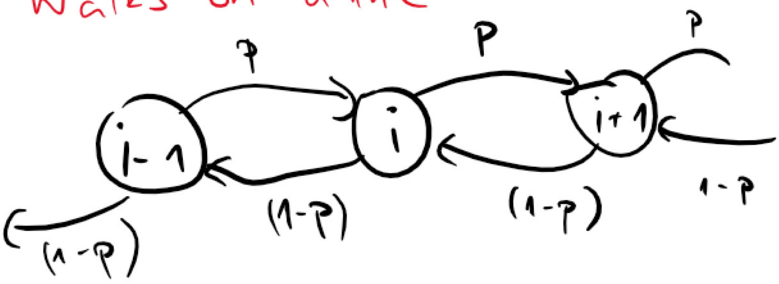
Tutorial X

23 April 2020 13:58

MARKOV CHAINS II

- → Walks on a line
- → randomized algorithm for 2-SAT
- → Fair 2-colorability of 3-colorable graphs.

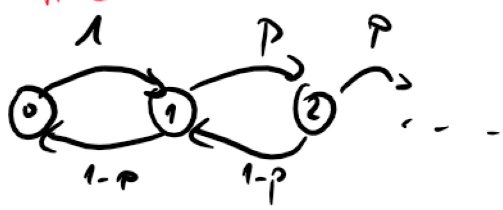
Walks on a line



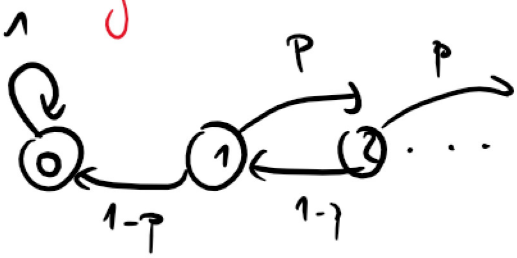
If the line is not infinite in both directions it contains at least one barrier.

TYPES OF BARRIERS

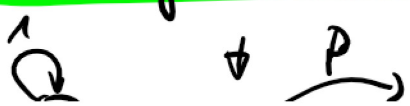
Reflective

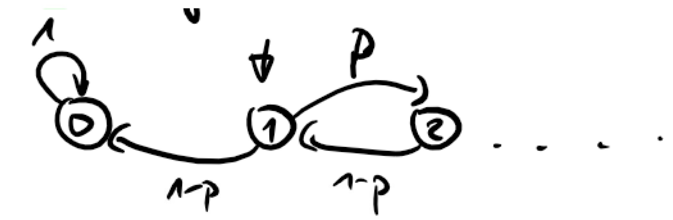


Absorbing

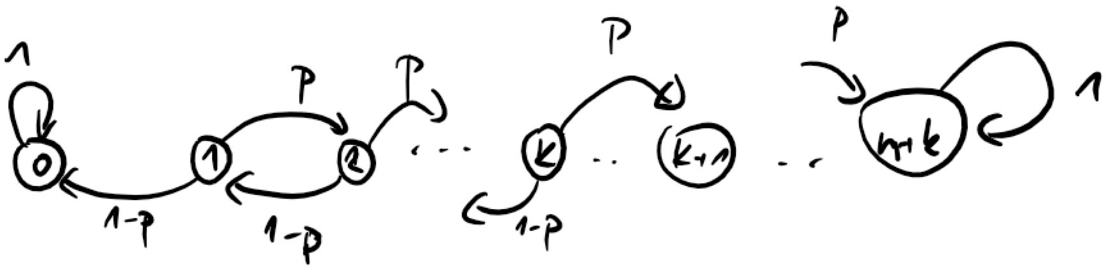


Monkey on a cliff

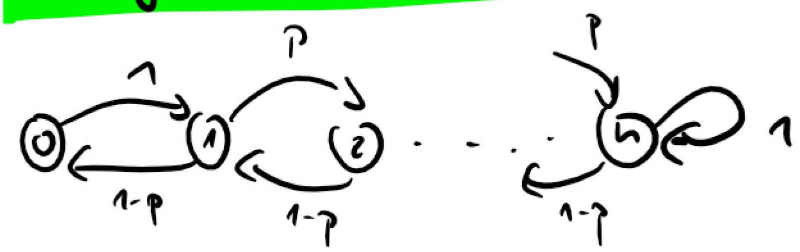




Gambler's ruin



Today we will discuss



→ What is the expected time to get from 0 to n.

