# IA159 Formal Verification Methods

## Introduction

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# Agenda

Agenda

- basic information about the course
- quick overview of formal methods
- selected topics

# What does “Formal Verification Methods” mean?

Formal methods
: are a collection of notations and techniques for describing and analyzing systems. Methods are **formal** in the sense that they are based on some mathematical theories, such as logic, automata or graph theory. [Pel01]

Verification
: is the process of applying a manual or an automatic technique that is supposed to establish whether the code either satisfies a given property or behaves in accordance with some higher-level description of it. [Pel01]

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In the context of this course, formal verification methods are techniques (usually based on mathematical theories) for analysing systems with the aim to improve their quality and reliability.

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...typically methods that can find a bug or prove its absence.

# Focus of the course

- The course focuses on theoretical and algorithmic bases of selected verification methods.
- The software engineering aspects of verification methods are beyond the scope of this course.

# Literature

- Books (cover only some topics of the course):
  - *D. A. Peled: Software Reliability Methods, Springer, 2001.*
  - *E. M. Clarke, O. Grumberg, D. Kroening, D. Peled, and R. Bloem: Model Checking, Second Edition, MIT, 2018.*
  - *Ch. Baier and J.-P. Katoen: Principles of Model Checking, MIT, 2008.*
  - *E. M. Clarke, T. A. Henzinger, H. Veith, and R. Bloem: Handbook of Model Checking, Springer, 2018.*
  - *D. S. Scott: The Seventeen Provers of the World, Springer, 2006.*
  - …
- Other sources (mainly journal or conference papers) will be referred and available in Study materials in IS.

# Connections to other courses

Mandatory prerequisites

- IA169 System Verification and Assurance or
- IV113 Introduction to Validation and Verification († 2018)

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Other relevant courses

- IA006 Selected Topics on Automata Theory (aka FJA II)
- IA040 Modal and Temporal Logics for Processes
- IV022 Design and Verification of Algorithms
- IV101 Seminar on Verification († 2015)
- IA072 Seminar on Verification

# Examination

- There will be an oral exam at the end.
- No intrasemestral tests, no written exams, no mandatory homeworks.

# Overview of verification methods

# Basic verification methods

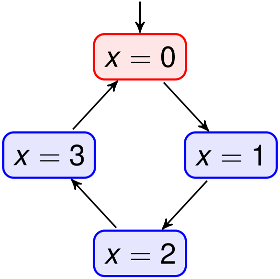
- testing
- deductive verification (with use of theorem provers)
- equivalence checking
- reachability analysis and model checking
- abstract interpretation and other static analyses
- symbolic execution

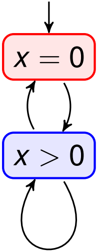
Other related techniques

- abstraction
- slicing
- SAT/SMT solving
- Craig interpolation

# Abstraction

- reduces the size of systems to be analyzed
- can transform an infinite-state system into a finite one
- the set of system behaviours is usually increased (source of false alarms)



# Slicing

- reduces the size of systems on the source code level
- the reduced system preserves values of given variables at given control locations
- M. Weiser: *Program Slicing*, IEEE Transactions on Software Engineering 10(4), 1984.

# Slicing: example

```
 1: char *copy(char *dst, char *src, int n, int *L) {
 2:     int i, len;
 3:     len = 0;
 4:     if (src != NULL && dst != NULL) {
 5:        len = n;
 6:        lock(L);
 7:     }
 8:     i = 0;
 9:     while (i < len) {
10:        dst[i] = src[i];
11:        i++;
12:     }
13:     if (len > 0) {
14:        unlock(L);
15:     }
16:     return dst;
17: }
```

Assume that we are interested only in values of lock `L` at the end of line 16.

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# SAT/SMT solving

**SAT problem** is to decide satisfiability of a given propositional logic formula.

**Satisfiability Modulo Theories (SMT) problem** is to decide satisfiability of a given first-order logic formula with respect to a given theory (e.g. theory of integers with addition and substraction).

- crucial for symbolic execution, bounded model checking, abstraction, deductive verification
- A. R. Bradley and Z. Manna: *The Calculus of Computation: Decision Procedures with Applications to Verification*, Springer, 2007.

# Craig interpolation

- if $\varphi \implies \psi$ then there exists an interpolant $\rho$ such that $\varphi \implies \rho \implies \psi$ and $\rho$ uses only propositional variables occurring in both $\varphi$ and $\psi$
- $\rho$ overapproximates $\varphi$ and it is usually smaller than $\varphi$
- crucial for PDR/IC3, Ultimate Automizer, and many methods/tools using abstraction refinement
- W. Craig: *Three uses of the Herbrand-Gentzen theorem in relating model theory and proof theory*, The Journal of Symbolic Logic 22(3), 1957.

