

$$\begin{aligned}
 [c_0, c_1, c_2, c_3] &\leftrightarrow [c_0 + c_1x + c_2x^2 + c_3x^3] = D(x) \\
 [d_0, d_1, d_2] &\leftrightarrow d_0 + d_1x + d_2x^2 = D(x) \\
 [c_0, c_1, c_2, c_3] &\leftrightarrow D(x) = c_0 \cdot d_0 + (c_1d_1 + c_2d_0)x + \dots \\
 &= E(x) = e_0 + e_1x + \dots \\
 \sum_{i=0}^{\infty} c_i d_{k-i} x^k & \quad (1-x)(1+x+x^2+\dots) = 1-x^{n+1} \\
 A(x) = \sum_{n=0}^{\infty} a_n x^n &= \lim_{n \rightarrow \infty} \frac{1-x^{n+1}}{1-x} = \frac{1}{1-x} \quad \text{für } |x| < 1
 \end{aligned}$$

dub 30-13:58

$$\begin{aligned}
 f(x) &= \ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots \\
 f'(0) &= \frac{1}{1+x} \Big|_0 = 1 \\
 f''(0) &= \frac{-1}{(1+x)^2} \Big|_0 = -1 \\
 f'''(0) &= 2 \cdot \frac{1}{(1+x)^3} \Big|_0 = 2 \\
 \ln \frac{1}{1-x} &= -\ln(1-x) = \sum_{n=1}^{\infty} \frac{1}{n} x^n \\
 g(x) &= (1+x)^r \quad g'(0) = 1, \quad g'(0) = r(1+x)^{r-1} \Big|_0 = r \\
 \frac{a_{r+1}}{a_r} &= \frac{\binom{r}{r+1}}{\binom{r}{r}} x = \frac{r-1}{r+1} x
 \end{aligned}$$

dub 30-14:23

binomische Reihe für $r = -n, n \in \mathbb{N}$

$$\begin{aligned}
 (1+x)^{-n} &= \sum_{\ell=0}^{\infty} \binom{-n}{\ell} x^\ell \\
 \binom{-n}{\ell} &= \frac{(-n)(-n-1)(-n-2)\dots(-n-\ell+1)}{\ell!} \\
 &= (-1)^\ell \frac{(n+\ell-1)(n+\ell-2)\dots(n)}{\ell!} \\
 &= (-1)^\ell \frac{(n+\ell-1)!}{\ell! (n-1)!} = (-1)^\ell \binom{n+\ell-1}{n-1} \\
 \Rightarrow (1-x)^{-n} &= \sum_{\ell=0}^{\infty} \binom{n+\ell-1}{n-1} x^\ell \quad \checkmark
 \end{aligned}$$

dub 30-14:38

$\{\text{Potenzreihe}\} \leftrightarrow \{\text{maximaler Rad}\}$

$$\begin{aligned}
 [a_0, a_1, a_2, \dots] &\leftrightarrow A(x) = \sum a_i x^i \\
 [b_0, b_1, b_2, \dots] &\leftrightarrow B(x) = \sum b_i x^i \\
 [0, \dots, 0, a_0, a_1, \dots] &\leftrightarrow x^\ell A(x) \\
 [a_2, a_3, a_4, \dots] &\leftrightarrow (A(x) - (a_0 + a_1 x)) x^{-2} \\
 [3, 4, 5, 6, \dots] &\leftrightarrow \left(\frac{1}{1-x} - 1 - 2x\right) \cdot x^{-2}
 \end{aligned}$$

dub 30-14:45

$$\begin{aligned}
 (1, \frac{1}{2}, \frac{1}{7}, \frac{1}{8}, \frac{1}{16}, \dots) &\leftrightarrow \frac{1}{1-\frac{1}{2}x} \\
 (1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \dots) &\leftrightarrow \frac{1}{1-\frac{1}{2}(-x)} = \frac{1}{1+\frac{1}{2}x} \\
 (a_0, a_1, a_2, \dots) &\leftrightarrow A(x) = \sum_{i=0}^{\infty} a_i x^i \\
 (a_0, 0, a_1, 0, a_2, 0, a_3, 0, \dots) &\leftrightarrow A(x^2) = \sum_{i=0}^{\infty} a_i x^{2i} \\
 (1, \frac{1}{2}, -\frac{1}{2}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{8}, -\frac{1}{8}, \frac{1}{16}, \frac{1}{16}, \dots) &\leftrightarrow \frac{1}{1+1/2x} + \frac{1}{1-1/2x} - 1 \cdot x^{-1} \\
 \frac{1}{1+1/2x} &\sim (1, 0, -\frac{1}{2}, 0, \dots) \quad \frac{1}{1-1/2x} \sim (1, 0, 1/2, 0, \dots)
 \end{aligned}$$

dub 30-14:54

$$\begin{aligned}
 \frac{1}{1-x} \cdot \frac{1}{1-x} &= \frac{1}{(1-x)^2} = \sum_{\ell=0}^{\infty} \binom{\ell+1}{1} x^\ell = \sum_{\ell=0}^{\infty} (\ell+1) x^\ell \\
 (1+\dots+x^\ell) &= \frac{1-x^{\ell+1}}{1-x} \\
 \binom{22}{2} - \binom{41}{2} - \binom{31}{2} - \binom{21}{2} &= 1061 \\
 \text{rd Resultate ergibt 10:} & \\
 (x^{10} + \dots + x^{20}) &= x^{10}(x + \dots + x^{20}) \\
 x^{20} (1 + \dots + x^{20}) &= (1 + \dots + x^{20})(1 + \dots + x^{10}) \\
 \binom{42}{2} - \binom{11}{2} &= \frac{42 \cdot 41}{2} - \frac{11 \cdot 10}{2}
 \end{aligned}$$

dub 30-15:08