Safety Model Checking with Complementary Approximations

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Complementary Approximate Reachability (CAR)

Technique for verifying invariant properties of boolean transition systems.

- Inspired by symbolic reachability and IC3/PDR
- Aims to find a property violation as fast as possible
- Works complementary to IC3/PDR

Preliminaries

We use propositional logic (boolean variables, connectives $\wedge, \vee, \neg, \implies, \iff$). A variable or its negation is called <u>literal</u>, conjunction of literals is <u>cube</u> and disjunction of literals is <u>clause</u>.

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A formula over variables $X = \{x_1, \ldots, x_n\}$ is satisfiable (SAT) if there exists an assignment $\alpha : X \to \{true, false\}$ to its variables that evaluates the formula to *true*. We allow α to be partial function, in which case we call the assignment partial assignment.

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If a formula ϕ is unsatisfiable (UNSAT), we can find its minimal unsat core (MUC) which is a subformula ψ of ϕ which is UNSAT but every subformula of ψ is SAT.

Preliminaries – cont.

Boolean transition system (BTS) is a tuple $(\bar{x}, I, T)$ consisting of

- state (boolean) variables $\bar{x} = \{x_1, x_2, ..., x_n\}$,
- a propositional formula $I(\bar{x})$ describing initial states,
- a propositional formula $T(\bar{x}, \bar{x}')$ describing transition relation

Where $\bar{x}'$ are the next-state versions of state variables: $\{x_1', x_2', ..., x_n'\}$.

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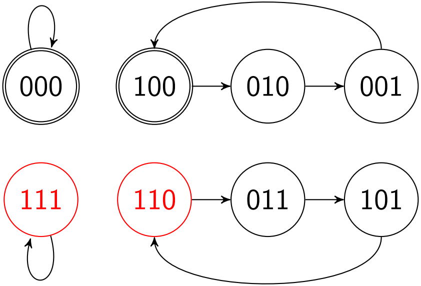
A predicate over $\bar{x}$ represents a set of states. Notably, a conjunction of n (different) literals represents a single <u>state</u>.

BTS Example



$$
\begin{aligned}
\bar{x} &= \{x_1, x_2, x_3\} \\
I(\bar{x}) &= \neg x_2 \wedge \neg x_3 \\
T(\bar{x}, \bar{x}') &= (x_1 \iff x_2') \wedge (x_2 \iff x_3') \wedge (x_3 \iff x_1') \\
P(\bar{x}) &= \neg x_1 \vee \neg x_2
\end{aligned}
$$

Preliminaries – cont.

From now on, we fix some BTS $(\bar{x}, I, T)$ and a set of states P called property.

Notation: we do not write arguments to named formulas. That is, we abbreviate $I(\bar{x})$ to I and $T(\bar{x}, \bar{x}')$ to T. Formula F' is the formula F with all variables primed. E.g., if $F = x \wedge y$ then $F' = x' \wedge y'$. If $F = z \wedge y'$, then $F' = z' \wedge y''$, etc.

Preliminaries – cont.

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A set of states is <u>invariant</u> of BTS if it is a superset of reachable states. A set S of states is <u>inductive</u> (closed under reachability) if

$$\begin{aligned} I &\implies S \\ S \wedge T &\implies S' \end{aligned}$$

Model Checking of BTSs

Assume we want to show that all states reachable from I satisfy a predicate P.

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Another option is to try to find an inductive invariant F such that $F \implies P$, i.e.,

- $I \implies F$
- $F \wedge T \implies F'$
- $F \implies P$

Model Checking of BTSs – cont.

A useful technique for finding inductive invariants for BTSs is IC3/PDR:

- IC3/PDR maintains an over-approximation F of states reachable in at most k steps (k is being increased if found insufficient)
- it iteratively blocks states that fail the inductiveness of F until F becomes inductive or a counter-example (real error) is found.
- IC3/PDR generalizes the blocked states to speed-up the convergence to an invariant.
- Problem 1: the generalization has a big over-head.
- Problem 2: it may take a long time to find a counter-example.

Model Checking of BTSs – cont.

CAR is inspired by IC3/PDR but tries to solve:

- Problem 1 (the generalization in IC3/PDR has a big over-head) by using MUC for the generalization. MUC are provided virtually free by SAT solvers.
- Problem 2 (it may take a long time to find a counter-example) by using also an under-approximation of states that reach bad states.

CAR Algorithm (main loop)

```
assume  B_i, F_i = false, true for all i > 0
F_0, B_0 ← I, ¬P

# check the first two steps from init
if sat(F_0 ∧ B_0) or sat(F_0 ∧ T ∧ B_0'):
    return unsafe (+ cex)

for i in  1, 2, ... :
    F_i ← P
    cex ← strengthen
    # a counter-example (real error) found
    if cex: return unsafe (+ cex)

    # some frame got inductive
    if ∃j ≤ i : F_j ⟹ ⋁_{m<j} F_m:
        return safe
```

CAR Algorithm (the core)

```
def strengthen():

    while sat((⋁_n F_n) ∧ T ∧ (⋁_n B'_n)):
        j ← the minimal j s.t. sat(F_j ∧ T ∧ B'_k) for some k
        c_1 ← partial_assignment(F_j ∧ T ∧ B'_k)|_x̄
        if j = 0: return c_1 # found error

        B_{k+1} ← B_{k+1} ∨ c_1
        φ ← F_{j-1} ∧ T ∧ c'_1
        if sat(φ):
            c_2 ← partial_assignment(φ)|_x̄
            B_{k+2} ← B_{k+2} ∨ c_2
        else:
            c_2 ← (unsat_core(φ)|_{c'_1})[x̄' ← x̄]
            F_j ← F_j ∧ ¬c_2
```

Final notes

- CAR can run also backwards (with T^{-1} from $\neg - P$ to I)
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Thank you!

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