



PA199 Advanced Game Design

Lecture 6 Collision Detection

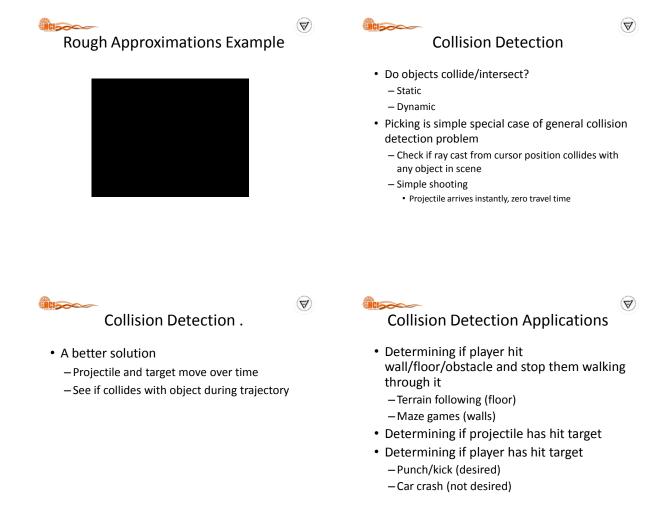
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Motivation

- Techniques for collision detection depend on the type of game
- For many games rough approximations are fine – i.e. Arcade-style games
- For more complex games need to be familiar with a variety of techniques ranging from simple to complex
 - i.e. 3D games





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Collision Detection Applications .

- Detecting points at which behavior should change
 - Car in the air returning to the ground
- Cleaning up animation
 - Making sure a motion-captured character's feet do not pass through the floor
- Simulating motion
 - Physics, or cloth, or something else



Simulating Motion

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Why it is Hard?

- · Complicated for two reasons
 - Geometry is typically very complexPotentially requiring expensive testing
 - Naïve solution is O(n2) time complexity
 - Since every object can potentially collide with every other object

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Basic Concepts

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From Simple to Complex

- Boundary check
 - -Perimeter of world vs. viewpoint or objects
 - 2D/3D absolute coordinates for bounds
 - Simple point in space for viewpoint/objects
- Set of fixed barriers
 - -Walls in maze game
 - 2D/3D absolute coordinate system

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From Simple to Complex .

- Set of moveable objects
 - One object against set of items
 Missile vs. several tanks
 - Multiple objects against each other
 - Punching game: arms and legs of players
 - Room of bouncing balls

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Naive General Collision Detection

- For each object i containing polygons p

 Test for intersection with object j containing polygons q
- For polyhedral objects, test if object i penetrates surface of j
 - Test if vertices of i straddle polygon q of j
 - If straddle, then test intersection of polygon q with polygon p of object i
- Very expensive! O(n2)

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Fundamental Design Principles

- Fast simple tests first, eliminate many potential collisions
 - Test bounding volumes before testing individual triangles
- Exploit locality, eliminate many potential collisions
 - Use cell structures to avoid considering distant objects

Fundamental Design Principles .

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- Use as much information as possible about geometry
 - Spheres have special properties that speed collision testing
- Exploit coherence between successive tests
 - Things don't typically change much between two frames

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Example: Player-Wall Collisions

- 'First person' games must prevent the player from walking through walls and other obstacles
- Most general case
 Dever and walks are poly
 - Player and walls are polygonal meshes
- Each frame, player moves along path not known in advance
 - Assume piecewise linear
 - Straight steps on each frame
 - Assume player's motion could be fast



Simple Approach

- On each step, do a general mesh-to-mesh intersection test to find out if the player intersects the wall
- If they do, refuse to allow the player to move
- Problems with this approach? how can we improve:
 - In response?
 - In speed?

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Collision Response

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- Frustrating to just stop
 - For player motions, often best thing to do is move player tangentially to obstacle
- Do recursively to ensure all collisions caught
 - Find time and place of collision
 - Adjust velocity of player
 - Repeat with new velocity, start time, start position (reduced time interval)
- Handling multiple contacts at same time
 - Find a direction that is tangential to all contacts

Collision Detection Approaches

- Two basic techniques:
 - Overlap testing
 - Detects whether a collision has already occurred
 - -Intersection testing
 - Predicts whether a collision will occur in the future



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Overlap Testing

Typical Approaches

- Facts
 - Most common technique used in games
 - Exhibits more error than intersection testing
- Concept
 - For every simulation step, test every pair of objects to see if they overlap
 - Easy for simple volumes like spheres, harder for polygonal models

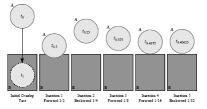
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Overlap Testing: Useful Results

- · Useful results of detected collision
 - Time collision took place
 - Collision normal vector

