Mining Data Streams

Advanced Search Techniques for Large Scale Data Analytics

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Data Streams

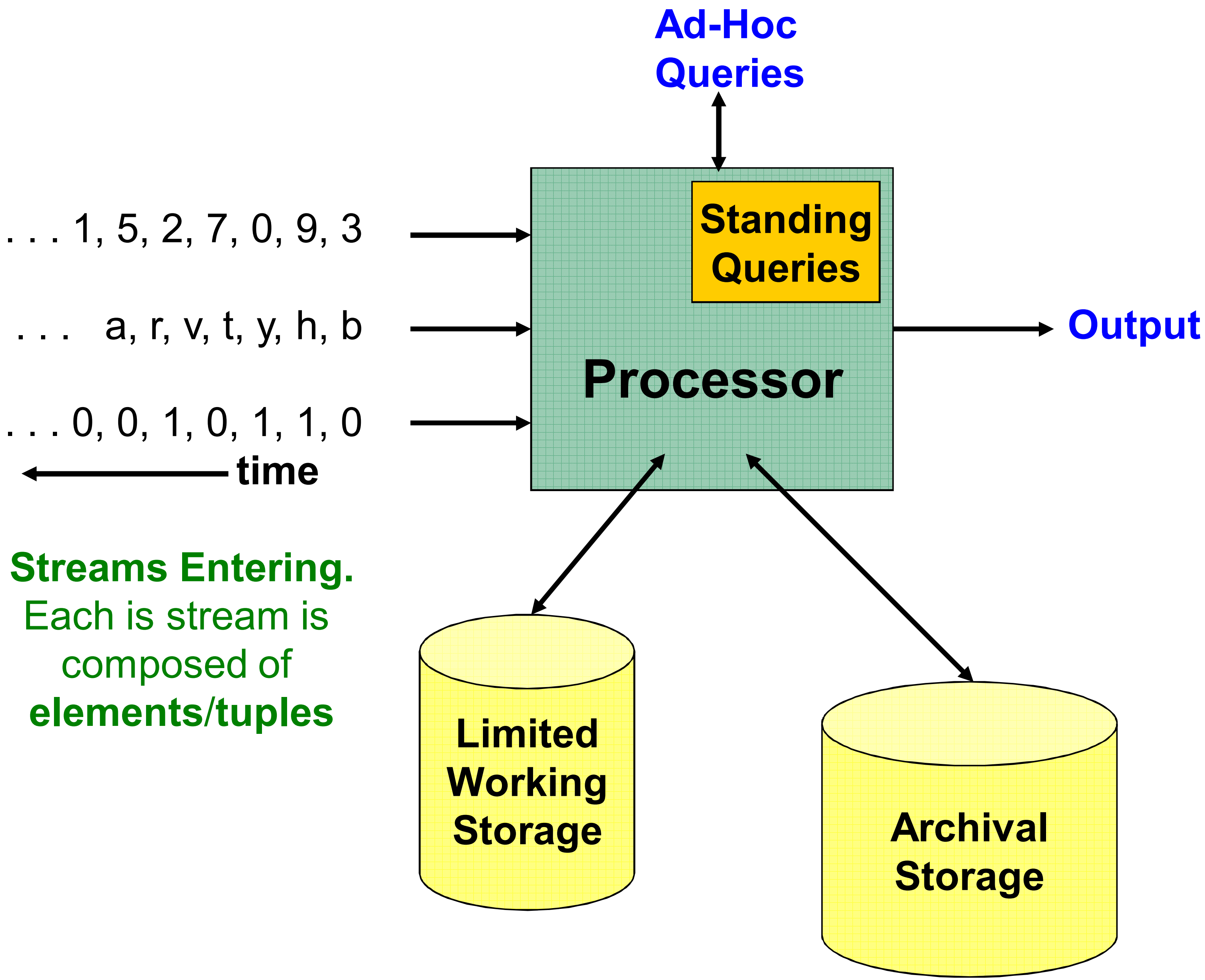
- **In many data mining situations, we do not know the entire data set in advance**

- **Stream Management** is important when the input rate is controlled **externally:**
  - Google queries
  - Twitter or Facebook status updates
- We can think of the **data** as **infinite** and **non-stationary** (the distribution changes over time)

The Stream Model

- Input **elements** enter at a rapid rate,
at one or more input ports (i.e., **streams**)
  - **We call elements of the stream tuples**
- **The system cannot store the entire stream accessibly**
- **Q: How do you make critical calculations about the stream using a limited amount of (secondary) memory?**

General Stream Processing Model



Streams Entering. Each is stream is composed of **elements/tuples**

Problems on Data Streams

- Types of queries one wants on answer on a data stream:
  - **Sampling data from a stream**
    - Construct a random sample
  - **Queries over sliding windows**
    - Number of items of type ***x*** in the last ***k*** elements of the stream
  - **Filtering a data stream**
    - Select elements with property ***x*** from the stream
  - **Counting distinct elements**
    - Number of distinct elements in the last ***k*** elements of the stream

