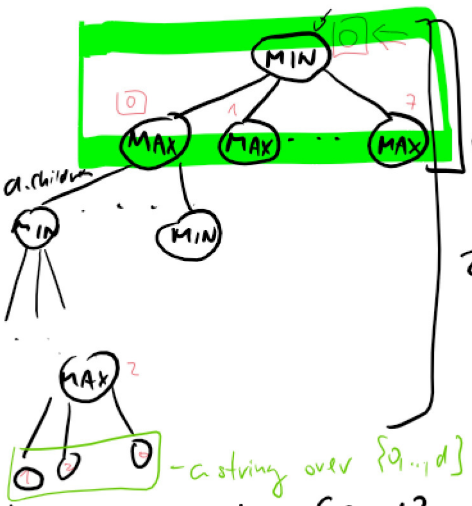
Tutorial IV

18 March 2020 10:03

Game Tree evaluation

MIN-MAX TREE



leaves are numbers {0,..,d}

$T_{k,d}$ - tree

k-levels

d-children

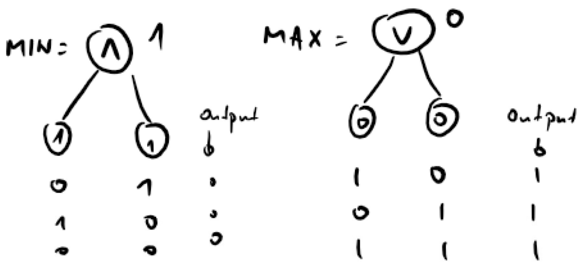
How many leaves does $T_{k,d}$ tree have? d^{2k}

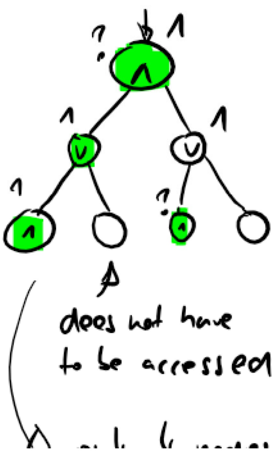
- → Each leaf contains a value
- → Each MIN node contains the smallest value of all it's children
- → Each MAX node contains the largest value of all it's children
- → Goal is to find the value of the root

$T_{k,2}$ ⇝ binary trees with input $2^{2k} = 4^k$ bits

min → $\wedge$ ⇝

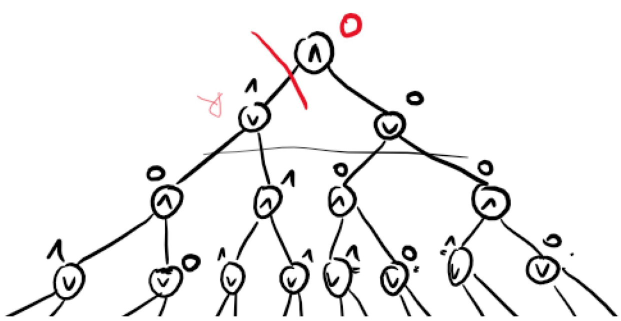
max → $\vee$





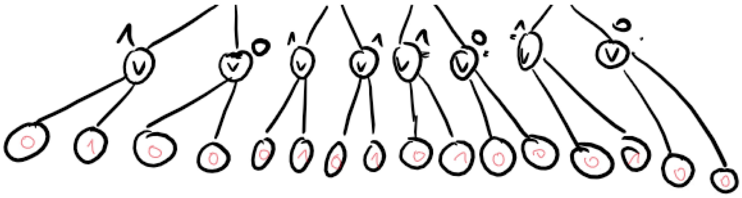
Deterministic order of node evaluation

'depth-first search', 'left child first'



to be accessed

only 4 nodes were evaluated



0100 0101 0100 0100 → difficult input

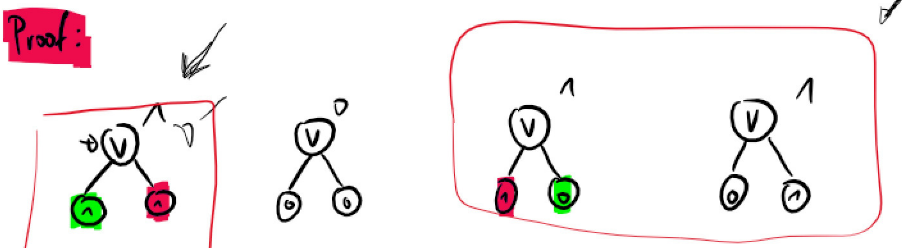
= all 32 nodes needed evaluation.