Overlap Testing: Collision Time

- Collision time calculated by moving object back in time until right before collision
 - Bisection is an effective technique





Overlap Testing: Limitations

- · Fails with objects that move too fast
 - Unlikely to catch time slice during overlap
- Possible solutions
 - Design constraint on speed of objects
 - Reduce simulation step size

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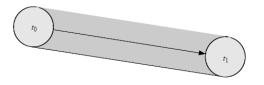
Intersection Testing

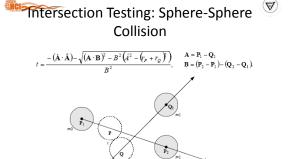
- Predict future collisions
- When predicted:
 - Move simulation to time of collision
 - Resolve collision
 - -Simulate remaining time step

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Intersection Testing: Swept Geometry

- Extrude geometry in direction of movement
- Swept sphere turns into a 'capsule' shape





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Intersection Testing: Limitations

- Issue with networked games
 - Future predictions rely on exact state of world at present time
 - Due to packet latency, current state not always coherent
- Assumes constant velocity and zero acceleration over simulation step
 - Has implications for physics model and choice of integrator



Complexity Issues

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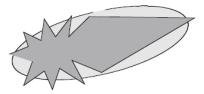
Dealing with Complexity

- Two common issues when dealing with complexity:
 - Complex geometry must be simplified
 Not so easy!
 - Reduce number of object pair tests
 Varies depending on the types of objects

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Simplified Geometry

- Approximate complex objects with simpler geometry
 - i.e. Ellipsoid shown below



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Minkowski Sum

- By taking the Minkowski Sum of two complex volumes and creating a new volume then overlap can be found
 - By testing if a single point is within the new volume

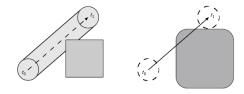
 $X \oplus Y = \{A + B : A \in X \text{ and } B \in Y\}$





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Minkowski Sum Example





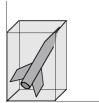
Bounding Volumes

- Bounding volume is a simple geometric shape
 - Completely encapsulates object
 - If no collision with bounding volume, no more testing is required
- Most common bounding volumes is box
 - More later on...

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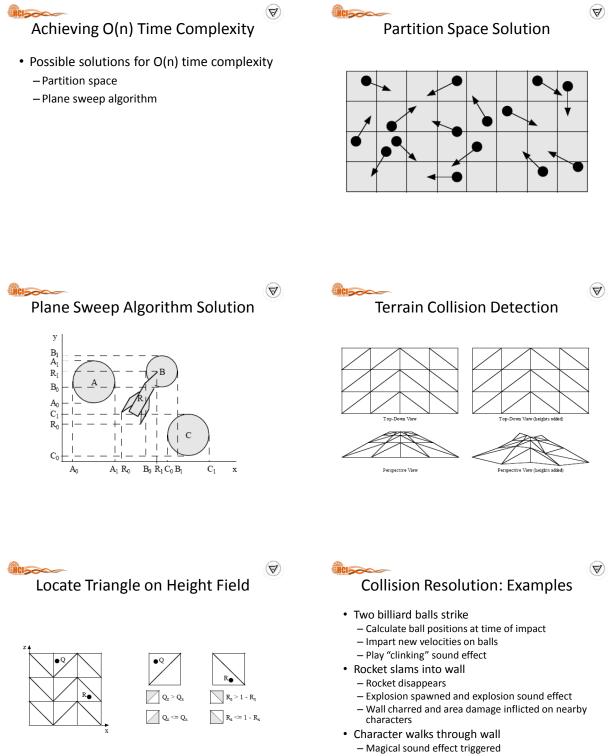
Box Bounding Volumes





Axis-Aligned Bounding Box

Oriented Bounding Box



– No trajectories or velocities affected

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Collision Resolution Components

- Resolution has three parts:
 - Prologue
 - Collision
 - Epilogue

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Prologue Stage

- Collision known to have occurred
- Check if collision should be ignored
- Other events might be triggered – Sound effects
 - -Send collision notification messages

Collision Stage

- Place objects at point of impact
- Assign new velocities using either – Physics
 - -Some other decision logic



Epilogue Stage

- Propagate post-collision effects
- Possible effects
 - Destroy one or both objects
 - Play sound effect
 - Inflict damage
- Many effects can be done either in the prologue or epilogue

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Resolving Overlap Testing

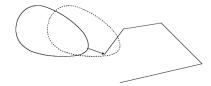
- Four common stages:
 - Extract collision normal
 - Extract penetration depth
 - Move the two objects apart
 - Compute new velocities

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Extract Collision Normal

- Find position of objects before impact
- Use two closest points to construct the collision normal vector



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Extract Collision Normal.