# Testing

- simple, feasible, very good cost/performance ratio
- very effective in early stages of debugging process
- applicable directly to real systems
- cannot guarantee that there are no errors
- in practice: standard technique for enhancing the quality of systems, wide tool support

# Deductive verification

Deductive verification is a method for proving that, for any input values satisfying a given initial condition, a given program terminates and resulting variable values satisfy a given final assertion.

If the initial condition $x2 > 0$ holds, then the execution of

```
y1=0;
y2=0;
while (y2 < x2) {
    y1 = y1 + x1;
    y2++;
}
```

always terminates and the resulting variable values satisfy final assertion

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}
```

always terminates and the resulting variable values satisfy final assertion $y1 = x1 * x2$.

# Deductive verification

- applicable to models or small parts of real systems
- needs a huge effort of an expert on both deductive verification and systems under verification
- can guarantee that (a model of) a real system satisfies a given property
- in practice: used rarely (e.g. partial correctness of FPU in AMD processors)
- tools: Coq, ACL2, Dafny, ...

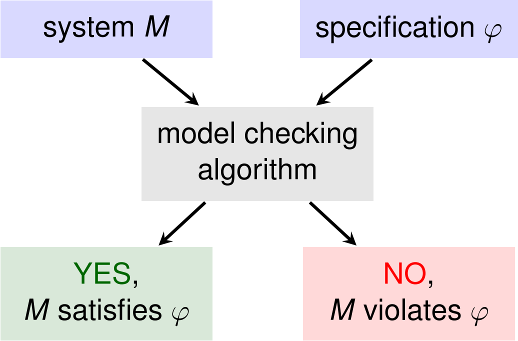
# Equivalence checking

Equivalence checking decides whether two given systems are equivalent with respect to a given equivalence.

- applicable mainly to models of real systems
- needs a detailed formal specification of a system under verification (or another “second system”)
- there are no algorithms for reasonable equivalences and infinite-state systems
- in practice: some specific applications (e.g. equivalence of different levels of hardware design)

# Reachability analysis and model checking

Reachability analysis decides whether any run of a given system can reach a given state. Model checking decides whether each run of a given system satisfies a given specification property (which is typically described by a temporal logic formula).



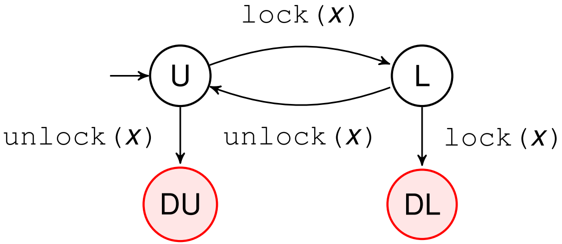
# Reachability analysis and model checking

- needs formal description of the property to be checked
- fully automatic, but feasible mainly for relatively small finite-state systems
- succesfull verification of real systems may require provision of a suitable abstraction
- bounded model checking examines only a part of the system (can find bugs, but not prove correctness)
- in practice: a standard technique for verification of simple hardware designs, used also for verification of small systems (e.g. communication protocols), bounded model checking applied to larger systems
- tools: DIVINE, SPIN, NuSMV, ...

# Abstract interpretation and other static analyses

Abstract interpretation and other static analyses are typically used to overapproximate or underapproximate a set of reachable values of selected program variables in each program location. The analyzed code is not executed.

Consider the following states of a lock $x$:




U = unlocked error states: DU = double unlock
L = locked DL = double lock

# Abstract interpretation and other static analyses

