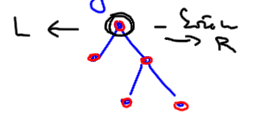
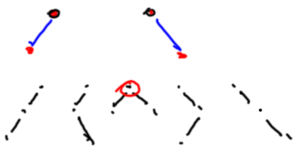
Stromy souvislé, bez cyklů



KOLIK JE BINÁRNÍCH pěstěných stromů?

$n=2$:

$n=3$:



$$b_n = \sum_{0\le k<n} b_k b_{n-k-1} + [n=0]1$$

kvě 7-13:56

$$\sum_{n\ge 0} b_n x^n = \sum_{n\ge 0} \sum_{0\le k<n} b_k b_{n-k-1} x^n$$

$$\boxed{B(x) = x\big(B(x)\big)^2 + 1}$$

$$B = xB^2 + 1$$

$$xB^2 - B + 1 = 0 \quad \text{kvadrat. r.}$$

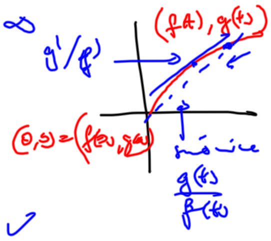
$$\Rightarrow \underline{2x\,B(x)} = 1 - \underline{(1-4x)^{1/2}}$$

kvě 7-14:19

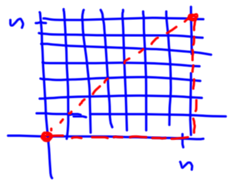
$$\lim_{x\to 0_+} \frac{1+\sqrt{1-4x}}{2x} = \infty$$

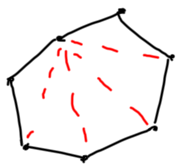
$$\lim_{x\to 0_+} \frac{1-\sqrt{1-4x}}{2x} =$$

$$\lim_{x\to 0_+} \frac{-\frac{1}{2}\frac{-4}{\sqrt{1-4x}}}{2} = 1 \quad ✓$$



kvě 7-14:28





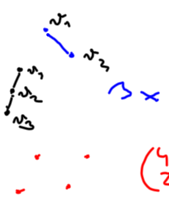
$$(1,1,1,\dots) \quad \begin{cases} \sum_{n\ge 0} x^n = \frac{1}{1-x} \\ \sum_{n\ge 0} \frac{1}{n!} x^n = e^x \end{cases}$$

kvě 7-14:40

$n \ge 2$:

$n=2 \quad 2^0 = 1$

$n=3 \quad 3^1 = 3$



$n=4 \quad 4^2 = 16$ ✓

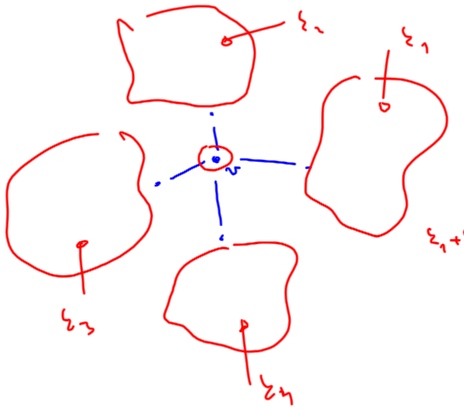
$\binom{4}{2} = 6$ hran

$\binom{6}{3} = 20$ trojic

$\binom{4}{0} = 4$ cyklů

$$\boxed{n^{n-2}}$$

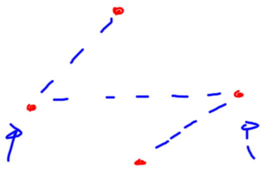
kvě 7-14:47



velikost n

$k_1 + k_2 + k_3 + k_4 = n-1$

kvě 7-14:57



$n = 4$

$n - 1 = 3$

$m = 1, 2, 3$

$$t_4 = \frac{3!}{3!} \cdot 3 \cdot t_3 + \cancel{\frac{1}{2}}\,\cancel{2}\,\frac{3!}{1!\,2!}\, 1 \cdot 2 \cdot t_1 t_2 + \frac{1}{3!} 3!\, t_1^{3}$$

$m = 1$ $\quad$ $m = 2$ $\quad$ $m = 3$

1) $k_1 = 1$, $k_2 = 2$

2) $k_1 = 2$, $k_2 = 1$

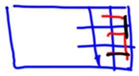
kvě 7-14:59

c_{n-2} $\quad$ 2 sloupce

r_n

3×6

$3 \times n$



$$c_n = c_{n-2} + 2 r_{n-1}$$

$$r_n = r_{n-2} + c_{n-1}$$

kvě 7-15:14

$$C = 2xR + x^2 C + 1$$

$$R = xC + x^2 R \Rightarrow R = \frac{xC}{1 - x^2}$$

$$C = 2\frac{x^2 C}{1 - x^2} + x^2 C + 1$$

$$\left(1 - \frac{2x^2}{1 - x^2} - x^2\right) C = 1$$

$$C = \frac{1 - x^2}{1 - x^2 - 2x^2 - x^2(1 - x^2)} = \frac{1 - x^2}{x^4 - 4x^2 + 1} \checkmark$$

$$R = \frac{x}{x^4 - 4x^2 + 1} \checkmark$$

kvě 7-15:31

$$D = \frac{1}{1 - 4z + z^2}$$

$$z_{1,2} = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$$

$d_n \leadsto$

kvě 7-15:35