- Sphere collision normal vector
 - Difference between centers at point of collision

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Resolving Intersection Testing

- · Simpler than resolving overlap testing -No need to find penetration depth or move objects apart
- · Simply just
 - Extract collision normal
 - -Compute new velocities

Acceleration Techniques

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Accelerating Collision Detection

- Two kinds of approaches (many others also) - Collision proxies / bounding volumes hierarchies - Spatial data structures to localize
- Used for both 2D and 3D
- Accelerates many things, not just collision detection
 - Raytracing
 - Culling geometry before using standard rendering pipeline

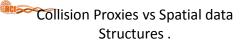
Collision Proxies vs Spatial data Structures

- **Collision Proxies:** - Object centric
- Spatial data Structures:
- Space centric
- Spatial redundancy



- Object redundancy





Collision Proxies:

- Object centric
- Spatial redundancy



- Spatial data Structures:
- Space centric
- Object redundancy



(\mathbf{A}) (a)Collision Proxies vs Spatial data Collision Proxies vs Spatial data Structures .. Structures ... Collision Proxies: Collision Proxies: Spatial data Structures: Spatial data Structures: - Object centric - Space centric - Object centric - Space centric - Spatial redundancy - Object redundancy - Spatial redundancy - Object redundancy (\mathbf{A}) (\mathbf{A}) **Collision Proxies** Collision Proxies . Proxy Good proxy - Something that takes place of real object - Cheap to compute collisions for, tight fit to - Cheaper than general mesh-mesh intersections the real geometry · Collision proxy (bounding volume) is piece of Common proxies geometry used to represent complex object -Sphere, cylinder, box, ellipsoid

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- Consider
 - Fat player, thin player, rocket, car ...

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object

Collision Proxies Example 1

for purposes of finding collision

- If proxy collides, object is said to collide

- Collision points mapped back onto original







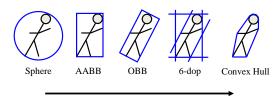
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Collision Proxies Example 3



Trade-off in Choosing Proxies



increasing complexity & tightness of fit

decreasing cost of (overlap tests + proxy update)

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Trade-off in Choosing Proxies .

- AABB
 - -Axis aligned bounding box
- OBB
 - Oriented bounding box, arbitrary alignment
- k-dops
 - Shapes bounded by planes at fixed orientations
 - Discrete orientation

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Pair Reduction

- Want proxy for any moving object requiring collision detection
- Before pair of objects tested in any detail, quickly test if proxies intersect
- When lots of moving objects, even this quick bounding sphere test can take too long:
 N² times if there are N objects
- Reducing this N² problem is called pair reduction
 - Pair testing isn't a big issue until N>50 or so...

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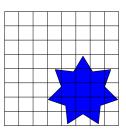
Spatial Data Structures

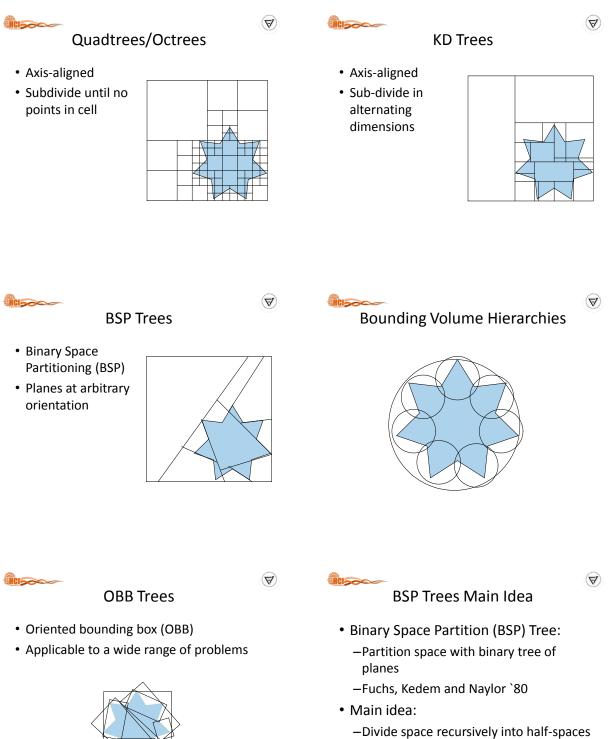
- Can only hit something that is close
- Spatial data structures tell you what is close to object
 - Uniform grid, octrees, kd-trees, BSP trees
 - Bounding volume hierarchies
 OBB trees
 - For player-wall problem, typically use same spatial data structure as for rendering
 - BSP trees most common





