

Významní funkce:
 (a_0, a_1, \dots) \leftrightarrow $\sum_{n \geq 0} a_n x^n$
 A
 operace:
 shift doprava \leftrightarrow násobení x
 $(0, a_0, a_1, \dots)$ \leftrightarrow $x A = \sum_{n \geq 0} a_n x^{n+1}$

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Lineární rekurence:
 $a_{n+2} = c_1 a_{n+1} + c_0 a_n$
 $a_{n+1} = c a_n \Rightarrow (a_0, c a_0, c^2 a_0, \dots, c^n a_0)$
 lineární rovnice: vezmeme $a_n = x^n$
 $\Rightarrow x^n(x^2 - c_1 x - c_0) = 0$
 $x^n(x^2 - c_1 x - c_0) = 0$

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$(0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \dots)$
 $F_0, F_1, F_2, F_3, \dots$ \leftrightarrow $F(x)$
 $\dots + [n=3] \cdot x + \dots$
 $F_{n+2} = F_{n+1} + F_n$ $F_0=0, F_1=1$
 $\sum_{n \geq 2} F_{n+2} x^{n+2} = x \sum_{n \geq 1} F_{n+1} x^{n+1} + x^2 \sum_{n \geq 0} F_n x^n$
 $x^0: F_0 = 0 + 0 = 0$
 $x^1: F_1 = F_0 + 0 = 1$
 $\Rightarrow F(x) - x F(x) - x^2 F(x) = x$

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$+ (F_0, F_1, F_2, F_3, \dots)$
 $- (0, F_0, F_1, F_2, F_3, \dots)$
 $- (0, 0, F_0, F_1, F_2, \dots)$
 $0 \quad 0 \quad 0 \quad 0 \quad \dots$
 $1 - x - x^2 = 0$ $x_{1,2} = \frac{1 \pm \sqrt{5}}{-2}$
 $\frac{1}{1-x} \leftrightarrow (1, 1, \dots)$ $\sum_{n \geq 0} 1 x^n$
 $\frac{1}{x-x} = \frac{1}{x(1-\frac{1}{x})} \leftrightarrow (1, x^{-1}, x^{-2}, x^{-3}, \dots)$ $\sum_{n \geq 0} (x^{-1})^n$

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mocninná řada:
 $(1, 1, \dots)$ \leftrightarrow $\frac{1}{1-x}$
 exponenciální:
 $(1, 1, \dots)$ \leftrightarrow $\sum_{n \geq 0} \frac{x^n}{n!} = e^x$
 $(a_0, a_1, a_2, a_3, \dots)$ \leftrightarrow $\sum_{n \geq 0} a_n \frac{1}{n!} x^n$
 integrace \leftarrow derivace
 $\sum_{n \geq 0} a_n \frac{1}{(n+1)!} x^{n+1}$ $\sum_{n \geq 1} a_n \frac{1}{(n-1)!} x^{n-1}$

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$a_0=0, a_1=1$
 $a_n = 5a_{n-1} - 6a_{n-2}$
 ① $a_n = 5a_{n-1} - 6a_{n-2} + [n=1]1$ $n=0 \checkmark$ $n=1 \checkmark$
 ② $\sum_{n \geq 0} a_n x^n = 5 \sum_{n \geq 0} a_{n-1} x^n - 6 \sum_{n \geq 0} a_{n-2} x^n + x$
 $\sum_{n \geq 0} a_n x^n = 5x \sum_{n \geq 1} a_{n-1} x^{n-1} - 6x^2 \sum_{n \geq 2} a_{n-2} x^{n-2} + x$
 ③ $A(x) = \frac{x}{1-5x+6x^2}$

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$$\frac{x}{1-5x+6x^2} = \frac{c}{1-3x} + \frac{d}{1-2x}$$

$$(1-3x)(1-2x)$$

$$x = c(1-2x) + d(1-3x)$$

$$x^0: 0 = c + d \quad \left| \begin{array}{l} 1 = -d \\ 1 = c \end{array} \right.$$

$$x^1: 1 = -2c - 3d \quad \left| \begin{array}{l} 1 = c \end{array} \right.$$

$$a_0 = 0$$

$$a_1 = 1$$

$$a_2 = 5$$

$$a_3 = 19$$

① $A(x) = \frac{1}{1-3x} - \frac{1}{1-2x}$

$$\Rightarrow a_n = 3^n - 2^n$$

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$$C_n = n-1 + \sum_{k=1}^n \frac{1}{n} (C_{k-1} + C_{n-k})$$

$$2 \cdot C_2 = 2 + 2(C_0 + C_1) = 2 \quad C_2 = 1$$

$$3 \cdot C_3 = 6 + 2(C_0 + C_1 + C_2) = 8 \quad C_3 = \frac{8}{3}$$

$C(x)$ nicht für C_n !

$$x C'(x) = \sum_{n \geq 1} n C_n x^{n-1}$$

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$$\frac{1}{1-x} \sim \sum_{n \geq 0} x^n$$

↓ deriv

$$\frac{1}{(1-x)^2} \sim \sum_{n \geq 1} n x^{n-1}$$

↓ deriv

$$\frac{2}{(1-x)^3} \sim \sum_{n \geq 2} n(n-1) x^{n-2}$$

mitte: $C' = \frac{2x}{(1-x)^3} + \frac{2}{1-x} C$

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