⇒ The difficulty of $\boxed{\text{any}}$ deterministic algorithm is $O(4^k)$. (worst case). $\boxed{n = 2^{2k}}$

$\boxed{\text{Randomized algorithm.}}$ → Choose the child to evaluate at random.

Claim: Complexity (expected) is $O(3^k)$ $\boxed{n^{0,79\ldots}}$

Proof:



1 evaluation

2 evaluations

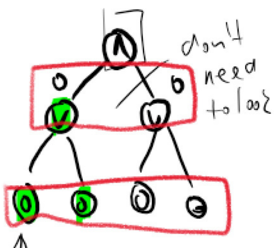
Red first - 1 evaluation

Green first - 2 evaluations

Avg: 3/2 evaluations

3/2 evaluations

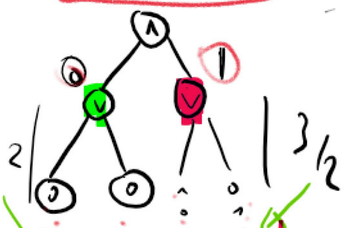
1 input | 3 different inputs | 3 inputs | 9 inputs



don't need to look

2 evaluations

$\boxed{\leq 3}$

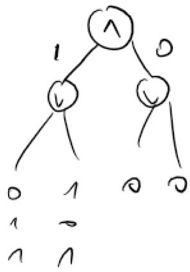


$\frac{1}{2} \cdot 2 + \frac{1}{2} \cdot \left(\frac{3}{2} + 2\right)$

evaluate ∨ (red)

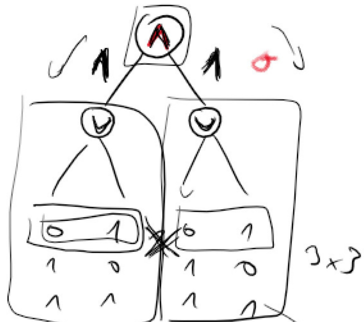
evaluate ∨ (green)

$$\boxed{\begin{gathered}\frac{1}{2} \cdot 2 + \frac{1}{2} \cdot \left(\frac{7}{2}\right) \\ 1 + \frac{7}{4} \leq 3\end{gathered}}$$



Symmetric to

$\boxed{\leq 3}$



$\boxed{\frac{3}{2} + \frac{3}{2} = 3}$

both take 3/2 evaluations on average

$\boxed{\leq 3}$

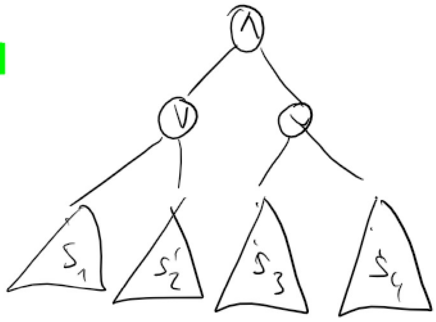
1 level trees take $\boxed{\leq 3}$ evaluations on average

1 level trees take $\boxed{\leq 3}$ evaluations on average

The basis of the induction.

I.H. $T_{k-1,2}$ take 3^{k-1} evaluations (at most)

I.S.



Same argument as in the basis case, but now each $S_1 \dots S_4$ takes 3^{k-1} steps (by I.H.) evaluate (not 1 as in the basis case)

therefore the expectation is less than $3 \cdot 3^{k-1} = 3^k$.