Applications (1)

- **Mining query streams**
  - Google wants to know what queries are more frequent today than yesterday
- **Mining click streams**
  - Yahoo wants to know which of its pages are getting an unusual number of hits in the past hour
- **Mining social network news feeds**
  - E.g., look for trending topics on Twitter, Facebook

Applications (2)

- **Sensor Networks**
  - Many sensors feeding into a central controller
- **Telephone call records**
  - Data feeds into customer bills as well as settlements between telephone companies
- **IP packets monitored at a switch**
  - Gather information for optimal routing
  - Detect denial-of-service attacks

Sampling from a Data Stream: Sampling a fixed proportion

As the stream grows the sample also gets bigger

Sampling from a Data Stream

- Since **we can not store the entire stream**, one obvious approach is to store a **sample**
- **Two different problems:**
  - **(1)** Sample a **fixed proportion** of elements in the stream (say 1 in 10)
  - **(2)** Maintain a **random sample of fixed size** over a potentially infinite stream
    - At any "time" ***k*** we would like a random sample of ***s*** elements
      - **What is the property of the sample we want to maintain?** For all time steps ***k***, each of ***k*** elements seen so far has equal prob. of being sampled

Sampling a Fixed Proportion

- **Problem 1: Sampling fixed proportion**
- **Scenario:** Search engine query stream
  - **Stream of tuples:** (user, query, time)
  - **Answer questions such as: How often did a user run the same query in a single days**
  - Have space to store **1/10th** of query stream
- **Naïve solution:**
  - Generate a random integer in **[0..9]** for each query
  - Store the query if the integer is **0**, otherwise discard

Problem with Naïve Approach

- **Simple question: What fraction of queries by an average search engine user are duplicates?**
  - Suppose each user issues ***x*** queries once and ***d*** queries twice (total of ***x*+2*d*** queries)
    - **Correct answer:** ***d*/(*x*+*d*)**
  - **Proposed solution: We keep 10% of the queries**
    - Sample will contain ***x*/10** of the singleton queries and **2*d*/10** of the duplicate queries at least once
    - But only ***d*/100** pairs of duplicates
      - **d/100 = 1/10 · 1/10 · d**
    - Of ***d*** “duplicates” ***18d/100*** appear exactly once
      - **18d/100 = ((1/10 · 9/10)+(9/10 · 1/10)) · d**
  - **So the sample-based answer is**

Solution: Sample Users

Solution:

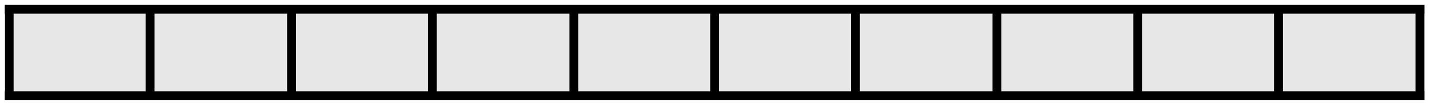
- Pick **1/10th** of **users** and take all their searches in the sample

- Use a hash function that hashes the user name or user id uniformly into 10 buckets

Generalized Solution

- **Stream of tuples with keys:**
  - Key is some subset of each tuple's components
    - e.g., tuple is (user, search, time); key is **user**
  - Choice of key depends on application

- **To get a sample of *a/b* fraction of the stream:**
  - Hash each tuple's key uniformly into ***b*** buckets
  - Pick the tuple if its hash value is at most ***a***



Hash table with **b** buckets, pick the tuple if its hash value is at most **a.**
How to generate a 30% sample?
Hash into b=10 buckets, take the tuple if it hashes to one of the first 3 buckets

Sampling from a Data Stream: Sampling a fixed-size sample

As the stream grows, the sample is of fixed size

Maintaining a fixed-size sample

- **Problem 2: Fixed-size sample**
- **Suppose we need to maintain a random sample *S* of size exactly *s* tuples**
  - E.g., main memory size constraint
- **Why?** Don't know length of stream in advance
- **Suppose at time *n* we have seen *n* items**
  - **Each item is in the sample *S* with equal prob. *s/n***

How to think about the problem: say s = 2

Stream: a x c y z k q d e g…

At **n= 5,** each of the first 5 tuples is included in the sample **S** with equal prob.

At **n= 7,** each of the first 7 tuples is included in the sample **S** with equal prob.