```
char *copy(char *dst, char *src, int n, int *L) {
    int i, len;                                        U
    len = 0;                                           U
    if (src != NULL && dst != NULL) {                  U
        len = n;                                       U
        lock(L);                                       L
    }                                                  U,L
    i = 0;                                             U,L
    while (i < len) {                                  U,L
        dst[i] = src[i];                               U,L
        i++;                                           U,L
    }                                                  U,L
    if (len > 0) {                                     U,L
        unlock(L);                                     DU,U
    }                                                  U,L
    return dst;                                        U,L
}
```

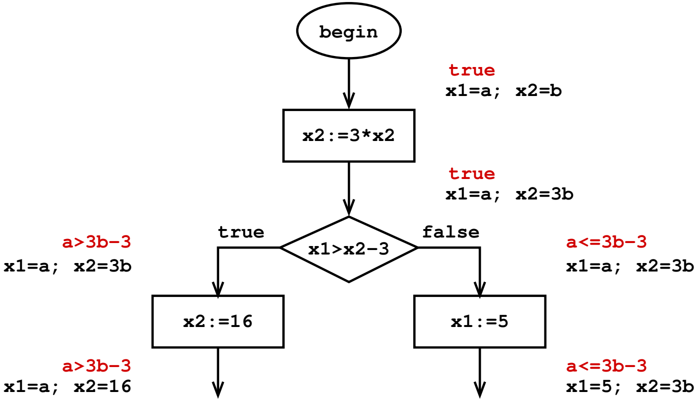
The indicated double unlock error is a false positive.

# Abstract interpretation and other static analyses

- applicable directly to source code of real systems (or directly to executables)
- feasible
- can verify only a specific class of properties (including many interesting properties)
- may produce false alarms
- fully automatic
- in practice: some static analysis is performed by almost every compiler, there are many efficient tools able to work with big pieces of real software (e.g. Linux kernel)
- tools: Coverity, CodeSonar, ...

# Symbolic execution

Symbolic execution executes the code on abstract symbols instead of input values.



# Symbolic execution

- can be seen as exhaustive testing
- applicable directly to source code of real systems (or directly to executables)
- fully automatic
- does not report false alarms
- feasible, but the computation usually did not finish due to large or even infinite number of execution paths
- in practice: several successful applications, but computational cost of pure symbolic execution is too high
- tools: Klee, ...

# Combined methods

- popular combinations:
  - model checking + abstraction + counter-example guided abstraction refinement (CEGAR)
  - abstract interpretation + CEGAR
  - testing + model checking
  - testing + symbolic execution + Craig interpolation
  - bounded model checking + $k$-induction
- the aim is to develop methods which are automatic (as much as possible) and applicable directly to sources or binaries of real systems
- may be incomplete and/or produce some false alarms
- in practice: already has some specific applications in verification (e.g. verification of Windows drivers by Static Driver Verifier, CPAchecker, Ultimate Automizer) and many applications in test-generation and bug-finding (e.g. SAGE, PEX, CBMC)
- the most promising approaches usually combine several basic techniques

# Verification of infinite-state systems

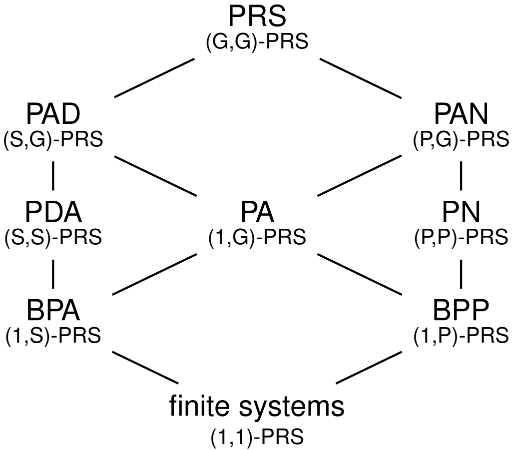
# Finite vs. infinite-state systems