- Axis-aligned
- Divide space uniformly





by choosing splitting planes that

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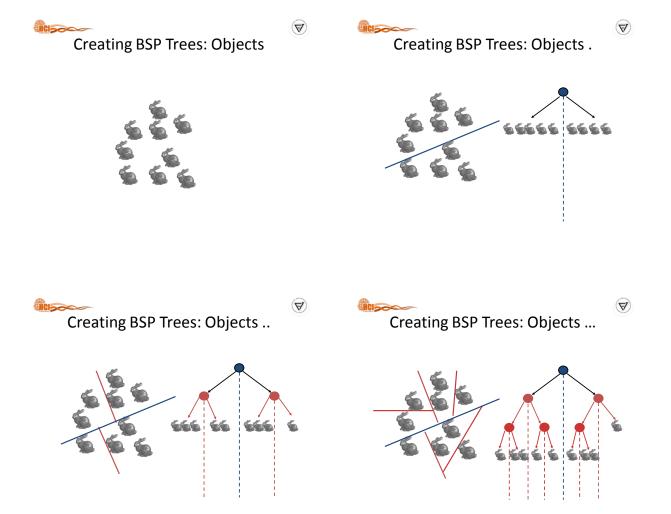
BSP Trees Methods

- More general, can deal with inseparable objects
- Automatic, uses as partitions planes defined by the scene polygons
- Method has two steps:
 - Building of the tree independently of viewpoint
 - Traversing the tree from a given viewpoint to get visibility ordering

- BSP Trees Methods .
- First step

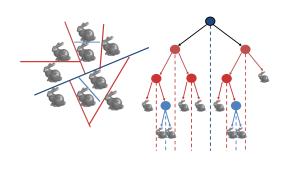
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- Preprocessing
 - Create binary tree of planes
- Second step
 - -Runtime
 - Correctly traversing this tree enumerates objects from back to front



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Splitting Objects

- No bunnies were harmed in previous example
- But what if a splitting plane passes through an object?
 - -Split the object; give half to each node



Traversing BSP Trees

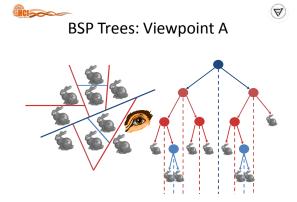
- Tree creation independent of viewpoint – Preprocessing step
- Tree traversal uses viewpoint

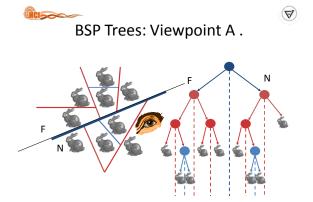
 Runtime, happens for many different viewpoints
- Each plane divides world into near and far
 - For given viewpoint, decide which side is near and which is far
 - Check which side of plane viewpoint is on independently for each tree vertex
 - Tree traversal differs depending on viewpoint!
 - Recursive algorithm
 - Recurse on far sideDraw object
 - Recurse on near side