Impractical solution would be to store all the *n* tuples seen so far and out of them pick *s* at random

Solution: Fixed Size Sample

- **Algorithm (a.k.a. Reservoir Sampling)**
  - Store all the first ***s*** elements of the stream to ***S***
  - Suppose we have seen ***n-1*** elements, and now the n^{th} element arrives ($n > s$)
    - With probability s/n, keep the n^{th} element, else discard it
    - If we picked the n^{th} element, then it replaces one of the ***s*** elements in the sample ***S***, picked uniformly at random
- **Claim:** This algorithm maintains a sample ***S*** with the desired property:
  - After ***n*** elements, the sample contains each element seen so far with probability s/n

Proof: By Induction

- **We prove this by induction:**
  - Assume that after ***n*** elements, the sample contains each element seen so far with probability ***s/n***
  - We need to show that after seeing element ***n+1*** the sample maintains the property
    - Sample contains each element seen so far with probability ***s/(n+1)***
- **Base case:**
  - After we see **n=s** elements the sample **S** has the desired property
    - Each out of **n=s** elements is in the sample with probability ***s/s = 1***

Proof: By Induction

- **Inductive hypothesis:** After ***n*** elements, the sample ***S*** contains each element seen so far with prob. ***s/n***
- **Now element *n+1* arrives**
- **Inductive step:** For elements already in ***S***, probability that the algorithm keeps it in ***S*** is:

$$\underbrace{\left(1-\frac{s}{n+1}\right)}_{\text{Element } \mathbf{n+1} \text{ discarded}}+\underbrace{\left(\frac{s}{n+1}\right)}_{\text{Element } \mathbf{n+1} \text{ not discarded}}\underbrace{\left(\frac{s-1}{s}\right)}_{\text{Element in the sample not picked}}=\frac{n}{n+1}$$

- So, at time ***n,*** tuples in ***S*** were there with prob. **s/n**
- Time ***n→n+1***, tuple stayed in ***S*** with prob. **n/(n+1)**
- So prob. tuple is in ***S*** at time ***n+1*** **=**

Queries over a (long) Sliding Window

Sliding Window: 1 Stream

- **Sliding window on a single stream:** **N = 6**

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

← Past Future →

Sliding Windows

- A useful model of stream processing is that queries are about a ***window*** of length ***N*** – the ***N*** most recent elements received

- **Interesting case:** ***N*** is so large that the data cannot be stored in memory, or even on disk
  - Or, there are so many streams that windows for all cannot be stored
- **Amazon example:**
  - For every product **X** we keep 0/1 stream of whether that product was sold in the **n**-th transaction
  - We want answer queries, how many times have we sold **X** in the last **k** sales

Counting Bits (1)

- **Problem:**
  - Given a stream of **0**s and **1**s
  - Be prepared to answer queries of the form
    How many 1s are in the last *k* bits? where $k \leq N$
- **Obvious solution:**
  Store the most recent ***N*** bits
  - When new bit comes in, discard the ***N*+1**st bit

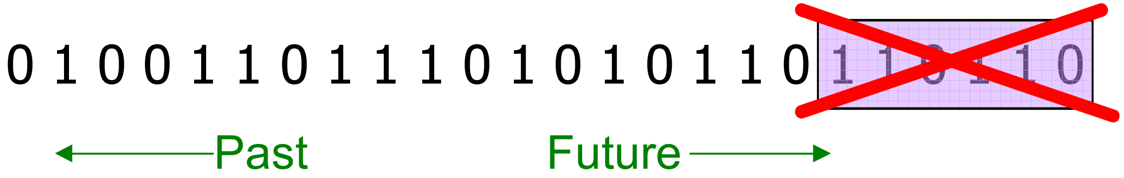
0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 $\boxed{1\ 1\ 0\ 1\ 1\ 0}$ Suppose N=6

← Past Future →

Counting Bits (2)

- **You can not get an exact answer without storing the entire window**

- **Real Problem:**
  What if we cannot afford to store *N* bits?
  - **E.g.**, we're processing 1 billion streams and
    ***N* = 1 billion**



- **But we are happy with an approximate answer**

An attempt: Simple solution

- Q: **How many 1s are in the last *N* bits?**
- A simple solution that does not really solve our problem: **Uniformity assumption**

N

0 1 0 0 1 1 1 0 0 0 1 0 1 0 0 1 0 0 0 1 0 1 1 0 1 1 0 1 1 1 0 0 1 0 1 0 1 1 0 0 1 1 0 1 0

Past Future

- **Maintain 2 counters:**
  - ***S***: number of 1s from the beginning of the stream
  - ***Z***: number of 0s from the beginning of the stream
- **How many 1s are in the last N bits?**
- **But, what if stream is non-uniform?**
  - What if distribution changes over time?

