

# Graphical Representation of LTL Temporal Operators



Note that missing formula stands for *true*.

Note that a system satisfies an LTL formula iff **all** its runs satisfy it.

## └ LTL examples

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▪  $X \text{ rain}$ 
▪  $F \text{ rain}$ 
▪  $\text{pick-up } R \text{ kin-gar}$ 
▪  $G(\text{drop-off} \implies (\text{kin-gar } U \text{ pick-up}))$ 
▪  $G(\neg(cs_1 \wedge cs_2))$ 
▪  $G(\text{req} \implies F \text{ resp}) \dots$  Does it guarantee that  $\#_{\text{req}} = \#_{\text{resp}}$ ?
▪  $G F \text{ chocolate}$ 
▪  $(G F \text{ req}) \implies (G F \text{ resp})$ 
▪  $\text{sin} \implies (F G \text{ hell})$ 
▪  $F(\text{sin} \wedge (\neg \text{confession } U \text{ death})) \implies (F G \text{ hell})$ 

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- $X \text{ rain}$  - it will rain tomorrow
- $F \text{ rain}$  - it will rain eventually
- $\text{pick-up } R \text{ kin-gar}$  - pick-up releases a child from kindergarten
- $G(\text{drop-off} \implies (\text{kin-gar } U \text{ pick-up}))$  - contrary to the previous one  $\text{pick-up}$  can be out of  $\text{kin-gar}$
- $G(\neg(cs_1 \wedge cs_2))$  - mutual exclusion (in critical sections)
- $G(\text{req} \implies F \text{ resp}) \dots$  Does it guarantee that  $\#_{\text{req}} = \#_{\text{resp}}$ ? - no
- $G F \text{ chocolate}$  - chocolate infinitely many times
- $(G F \text{ req}) \implies (G F \text{ resp})$  - only infinitely many requests cause i. m. responses
- $\text{sin} \implies (F G \text{ hell})$  -  $\text{sin}$  causes  $\text{hell}$
- $F(\text{sin} \wedge (\neg \text{confession } U \text{ death})) \implies (F G \text{ hell})$  - improved with confession

## └ LTL properties - example

You have two fishes, say Alice (A) and Bob (B). There is an aquarium divided into two parts: left (L) and right (R). Both fish start on the right side of the aquarium and do the following sequence of steps (independently). They move to the left, eat, move back to the right. Formulate using LTL:

- Whenever Alice eats, she is on the left.
- Whenever Bob is on the left, he will eat eventually.
- Whenever Bob eats, he will immediately go to the left.
- If Alice do not eat before Bob, she will never eat.
- Alice and Bob will never be on the same side from some point.
- Bob chases Alice until they both eat together.

- Whenever Alice eats, she is on the left  
 $G (ae \implies al)$
- Whenever Bob is on the left, he will eat eventually  
 $G (bl \implies F be)$
- Whenever Bob eats, he will immediately go to the left  
 $G (be \implies X bl)$
- If Alice do not eat before Bob, she will never eat  
 $((\neg ae) U be) \implies G (\neg ae)$
- Alice and Bob will never be on the same side from some point  
 $FG((al \wedge br) \vee (ar \wedge bl))$
- Bob chases Alice until they both eat together  
 $((al \implies X bl) \wedge (ar \implies X br)) W (ae \wedge be)$

# └ LTL properties - distributivity questions

Is it true that ...		
•	$X(\varphi \vee \psi) \stackrel{?}{=} X\varphi \vee X\psi$	
•	$X(\varphi \wedge \psi) \stackrel{?}{=} X\varphi \wedge X\psi$	
•	$F(\varphi \vee \psi) \stackrel{?}{=} F\varphi \vee F\psi$	$GF(\varphi \vee \psi) \stackrel{?}{=} GF\varphi \vee GF\psi$
•	$F(\varphi \wedge \psi) \stackrel{?}{=} F\varphi \wedge F\psi$	$GF(\varphi \wedge \psi) \stackrel{?}{=} GF\varphi \wedge GF\psi$
•	$G(\varphi \vee \psi) \stackrel{?}{=} G\varphi \vee G\psi$	$FG(\varphi \vee \psi) \stackrel{?}{=} FG\varphi \vee FG\psi$
•	$G(\varphi \wedge \psi) \stackrel{?}{=} G\varphi \wedge G\psi$	$FG(\varphi \wedge \psi) \stackrel{?}{=} FG\varphi \wedge FG\psi$
•	$\varphi U(\psi_1 \vee \psi_2) \stackrel{?}{=} (\varphi U \psi_1) \vee (\varphi U \psi_2)$	
•	$\varphi U(\psi_1 \wedge \psi_2) \stackrel{?}{=} (\varphi U \psi_1) \wedge (\varphi U \psi_2)$	
•	$(\varphi_1 \vee \varphi_2) U \psi \stackrel{?}{=} (\varphi_1 U \psi) \vee (\varphi_2 U \psi)$	
•	$(\varphi_1 \wedge \varphi_2) U \psi \stackrel{?}{=} (\varphi_1 U \psi) \wedge (\varphi_2 U \psi)$	

$$\bullet X(\varphi \vee \psi) \equiv X\varphi \vee X\psi$$

$$\bullet X(\varphi \wedge \psi) \equiv X\varphi \wedge X\psi$$

$$\bullet F(\varphi \vee \psi) \equiv F\varphi \vee F\psi$$

$$\bullet F(\varphi \wedge \psi) \not\equiv F\varphi \wedge F\psi$$

$$\bullet G(\varphi \vee \psi) \not\equiv G\varphi \vee G\psi$$

$$\bullet G(\varphi \wedge \psi) \equiv G\varphi \wedge G\psi$$

$$\bullet \varphi U(\psi_1 \vee \psi_2) \equiv (\varphi U \psi_1) \vee (\varphi U \psi_2)$$

$$\bullet \varphi U(\psi_1 \wedge \psi_2) \not\equiv (\varphi U \psi_1) \wedge (\varphi U \psi_2)$$

$$\bullet (\varphi_1 \vee \varphi_2) U \psi \not\equiv (\varphi_1 U \psi) \vee (\varphi_2 U \psi)$$

$$\bullet (\varphi_1 \wedge \varphi_2) U \psi \equiv (\varphi_1 U \psi) \wedge (\varphi_2 U \psi)$$

$$GF(\varphi \vee \psi) \equiv GF\varphi \vee GF\psi$$

$$GF(\varphi \wedge \psi) \not\equiv GF\varphi \wedge GF\psi$$

$$FG(\varphi \vee \psi) \not\equiv FG\varphi \vee FG\psi$$

$$FG(\varphi \wedge \psi) \equiv FG\varphi \wedge FG\psi$$