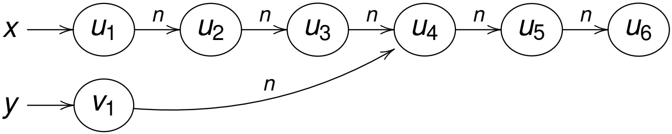


Suggest an instrumentation predicate that would make shape graphs for the two configurations different.

Solution: $is[n](v)$ defined by $\exists v_1, v_2 . n(v_1, v) \wedge n(v_2, v) \wedge v_1 \neq v_2$

3-valued logic approach

Algorithm – the first look

Algorithm – the first look

- there are only finitely many different shape graphs for a fixed finite set of core and instrumentation predicates
- the algorithm is a standard abstract interpretation

Algorithm

input: a program and shape graphs describing possible initial memory configurations

1. assign input shape graphs to the initial program location
2. for each program statement, take the shape graphs assigned to the location before the statement and update shape graphs in the locations after the statement
3. repeat step 2 until a fixpoint is reached

Step 2

- for each core predicate and each program statement, there is a predicate-update formula describing the values of the predicate after the statement using the values of core predicates before the statement
- using the predicate-update formulae, it is easy to compute the effect of the statement on concrete memory configurations
- to compute the effect of a statement on shape graphs is harder: values of instrumentation predicates are given by their definition formulas and values of core predicates, but this approach would quickly lead to loss of precision (values $1/2$)
- to get better results, we define also predicate-update formulas for instrumentation predicates, which may use values of both core and instrumentation predicates before the statement

3-valued logic approach

TVLA and (semi)demo

TVLA

= Three Valued Logic Analysis Engine

- developed at Tel Aviv University under supervision of Mooly Sagiv
- written in Java
- currently in version 3 (extended with heap decomposition)
- available for academic purposes
- `http://www.cs.tau.ac.il/~tvla/`

Input

Program has to be specified in four parts

1. declaration of predicates and integrity constraints
   - core predicates are just declared
   - instrumentation predicates have to be defined by formulas
2. operation semantics of all program statements
   - for each statement used in the program, the corresponding predicate-update formulas have to be given
   - each statement can be accompanied by an error detection formula (e.g. null dereference)
3. program flowgraph (including asserts)
4. the list of locations for which we want to get all reachable shape graphs

- parts 1 and 2 can be used repeatedly and they are available for certain classes of programs (e.g. for programs manipulating linked lists or trees)
- part 4 is optional

Input

Initial shape graphs

- described using a simple text format

Execution and output

```
tvla <program> <initial_graphs>
```

Output file contains

- picture of the program flowgraph
- reachable shape graphs for specified locations
- potential error messages

Example

```
typedef struct node {
  struct node *n;
  int data;
} *List;

List reverse(List x) {
  List y, t;
  y = NULL;
   (x != NULL) {
    t = x->n;
    x->n = y;
    y = x;
    x = t;
  }
  return y;
}
```

(SEMI)DEMO

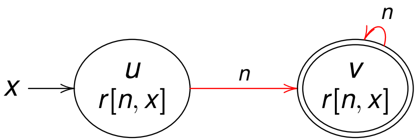
3-valued logic approach

Algorithm – a closer look

Computing the effect of a statement on a shape graph

1. operation Focus
2. evaluation of statement guards
3. computing new values of predicates
4. operation Coerce
5. operation Blur

We will compute the effect of `t = x->n` on the shape graph:

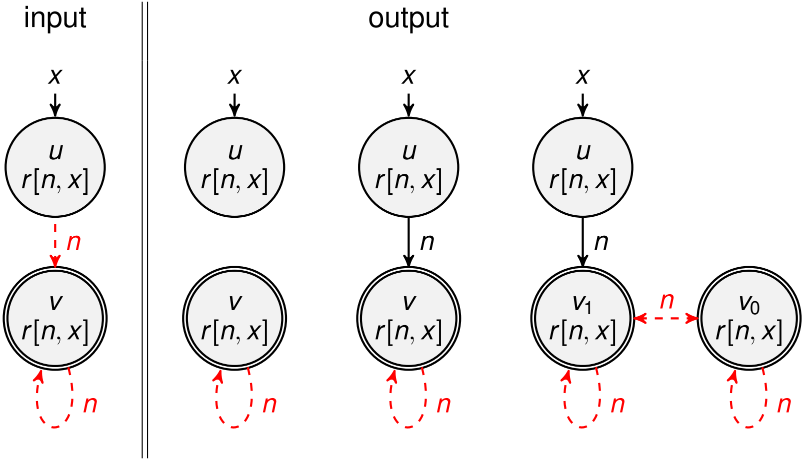


Operation Focus

- applied on statements with defined focus formula, which is a formula with exactly one free variable
- operation Focus takes the shape graph and returns the set of shape graphs representing the same configurations and such that the focus formula is not evaluated to $1/2$ on any node of any of the graphs.
- operation Focus modifies only values of predicates in the focus formula, values of other predicates are not recomputed
- hence, some resulting graphs may not satisfy integrity constraints