[Datar, Gionis, Indyk, Motwani]

DGIM Method

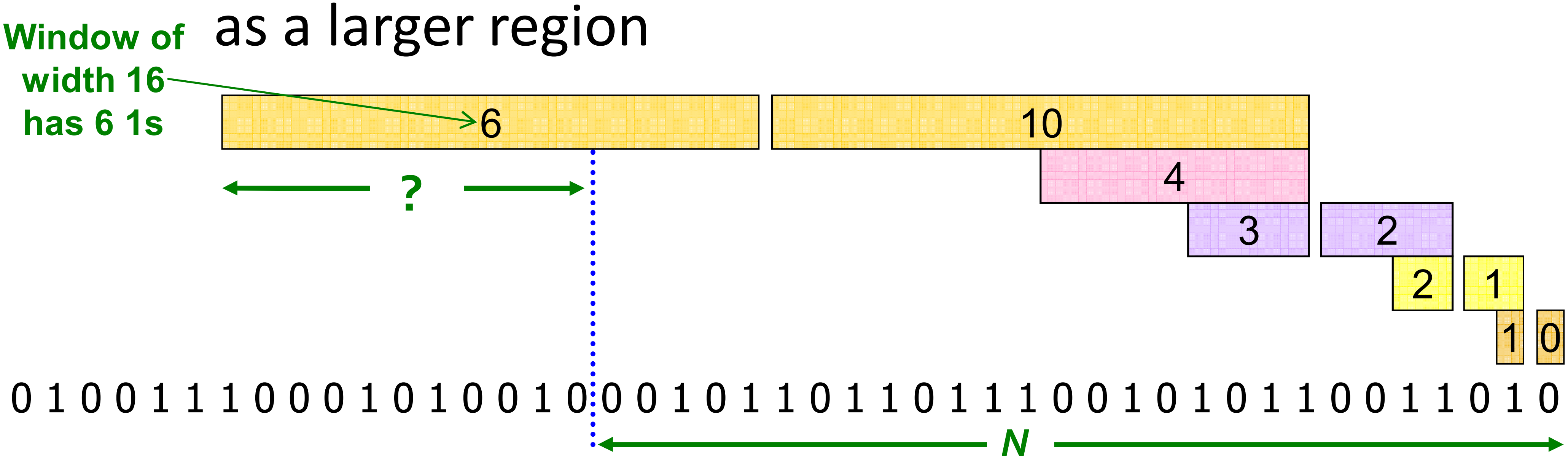
- **DGIM solution that does <u>not</u> assume uniformity**

- We store bits per stream

- **Solution gives approximate answer, never off by more than 50%**
  - Error factor can be reduced to any fraction > 0, with more complicated algorithm and proportionally more stored bits

Idea: Exponential Windows

- **Solution that doesn't (quite) work:**
  - Summarize **exponentially increasing** regions of the stream, looking backward
  - Drop small regions if they begin at the same point as a larger region



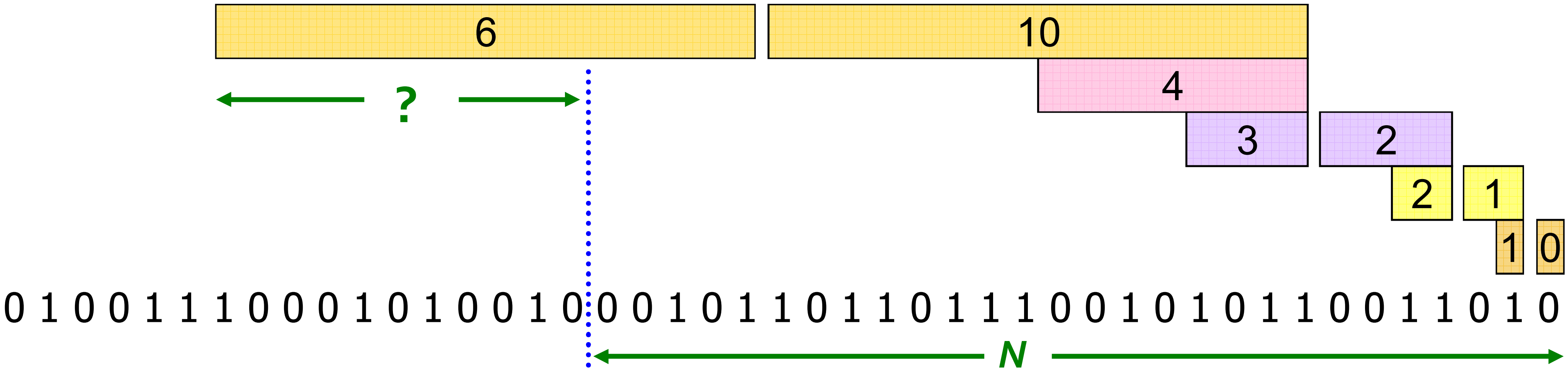
We can reconstruct the count of the last ***N*** bits, except we are not sure how many of the last **6 1s** are included in the ***N***

What's Good?