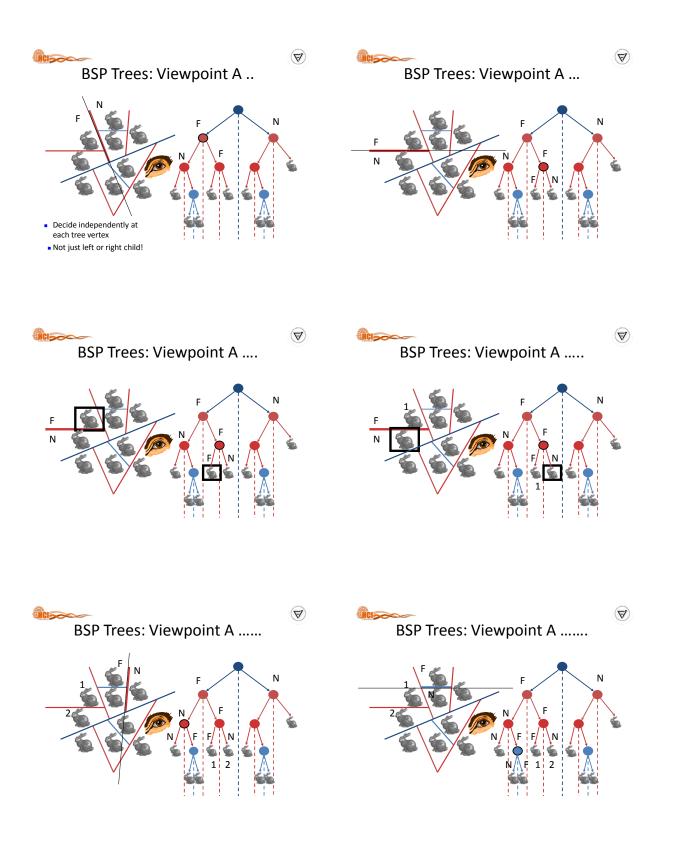


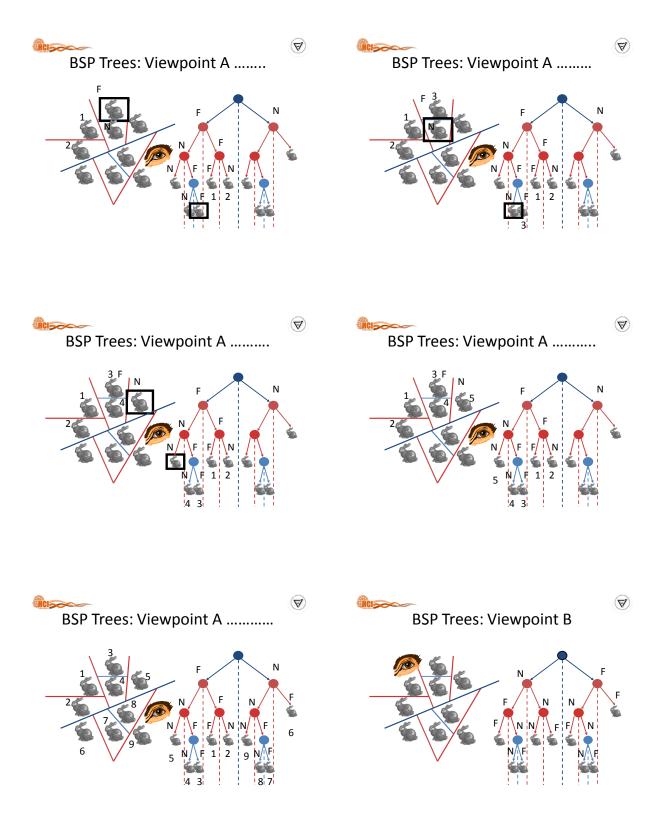
 Query: given a viewpoint, produce an ordered list of (possibly split) objects from back to front

```
renderBSP(BSPtree *T)
BSPtree *near, *far;
if (eye on left side of T->plane)
    near = T->left; far = T->right;
else
    near = T->right; far = T->left;
renderBSP(far);
if (T is a leaf node)
    renderBSP(near);
```

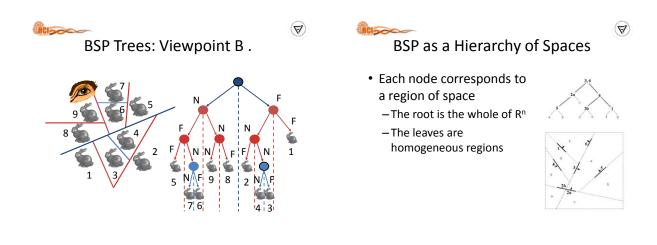








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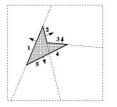
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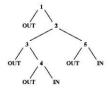
BSP Tree Traversal: Polygons

- Split along the plane defined by any polygon from scene
- Classify all polygons into positive or negative half-space of the plane
 - If a polygon intersects plane, split polygon into two and classify them both
- Recurse down the negative half-space
- Recurse down the positive half-space



Representation of Polygons

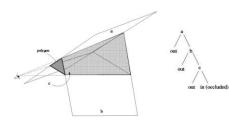




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Representation of Polyhedra





BSP Trees for Dynamic Scenes

- When an object moves the planes that represent it must be removed and reinserted
- Some systems only insert static geometry into the BSP tree
- Otherwise must deal with merging and fixing the BSP cells

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BSP Trees Pos

- Simple, elegant scheme
- Correct version of painter's algorithm back-to-front rendering approach
- Popular for video games

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BSP Trees Cons

- Slow to construct tree
 O(n log n) to split, sort
- Splitting increases polygon count -O(n2) worst-case
- Computationally intense preprocessing stage restricts algorithm to static scenes

BSP Demo

<u>http://www.symbolcraft.com/graphics/bsp/</u>



BSP Videos

- <u>https://www.youtube.com/watch?v=WAd7vzw</u> knF0
- <u>https://www.youtube.com/watch?v=jF2a4imSu</u> <u>vl</u>
- <u>http://www.youtube.com/watch?v=JJjyXRvokE4</u>