Operation Focus – example

- focus formula for `t = x->n` is $f(w) = \exists v_1 . x(v_1) \wedge n(v_1, w)$
- formula ensures that after the statement, the predicate $t(v)$ cannot have value $1/2$



Evaluation of statement guards

- for each statement, there can be defined a guard, which is again a formula
- the statements is not performed on the shape graphs for which the guard evaluates to 0
- it is typically used to handle program branching
- statement `t = x->n` has no guard

Computing new values of predicates

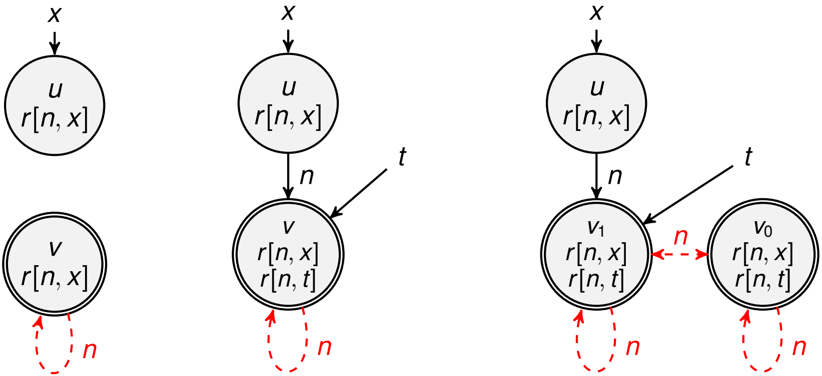
- we use predicate-update formulas corresponding to the statement to compute new predicate values
- predicates with no predicate-update formulas keep their value

Computing new values of predicates – example

Predicate-update formulas for `t = x->n`

predicate	predicate-update formula
$t(v)$	$\exists v_1 . x(v_1) \wedge n(v_1, v)$
$r[n,t](v)$	$r[n,x](v) \wedge (c[n](v) \vee \neg x(v))$

Output

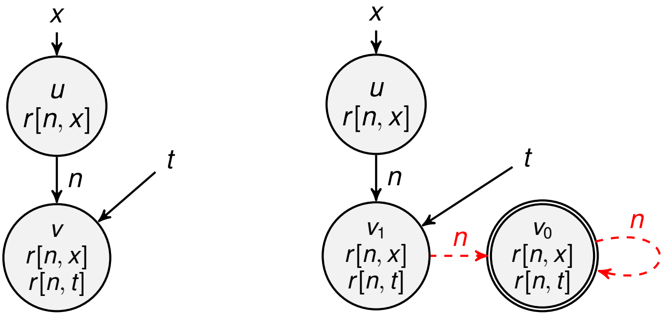


Operation Coerce

- removes shape graphs not satisfying integrity constraints
- makes values of some predicates more precise

Operation Coerce – example

- shape graph on the left is corrupted as $r[n,x](v)$ cannot hold $\implies$ the graph is removed
- in the shape graph in the middle, v cannot be a summary node as $t(v)$ holds
- on the right, v_1 cannot be a summary node for the same reason, and moreover $c[n](v_1)$ does not holds and thus $n(v_1, v_1), n(v_0, v_1)$ cannot hold



Operation Blur

- can further merge nodes with same values of unary predicates
- consequently, some shape graphs can become identical
- in our example, Blur has no effect

Notes

- TVLA works automatically, but the user has to
  - provide semantics of program statements
  - select/supply suitable instrumentation predicates
  - process the results and filter out false alarms

- Studied extensions and applications
  - interprocedural shape analysis (can handle also recursive programs)
  - lazy shape analysis
  - shape analysis and CEGAR
  - shape analysis for parallel processes
  - mix of shape analysis and data-related abstract interpretation (can be used e.g. to prove that sorting algorithms output sorted linked lists)
  - can be used also to analyse liveness of java objects and their timely deallocation
  - …

Other approaches and tools

Other approaches and tools

Other approaches to analysis of dynamically allocated memory are based on

- separation logic and (bi-)abduction (Infer)
- translation to first-order logic and automated theorem proving (HAVOC)
- symbolic memory graphs (Predator)
- tree automata (Forester)
- ...

Coming next week

Verification via automata, symbolic execution, and interpolation

- Try to hit an error location and learn from failure.
- Implemented in Ultimate Automizer,
  the winner of SV-COMP 2016 and 2017.