- **Stores only $O(\log^2 N)$ bits**
  - counts of bits each
- **Easy update as more bits enter**
- Error in count no greater than the number of **1s** in the "**unknown**" area

What's Not So Good?

- As long as the **1s** are fairly evenly distributed, the error due to the unknown region is small – **no more than 50%**
- But it could be that all the **1s** are in the unknown area at the end
- In that case, **the error is unbounded!**

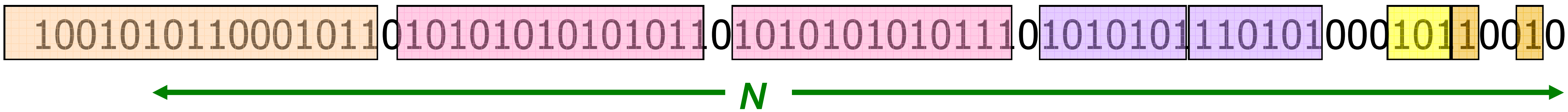


[Datar, Gionis, Indyk, Motwani]

Fixup: DGIM method

- **Idea:** Instead of summarizing fixed-length blocks, summarize blocks with specific number of **1s**:
  - Let the block ***sizes*** (number of **1s**) increase exponentially

- **When there are few 1s in the window, block sizes stay small, so errors are small**



DGIM: Timestamps

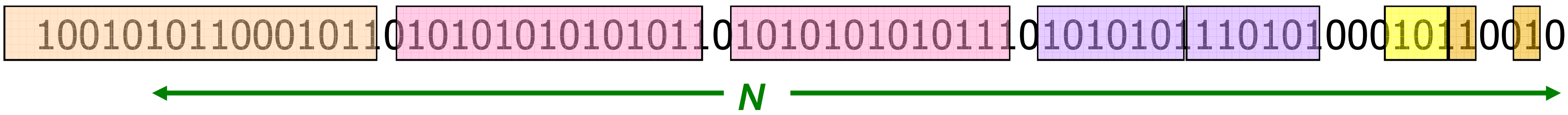
- Each bit in the stream has a ***timestamp***, starting **1**, **2,** …

- Record timestamps modulo ***N*** (**the window size**), so we can represent any **relevant** timestamp in bits

DGIM: Buckets

- A ***bucket*** in the DGIM method is a record consisting of:
  - **(A) The timestamp of its end** [O(log *N*) bits]
  - **(B) The number of 1s between its beginning and end** [O(log log *N*) bits]

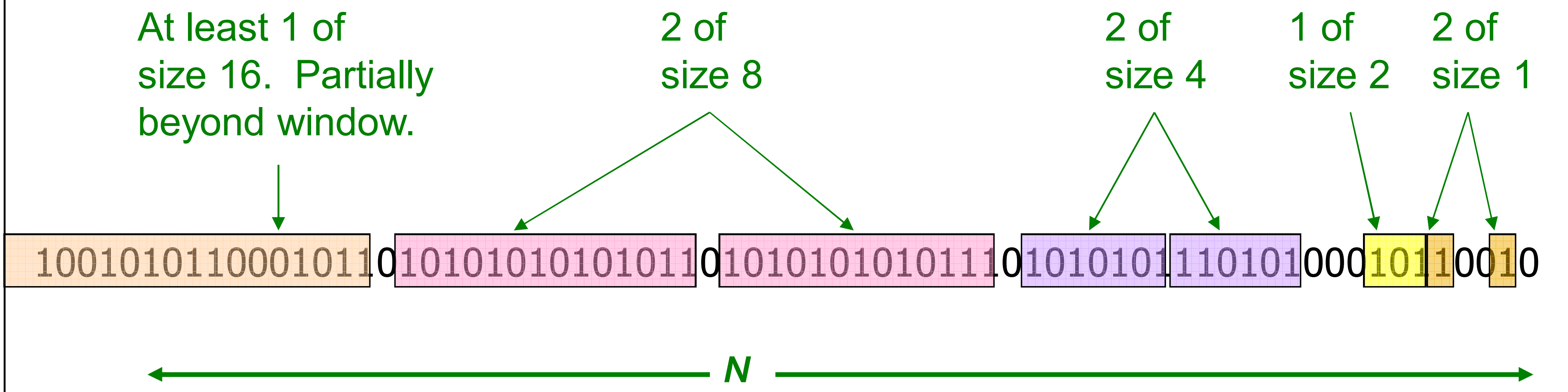
- **Constraint on buckets:**
  Number of **1s** must be a power of **2**
  - That explains the **O(log log *N*)** in **(B) above**



Representing a Stream by Buckets

- Either **one** or **two** buckets with the same **power-of-2 number** of **1s**
- **Buckets do not overlap in timestamps**
- **Buckets are sorted by size**
  - Earlier buckets are not smaller than later buckets
- Buckets disappear when their end-time is **>** ***N*** time units in the past