Collision Detection Approach

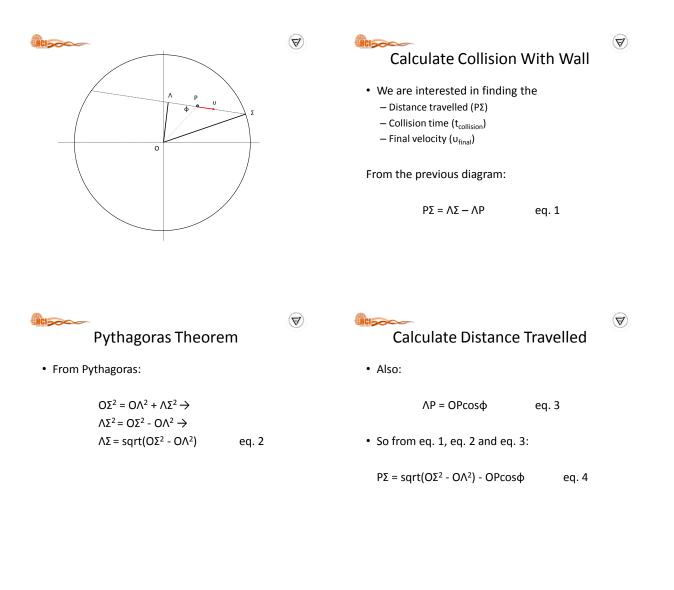
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Introduction to 3D Breakout

- Most important thing is ball-wall collision detection
- Can be used in:
 - Ball-wall collisions
 - -Ball-bat collisions
 - Apart from some cases
 - -Ball-Well collisions
 - Apart from some cases (similarly to ball-bat)



Calc	ulate Distance Travelled	(Calculate Distance Travelled	\blacksquare
• But:			• From eq. 4 and eq. 5	
	sinφ = OΛ/OP → OΛ = OPsinφ		$P\Sigma = sqrt(O\Sigma^2 - OP^2sin\phi^2) - OPcos\phi$ eq. 6	
• And:			• Also from:	
	$O\Lambda^2 = OP^2 sin \varphi^2$ eq. 5		$sin\phi^2 + cos\phi^2 = 1 \rightarrow sin\phi^2 = 1 - cos\phi^2$ eq. 7	

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Calculate Distance Travelled ...

- From eq. 6 and eq. 7
- $P\Sigma = sqrt(O\Sigma^2 OP^2 + OP^2 cos\varphi^2) OP cos\varphi$
- Since OP•υ = (OP)υ/|υ|cosφ, so the above equation will become:
- $$\begin{split} \mathsf{P}\Sigma &= \mathsf{sqrt}(\mathsf{O}\Sigma^2 \mathsf{O}\mathsf{P}^2 + (\mathsf{O}\mathsf{P}\upsilon/|\upsilon|\mathsf{cos}\varphi)^2) \\ & (\mathsf{O}\mathsf{P})\upsilon/|\upsilon|\mathsf{cos}\varphi \end{split}$$

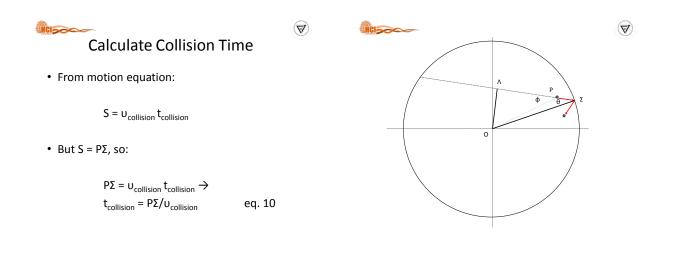
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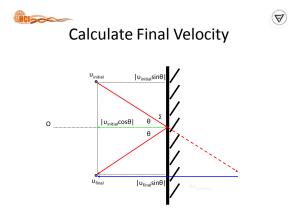
Calculate Distance Travelled

• From the dot product the previous equation will become

 $\begin{aligned} \mathsf{P}\Sigma &= \mathsf{sqrt}(\mathsf{O}\Sigma^2 - \mathsf{O}\mathsf{P}^2 + (\mathsf{O}\mathsf{P} \bullet \upsilon / |\upsilon|)^2) - \mathsf{O}\mathsf{P} \bullet \upsilon / |\upsilon| \\ & \mathsf{eq. 8} \end{aligned}$

- Must take absolute value in case $\phi > 90$
- $P\Sigma = |(\operatorname{sqrt}(O\Sigma^2 OP^2 + (OP \bullet \upsilon / |\upsilon|)^2) OP \bullet \upsilon / |\upsilon| |$ eq. 9





Calculate Final Velocity.

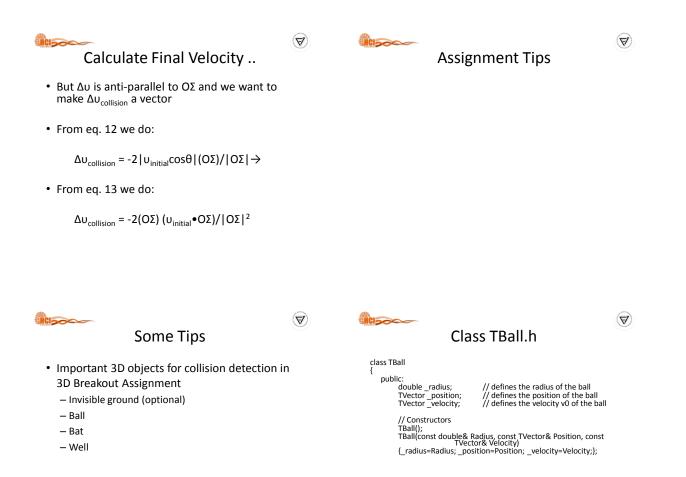
• The change in ball velocity from the collision:

 $|\Delta v_{\text{collision}}| = |v_{\text{final}} - v_{\text{initial}}|$ eq. 11

• From the above figure:

 $|\Delta v_{collision}| = 2 |v_{initial} \cos \theta|$ or eq. 12

 $|\Delta v_{collision}| = 2v_{initial} \bullet (O\Sigma / |O\Sigma|)$ eq. 13





Class TBall.h .

// Selectors double GetBallRadius() const {return _radius;}; TVector GetBallPosition() const {return _position;}; TVector GetBallVelocity() const {return _velocity;};

void DrawBall(); // Draws the ball

void CalculateVelocity(const TVector& velocity, const double& seconds); // Assigns the ball a velocity

TVector CalcDistanceTravelled(const double& seconds) const; // Calculates the distance traveled

void MoveBall(const double& seconds); // Moves the ball

Default Constructor for the Ball

TBall::TBall()

{

}

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// Assign default values for the attributes
// of the ball
_radius = 4.0;
_position = TVector(0.0, 0.0, 0.0);
_velocity = TVector(1.0, 0.0, 0.0);

Function to Draw the Ball

```
{
    glPushMatrix();
    glTranslatef(_position.X(),
    _position.Y(), _position.Z());
    glutSolidSphere(_radius, 20, 20);
    glPopMatrix();
}
```

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More Functions

- Functions for the TBall Class:
 - -CalculateVelocity
 - CalcDistanceTravelled
 - -MoveBall

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- Function for TDisplayImp – Idle
- TBat Class

(\mathbf{A}) (\mathbf{A}) iici 🤝 CalcDistanceTravelled Function CalculateVelocity Function TVector TBall::CalcDistanceTravelled(const double& void TBall::CalculateVelocity(const TVector& seconds) const velocity, const double& seconds) { TVector new_velocity, new_position; { _velocity = velocity; new_velocity = _velocity; new_position = _position + new_velocity*seconds; } return new_position; } (\mathbf{A}) (\mathbf{A}) **MoveBall Function Idle Function** void TBall::MoveBall(const double& seconds) void TDisplayImp::idle(void) { { _position = CalcDistanceTravelled(seconds); // Set the time for the simulation } _scene->CalculateSimulations(); glutPostRedisplay();

}

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{

Class TBat

class TBat public: TVector _points[16]; first bat TVector normal[15];

// points for the // normal of the ground

public: // Default constructor TBat(){}; TBat(double rotation_angle);

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Class TBat.

void DrawBat(); // Draws the bats void MoveBatRight(); // Moves bat on the right void MoveBatLeft(); // Moves the bat on the right int BatCollisions(const TBall &ball, const double& seconds); int BatCollisionsSides(const TBall & ball, const double& seconds); int BatCollisionsEdges(const TBall & ball, const double& seconds);

Class TBat ..

TVector Bat_Faces_Reflection(TBall &ball, const double& seconds, const double& distance);

TVector Bat_Left_Side_Reflections(TBall const double& seconds, uble& parameter); &ball, const double&

TVector Bat_Right_Side_Reflections(TBall &ball, Const double& seconds, const double& parameter);

TBat Constructor

// Define a vector for the construction of the ground points of the bats initial_vector = TVector(1.0, 0.0, 0.0);

// Define a vector for the construction of the upper points of the bats upper_vector = TVector(0.0, 10.0, 0.0);



};

Class TBat ...

TVector Bat_Edge12_Reflections(TBall & ball, const double& seconds);

TVector Bat_Edge15_Reflections(TBall &ball, const double & seconds);

TVector Bat_Edge13_Reflections(TBall & ball, const double& seconds);

TVector Bat_Edge11_Reflections(TBall & ball, const double& seconds);

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TBat Constructor.

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// Define the rotation axis TVector rotation_axis(0.0,1.0,0.0);

// Define the three rotation matrices for the bats TMatrix33 bat_construction = TMatrix33(rotation_axis, rotation_angle);

// Define the vector used for the construction of the bats construction_vector = bat_construction*initial_vector;

// Define the rotation matrix for the constuction of the bats TMatrix33 bat_rotation = TMatrix33(rotation_axis, angle);

TBat::TBat(double rotation_angle)

TVector initial vector, upper_vector, construction_vector;

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TBat Constructor..



