

rekurencie: ξ -tyj řada

$$\boxed{x_0, x_1, \dots, x_n, \dots}$$

$$x_{n+1} = F(x_{n+1-1}, \dots, x_n)$$

řada x a y \rightarrow mocninová řada

$$f(x) = \sum_{\xi=0}^{\infty} x_{\xi} y^{\xi}$$

$$a_0 + a_1 x + a_2 x^2 + \dots = \sum_{i=0}^{\infty} a_i x^i$$

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polynomi

$$(1, 1, 1, \dots)$$

$$a_{\xi} = \binom{\xi+1-1}{\xi-1} = \binom{\xi}{0} = 1$$

$$a_{\xi} = \binom{\xi+2-1}{\xi-1} = \binom{\xi+1}{1} = \xi+1$$

$$(1, 2, 3, 4, 5, \dots)$$

$$a_{\xi} = \binom{\xi+2}{2} = \frac{(\xi+2)(\xi+1)}{2}$$

$$(1, 3, 6, 10, \dots)$$

řada $f(x)$

$$1 + x + x^2 + x^3 + \dots = \frac{1-x^{k+1}}{1-x}$$

$$(1 + \dots + x^k)(1-x) = 1 - x^{k+1}$$

$$(1 + \dots + x^k) = \frac{1-x^{k+1}}{1-x}$$

$$|x| < 1 \rightarrow \frac{1}{1-x}$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(0) x^n$$

$$\frac{1}{(1-x)^2} \quad \frac{1}{(1-x)^3}$$

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$$\binom{r}{\xi} = \frac{r \cdot (r-1) \cdot \dots \cdot (r-\xi+1)}{\xi!}$$

$$\binom{m}{\xi} = \frac{m!}{\xi!(m-\xi)!}$$

$$(1+x)^r = \sum_{\xi \geq 0} \binom{r}{\xi} x^{\xi}$$

$$(1+x)^{-n} = \sum_{\xi \geq 0} \frac{(-n)(-n-1)\dots(-n-\xi+1)}{\xi!} (-x)^{\xi}$$

$$\binom{m+\xi-1}{\xi} = \binom{m+\xi-1}{m-1}$$

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$$(1, 1, 1, \dots)$$

$$4, (1, 1, 1, 1, \dots)$$

$$(0, 0, 1, 1, 1, \dots)$$

$$(4, 4, 5, 5, 5, \dots)$$

$$(0, 1, 2, 3, \dots)$$

$$(3, 4, 5, 6, \dots)$$

$$(1, 2, 3, 4, \dots)$$

$$(0, 1, 2, 3, \dots)$$

$$\frac{1}{1-x}$$

$$\frac{4+x^2}{1-x} = 4 \cdot \frac{1}{1-x} + x^2 \cdot \frac{1}{1-x}$$

$$\frac{x}{(1-x)^2}$$

$$\left(\frac{1}{(1-x)^2} - 1 - 2x \right) \cdot \frac{1}{x}$$

$$\frac{1 - (1+2x)(1-x)^2}{(1-x)^2} = \frac{1 - (1+2x)(1-x)^2}{(1-x)^2}$$

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$$a(x) = \sum_{\xi=0}^{\infty} a_{\xi} x^{\xi} \quad x=y^2$$

$$b(y) = \sum_{\xi=0}^{\infty} a_{\xi} y^{3\xi}$$

$$(a_0, a_1, a_2, \dots)$$

$$(a_0, 0, 0, a_1, 0, 0, a_2, \dots)$$

$$(100010001\dots) \left| \frac{1}{1-x^4} \right.$$

$$(1000200030\dots) \left| \frac{1}{(1-x^4)^2} \right.$$

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$$(1, -1, 1, -1, 1, -1, \dots)$$

$$(1, 2, 4, 8, 16, 32, \dots)$$

$$\left(\sum_{\xi=0}^{\infty} a_{\xi} x^{\xi} \right)' = \sum_{\xi=0}^{\infty} \xi \cdot a_{\xi} \cdot x^{\xi-1}$$

$$(1, 1, 1, \dots) \left| \left(\frac{1}{1-x} \right)' = + \frac{1}{(1-x)^2} \right.$$

$$(1, 2, 3, \dots) \left| \left(\frac{1}{(1-x)^2} \right)' = + 2 \frac{1}{(1-x)^3} \right.$$

$$(2, 6, 12, 20, \dots)$$

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$$\int_0^1 \sum_{k \geq 0} a_k x^k = \text{bgy} = \sum_{k \geq 0} \frac{1}{k+1} a_k x^{k+1}$$

$$\int_0^1 \frac{1}{1-x} dx = [-\ln(1-x)]_0^1 = \ln \frac{1}{1-y}$$

$(1, 1, \dots) \rightsquigarrow (0, 1, \frac{1}{2}, \frac{1}{3}, \dots)$
 $\frac{1}{1-x}$ $\xrightarrow{\ln \frac{1}{1-x}}$

$H_n = \text{sumet punit n recipos} \} \text{ \& \text{oblat} } \frac{1}{k}$

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$$\left(\sum_{k \geq 0} a_k x^k \right) \cdot \left(\sum_{l \geq 0} b_l x^l \right)$$

$$\parallel$$

$$\sum_{n \geq 0} \left(\sum_{k+l=n} a_k b_l \right) x^n$$

$$\parallel$$

$$c_n = \sum_{l=0}^n b_l$$

$a = (1, 1, 1, \dots) \rightsquigarrow c_n = \sum_{l=0}^n b_l$

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Pravila (sumaci pčet)

pod $a = (a_0, a_1, \dots)$

raznaka $\Delta a = (a_1 - a_0, a_2 - a_1, \dots, a_{k+1} - a_k)$

čističar $\sigma a = (a_0, a_0 + a_1, a_0 + a_1 + a_2, \dots)$

$a \mapsto \Delta a \mapsto \sigma \Delta a = (a_1 - a_0, a_2 - a_1, \dots, a_{k+1} - a_k)$

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