Example: Bucketized Stream



Three properties of buckets that are maintained:

- Either **one** or **two** buckets with the same **power-of-2** number of **1s**
- Buckets do not overlap in timestamps
- Buckets are sorted by size

Updating Buckets (1)

- When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to ***N*** time units before the current time
- **2 cases:** Current bit is **0** or **1**
- **If the current bit is 0:**
  no other changes are needed

Updating Buckets (2)

- **If the current bit is 1:**
  - **(1)** Create a new bucket of size **1**, for just this bit
    - **End timestamp = current time**
  - **(2)** If there are now **three buckets of size 1**, **combine the oldest two into a bucket of size 2**
  - **(3)** If there are now **three buckets of size 2**, **combine the oldest two into a bucket of size 4**
  - **(4) And so on …**

Example: Updating Buckets

Current state of the stream:

1001010110001011 0 1010101010101011 0 1010101010101110 1010101 110101 000 101 1 00 1 0

Bit of value 1 arrives

0010101100010110 1010101010101011 0 1010101010101110 1010101 110101 000 101 1 00 1 0 1

Two orange buckets get merged into a yellow bucket

0010101100010110 1010101010101011 0 1010101010101110 1010101 110101 000 101 1001 0 1

Next bit 1 arrives, new orange bucket is created, then 0 comes, then 1:

010110001011 0 1010101010101011 0 1010101010101110 1010101 110101 000 101 1001 0 1 1 0 1

Buckets get merged…

010110001011 0 1010101010101011 0 1010101010101110 1010101 110101 000 101 1001 0 11 0 1

State of the buckets after merging

010110001011 0 10101010101010111010101010101110 1010101110101 000 1011001 0 11 0 1

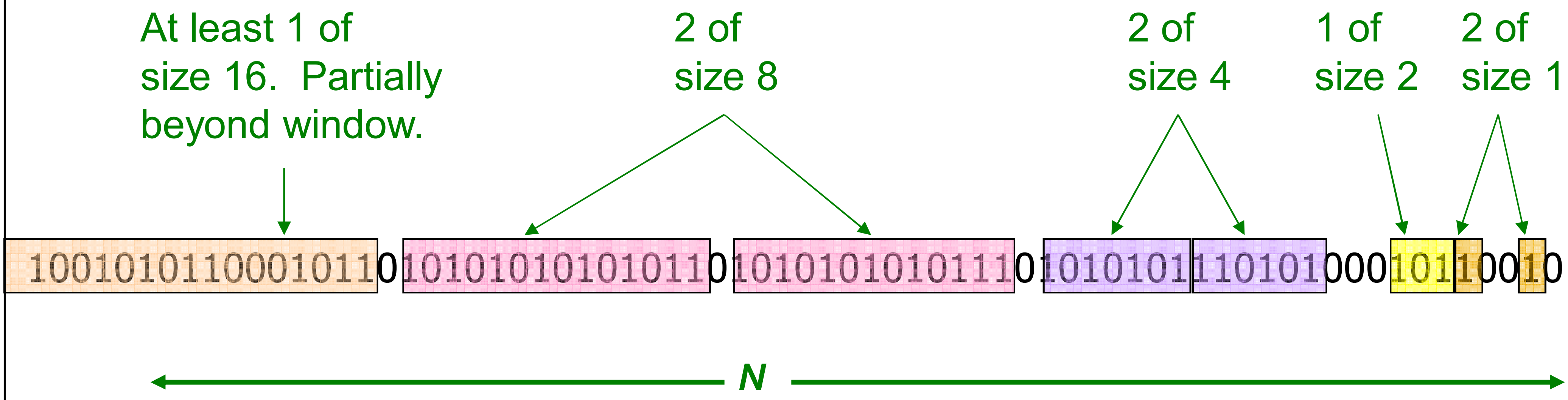
How to Query?

- **To estimate the number of 1s in the most recent *N* bits:**
  1. **Sum the sizes of all buckets but the last**

     (note "size" means the number of 1s in the bucket)
  2. **Add half the size of the last bucket**

- **Remember:** We do not know how many **1s** of the last bucket are still within the wanted window

Example: Bucketized Stream



(1) Filtering Data Streams

Filtering Data Streams

- **Each element of data stream is a tuple**
- Given a list of keys **S**
- **Determine which tuples of stream are in *S***

- **Obvious solution: Hash table**
  - But suppose we **do not have enough memory** to store all of ***S*** in a hash table
    - E.g., we might be processing millions of filters on the same stream