Drawing Front Side of Bats

glBegin(GL_QUAD_STRIP);

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// Front face, normal of first surface _normal[0] = ((_points[8] - _points[0])*(_points[1] - _points[0])).unit(); glNormal3f(_normal[0].X(), _normal[0].Y(), _normal[0].Z());

// Construct first quad glVertex3f(_points[0].X(), _points[0].Y(), _points[0].Z()); glVertex3f(_points[8].X(), _points[8].Y(), _points[8].Z());

// Front face, second surface
_normal[1] = ((_points[9] - _points[1])*(_points[2] - _points[1])).unit();
glNormal3f(_normal[1].X(), _normal[1].Y(), _normal[1].Z());

}

Drawing Front Side of Bats.

// Construct second quad glVertex3f(_points[1].X(), _points[1].Y(), _points[1].Z()); glVertex3f(_points[9].X(), _points[9].Y(), _points[9].Z());

// Front face, third surface _normal[2] = ((_points[10] - _points[2])*(_points[3] - _points[2])).unit();

glNormal3f(_normal[2].X(), _normal[2].Y(), _normal[2].Z());

 $\label{eq:construct_third_quad} glvertex3f(_points[2].X(),_points[2].Y(),_points[2].Z()); glvertex3f(_points[10].X(),_points[10].Y(),_points[10].Z()); glvertex3f(_normal[2].X(),_normal[2].Y(),_normal[P_$

// Construct fourth quad glVertex3f(_points[3].X(), _points[3].Y(), _points[3].Z()); glVertex3f(_points[11].X(), _points[11].Y(), _points[11].Z());

glEnd();

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Drawing the Rest of the Bats

- In the same way you will have to draw the:
 - -Left side of the bat
 - -Back side of the bat
 - -Right side of the bat
 - Up side of the bat

HCI

Bat Collisions

- At least three checks:
 - Check for collisions between the ball and the three bats like ball-wall
 - Check for collisions between the ball and the side of the bats
 - Check for collisions between the ball and the edges of the bats
- Repeat the same procedure for reflections of the ball after collisions

Calculate the reflection of the ball after collision

double TBounds::Ball_Reflection(TBall & ball, const double& seconds)

TVector ball_velocity_after_collision, previous_ball_position, collision_vector, final_velocity;

// Perform calculations for the previous time step
previous_ball_position = ball.GetBallPosition() ball.GetBallVelocity()*seconds;

double absBallVelocity =
sqrt(ball.GetBallVelocity().dot(ball.GetBallVelocity()));

 (\mathbf{A})

Calculate the reflection of the ball

after collision .

// Calculate the Ri*V to calculate the collision time double RV = previous ball_position.dot(ball.GetBallVelocity())/absBallVelocity;

 (\mathbf{A})

// Absolute RV double abs_RV = abs(RV);

// Define the initial distance = 100 - 4 = 96
double initial_distance = 100.0 ball.GetBallRadius();

Calculate the reflection of the ball after collision ..

// Calculate the determinant double Determinant = ((RV*RV) previous ball_position.dot(previous_ball_position) + initial_distance*initial_distance);

// Calculate the collision time double collision_time = abs(-abs_RV + sqrt(Determinant))/absBallVelocity;

// Calculate the collision vector (normal vector) from: R = r + collision vector = provings half position +

collision_vector = previous_ball_position + ball.GetBallVelocity()*collision_time;

// Make the collision vector (normal vector) unit vector TVector unit_collision_vector = TVector::unit(collision_vector);

Calculate the reflection of the ball after collision ...

// Define velocity by: Vreflected = (Vinitial*Normal.unit)*Normal.unit ball velocity_after_collision = unit_collision_vector*(ball.GetBallVelocity().dot(unit_col lision_vector));

// Calculate the velocity of the ball after collision with the invisible wall final_velocity = ball.GetBallVelocity() - ball_velocity_after_collision*2.0;

ball.CalculateVelocity(final_velocity, collision_time);

return collision_time;



References

- <u>http://www.cs.wisc.edu/~schenney/courses/c</u> s679-f2003/lectures/cs679-22.ppt
- <u>http://graphics.ucsd.edu/courses/cse169_w05</u> /CSE169_17.ppt



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Links

- <u>http://en.wikipedia.org/wiki/Bounding_volume</u>
- <u>http://nehe.gamedev.net/data/lessons/lesson.asp?</u> <u>lesson=30</u>
- <u>http://web.cs.wpi.edu/~matt/courses/cs563/talks/</u> <u>bsp/bsp.html</u>
- <u>http://www.devmaster.net/articles/bsp-trees/</u>
- <u>http://maven.smith.edu/~mcharley/bsp/createbspt</u> ree.html
- <u>http://www.cs.unc.edu/~geom/</u>
- <u>http://www.cs.ox.ac.uk/stephen.cameron/distances/</u>