```
y1=0;
y2=0;
while (y2 < x2) {
   y1 = y1 + x1;
   y2++;
}
```

- verification of algorithm vs. verification of programs
- all verification problems are decidable for finite systems
- for infinite-state systems, decidability depends on the problem and type of the system
- explicit and symbolic (BDD-based) model checking applicable only to finite systems

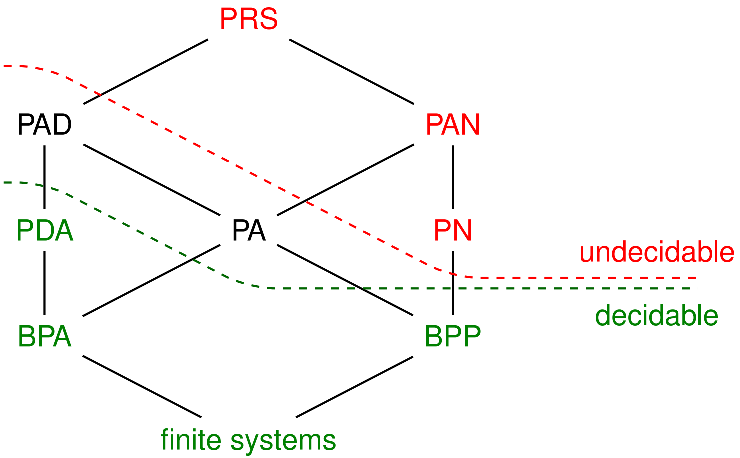
# PRS-hierarchy of infinite-state systems

The hierarchy compares expressive power of many classes of infinite-state systems including BPA, BPP, PA, Petri nets (PN), and pushdown processes (PDA). systems.



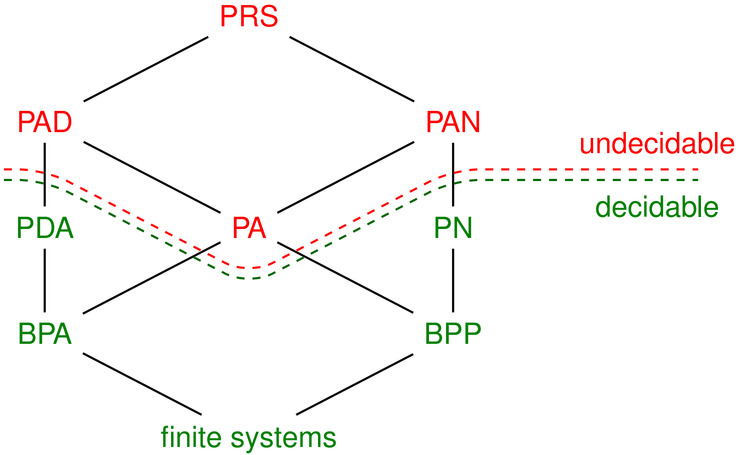
# Decidability of equivalence checking

The decidability boundary of strong bisimulation in the PRS-hierarchy.



# Decidability of model checking

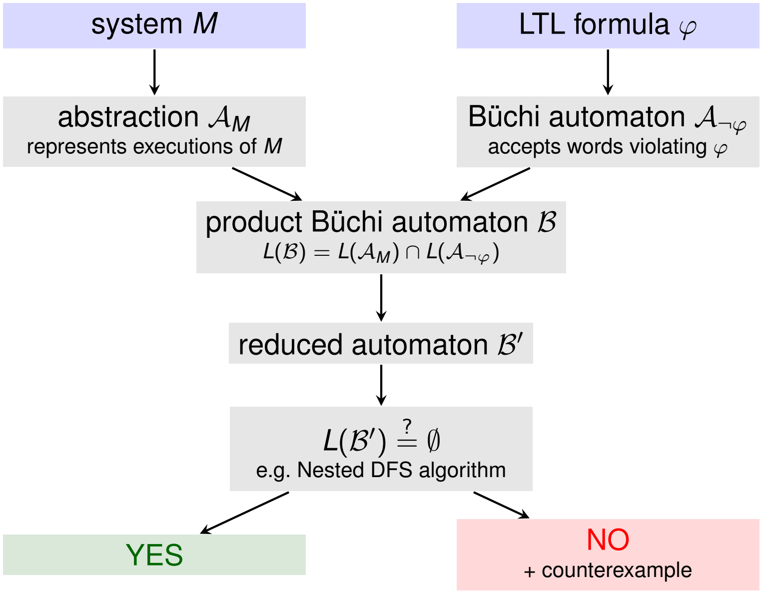
The decidability boundary of the action-based LTL model checking in the PRS-hierarchy.



# Actual topics of the course

- deductive verification
  - theorem prover ACL2 + Demo
- reachability analysis & verification of infinite-state systems
  - reachability analysis of pushdown systems
  - LTL model checking of pushdown systems
- LTL model checking
  - ~~translation of LTL to Büchi automata (via alternating aut.)~~
  - partial order reduction
  - abstraction and CEGAR
- static analysis
  - abstract interpretation
  - shape analysis (abs. int. of dynamic memory operations)
- Ultimate Automizer: verification via automata, symbolic execution, and interpolation
- property directed reachability (PDR/IC3)
- ~~symbolic execution~~

# Automata-based LTL model checking of finite systems



# Automata-based LTL model checking of finite systems

system $M$

LTL formula $\varphi$

abstraction $\mathcal{A}_M$
represents executions of $M$

Büchi automaton $\mathcal{A}_{\neg\varphi}$
accepts words violating $\varphi$

product Büchi automaton $\mathcal{B}$
$L(\mathcal{B}) = L(\mathcal{A}_M) \cap L(\mathcal{A}_{\neg\varphi})$

reduced automaton $\mathcal{B}'$

$L(\mathcal{B}') \stackrel{?}{=} \emptyset$
e.g. Nested DFS algorithm

YES

NO
+ counterexample

# An extra piece of motivation

- Formal verification is used in Microsoft, Intel, facebook, Amazon, NASA, ...
- Formal verification is usually a supplementary method, the main methods are testing or simulation.
- In development of execution cluster of Core i7 (2008), formal verification has been used as a primary validation vehicle
  - simulation has been dropped
  - only 3 bugs escaped to silicon (2 other bugs were detected during the pre-silicon stage by full chip testing)
  - this number is usually about 40
  - the previous minimum is 11
  - More information in Kaivola et al: *Replacing Testing with Formal Verification in Intel Core i7 Processor execution Engine Validation*, CAV 2009, LNCS 5643, Springer, 2009.

# Coming next week

Theorem prover ACL2

`http://www.cs.utexas.edu/users/moore/acl2/`

- How it works?
- What is it good for?
- Including a live show!