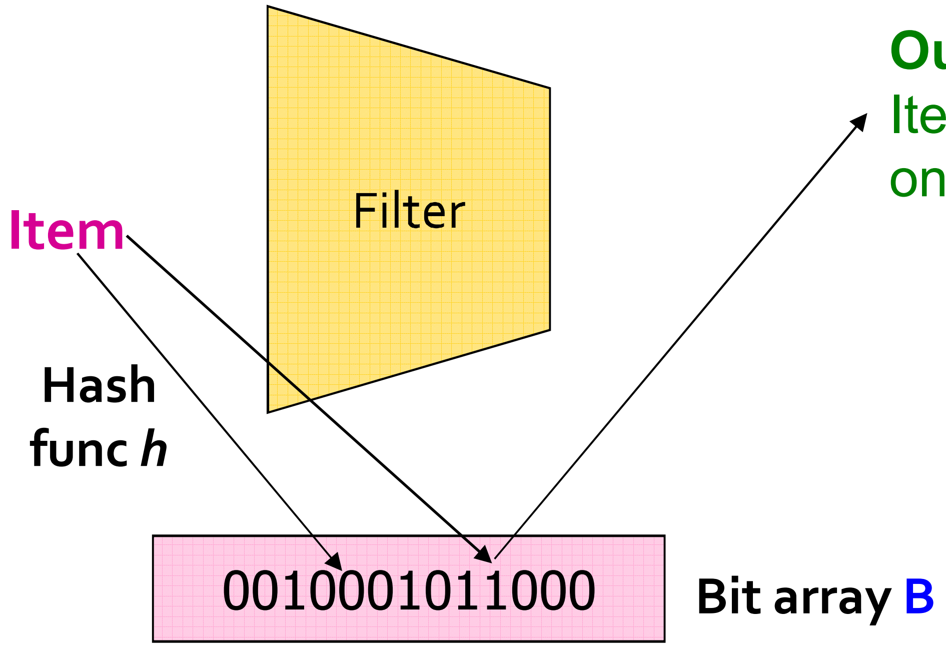
Applications

- **Example: Email spam filtering**
  - We know 1 billion "good" email addresses
  - If an email comes from one of these, it is **NOT** spam
- **Publish-subscribe systems**
  - You are collecting lots of messages (news articles)
  - People express interest in certain sets of keywords
  - Determine whether each message matches user's interest

First Cut Solution (1)

- **Given a set of keys *S* that we want to filter**
- Create a **bit array *B*** of ***n*** bits, initially all ***0*s**
- Choose a **hash function *h*** with range **[*0,n*)**
- Hash each member of $s \in S$ to one of n buckets, and set that bit to **1**, i.e., ***B[h(s)]=1***
- Hash each element ***a*** of the stream and output only those that hash to bit that was set to **1**
  - **Output *a* if B[h(a)] == 1**

First Cut Solution (2)



Output the item since it may be in *S*.
Item hashes to a bucket that at least one of the items in **S** hashed to.

Drop the item.
It hashes to a bucket set to **0** so it is surely not in ***S***.

- **Creates false positives but no false negatives**
  - If the item is in ***S*** we surely output it, if not we may still output it

First Cut Solution (3)

- **|S| = 1 billion email addresses**
  |B|= 1GB = 8 billion bits
- If the email address is in ***S***, then it surely hashes to a bucket that has the big set to **1**, so it always gets through (***no false negatives***)
- Approximately **1/8** of the bits are set to **1**, so about **1/8th** of the addresses not in ***S*** get through to the output (***false positives***)
  - Actually, less than **1/8th**, because more than one address might hash to the same bit