$E_{i,j}$ = expected time to reach j from i

$\forall_j \; E_{j,j} = 0 \qquad E_{0,n} = E_{0,1} + E_{1,2} + E_{2,3} + \dots + E_{n-1,n}$

$E_{0,1} = 1$

$\forall_i \; E_{i,i+2} = E_{i,i+1} + E_{i+1,i+2}$

$$\forall_i \; E_{i,i+1} = 1 + p \overbrace{E_{i+1,i+1}}^{0} + (1-p) \overbrace{E_{i-1,i+1}}^{\text{2 step expectation}}$$

$$= 1 + (1-p)\left(E_{i-1,i} + E_{i,i+1}\right)$$

$$= 1 + (1-p) E_{i-1,i} + (1-p) E_{i,i+1}$$

$$= 1 + (1-p)E_{i-1,i} + (1-p)E_{i,i+1}$$

$$p \cdot E_{i,i+1} = 1 + (1-p) E_{i-1,i}$$

$$E_{i,i+1} = \frac{1}{p} + \frac{(1-p)}{p} E_{i-1,i}$$

$$E_{i,i+1} = r_i$$

$$\boxed{\begin{aligned} r_0 &= 1 \\ r_i &= 1/p + \frac{(1-p)}{p} r_{i-1} \end{aligned}}$$

Linear recursive relation. For given p solutions are easy to find:

http://www.cs.unipr.it/purrs/

2-SAT

Logical formula with $x_1, \dots, x_n$

terms

$$(x_1 \vee x_2) \wedge (x_3 \vee \neg x_2) \wedge (x_4 \vee x_1) \wedge \dots \wedge (\neg x_1 \vee \neg x_5)$$

Is this satisfiable? - is there a assignment of truth values (0/1) to variables x_i such that all terms are satisfied?

To find assignment A which satisfies all terms do:

1. Assign values to variables at random

Repeat the following:

2.) Find an unsatisfied term (if it doesn't exist you have a solution)

with variables x_a and x_b. Randomly pick x_a or x_b and flip its assignment.

This is a Las Vegas algorithm. If it runs too long say "I don't know".

If A exists how long will it take to find it?

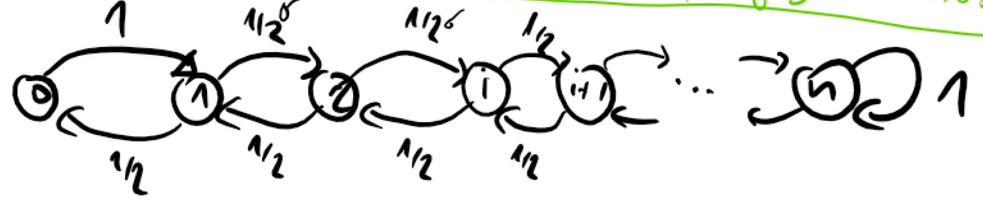
To analyze the running time we will use walk on a line

We count the number of variables in an intermediate solution (before step 2) with assignments identical to A.

A   1   1

$\wedge (x_a \vee x_b)$ → if this is not satisfied AT LEAST one of the assigned values differs from A. With probability AT LEAST 1/2 random flipping set us closer to A.

→ 0   0   0



The upper bound on the expected number of steps is equal to $E_{0,n}$.

E [illegible] $= 1$

$$E_{0,1} = 1$$

$$E_{i,i+1} = 1/p + \frac{(1-p)}{p} E_{i-1,i}$$

$$p = 1/2$$

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