Bloom Filter

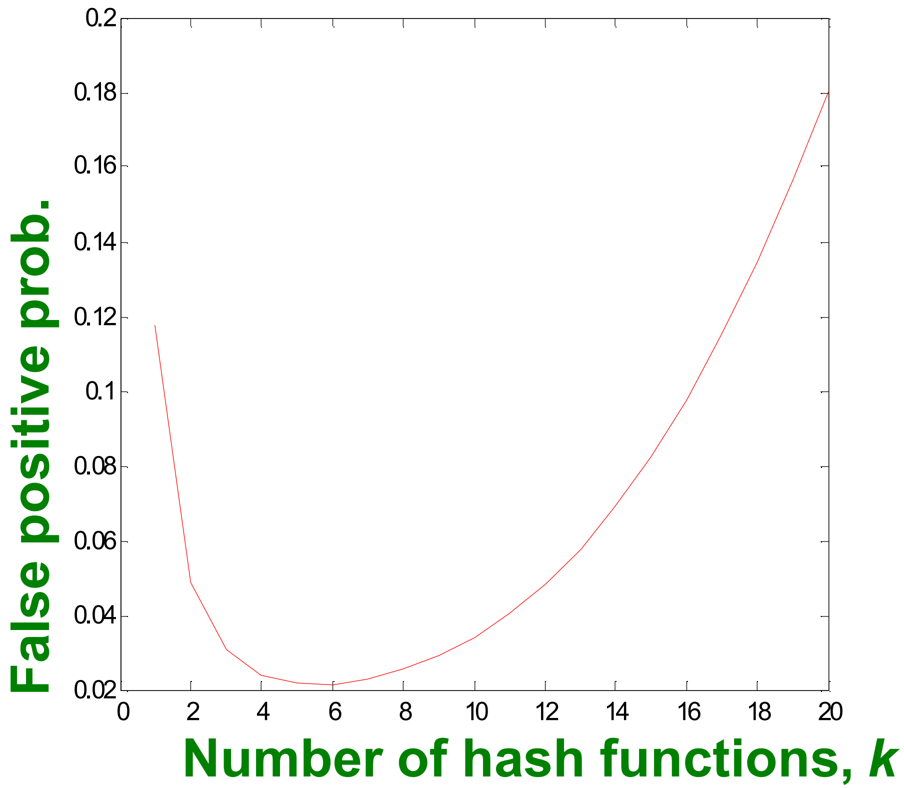
- Consider: **|S| = *m*, |B| = *n***
- Use ***k*** independent hash functions $h_1, \ldots, h_k$
- **Initialization:**
  - Set **B** to all **0s**
  - Hash each element $s \in S$ using each hash function h_i, set $B[h_i(s)] = 1$ (for each $i = 1,.., k$)

(**note:** we have a single array B!)

- **Run-time:**
  - When a stream element with key ***x*** arrives
    - If $B[h_i(x)] = 1$ for all $i = 1,\ldots, k$ then declare that ***x*** is in ***S***
      - That is, ***x*** hashes to a bucket set to **1** for every hash function $h_i(x)$
    - Otherwise discard the element ***x***

Bloom Filter – Analysis

- ***m* = 1 billion, *n* = 8 billion**
  - **k = 1**: $(1 - e^{-1/8})$ = **0.1175**
  - **k = 2**: $(1 - e^{-1/4})^2$ = **0.0493**



- **What happens as we keep increasing *k*?**

- "Optimal" value of ***k***: ***n/m* ln(2)**
  - **In our case:** Optimal **k = 8 ln(2) = 5.54 ≈ 6**

Bloom Filter: Wrap-up

- **Bloom filters guarantee no false negatives, and use limited memory**
  - Great for pre-processing before more expensive checks
- **Suitable for hardware implementation**
  - Hash function computations can be parallelized

- Is it better to have **1** big **B** or *k* small **B**s?
  - **It is the same:** $(1 - e^{-km/n})^k$ vs. $(1 - e^{-m/(n/k)})^k$
  - But keeping **1 big B** is simpler

(2) Counting Distinct Elements

Counting Distinct Elements

- **Problem:**
  - Data stream consists of a universe of elements chosen from a set of size ***N***
  - Maintain a count of the number of distinct elements seen so far
- **Obvious approach:**
  Maintain the set of elements seen so far
  - That is, keep a hash table of all the distinct elements seen so far

Applications

- **How many different words are found among the Web pages being crawled at a site?**
  - Unusually low or high numbers could indicate artificial pages (spam?)
- **How many different Web pages does each customer request in a week?**
- **How many distinct products have we sold in the last week?**

Using Small Storage

- **Real problem:** What if we do not have space to maintain the set of elements seen so far?
- Estimate the count in an unbiased way
- Accept that the count may have a little error, but limit the probability that the error is large

Flajolet-Martin Approach

- Pick a hash function ***h*** that maps each of the ***N*** elements to at least **$\log_2 N$** bits
- For each stream element ***a***, let ***r*(*a*)** be the number of trailing **0s** in ***h*(*a*)**
  - **r(a)** = position of first 1 counting from the right
    - E.g., say ***h(a) = 12***, then ***12*** is ***1100*** in binary, so ***r(a) = 2***
- Record ***R* = the maximum *r*(*a*) seen**
  - **R = $\max_a$ r(a)**, over all the items ***a*** seen so far
- **Estimated number of distinct elements = 2^R**

Why It Works: Intuition

- **Very very rough and heuristic intuition why Flajolet-Martin works:**
  - ***h(a)*** hashes ***a*** with **equal prob.** to any of ***N*** values
  - Then ***h(a)*** is a sequence of $\mathbf{log_2 N}$ bits, where ***2^{-r}*** fraction of all ***a***s have a tail of ***r*** zeros
    - About 50% of ***a***s hash to *****0**
    - About 25% of ***a***s hash to ****00**
    - So, if we saw the longest tail of ***r=2*** (i.e., item hash ending ***100**) then we have probably seen **about *4*** distinct items so far
  - **So, it takes to hash about *2^r* items before we see one with zero-suffix of length *r***