PyMC3

Very Brief Intro

to MCMC

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Bayes theorem recall

$$\underbrace{p(\theta \mid y)}_{\text{Posterior}} = \frac{\overbrace{p(y \mid \theta)}^{\text{Likelihood}} \cdot \overbrace{p(\theta)}^{\text{Prior}}}{\underbrace{p(y)}_{\text{Evidence}}}$$

Computational challenge

$$p(y) = \int_\Theta p(y \mid \theta)p(\theta)d\theta$$

$$= \int_\Theta p(y, \ \theta)d\theta$$

The application of **total probability theorem**

Monte Carlo

Invented by **Stanislaw Ulam**

Many applications, i.e. Monte Carlo integration

$$\mathbb{E}[f(x)] = \int_X f(x)p(x)dx$$

$$\simeq \frac{1}{n}\sum_{i=1}^{n} f(\cdot)$$

$n \rightarrow \infty$

Sampling

Pi approximation

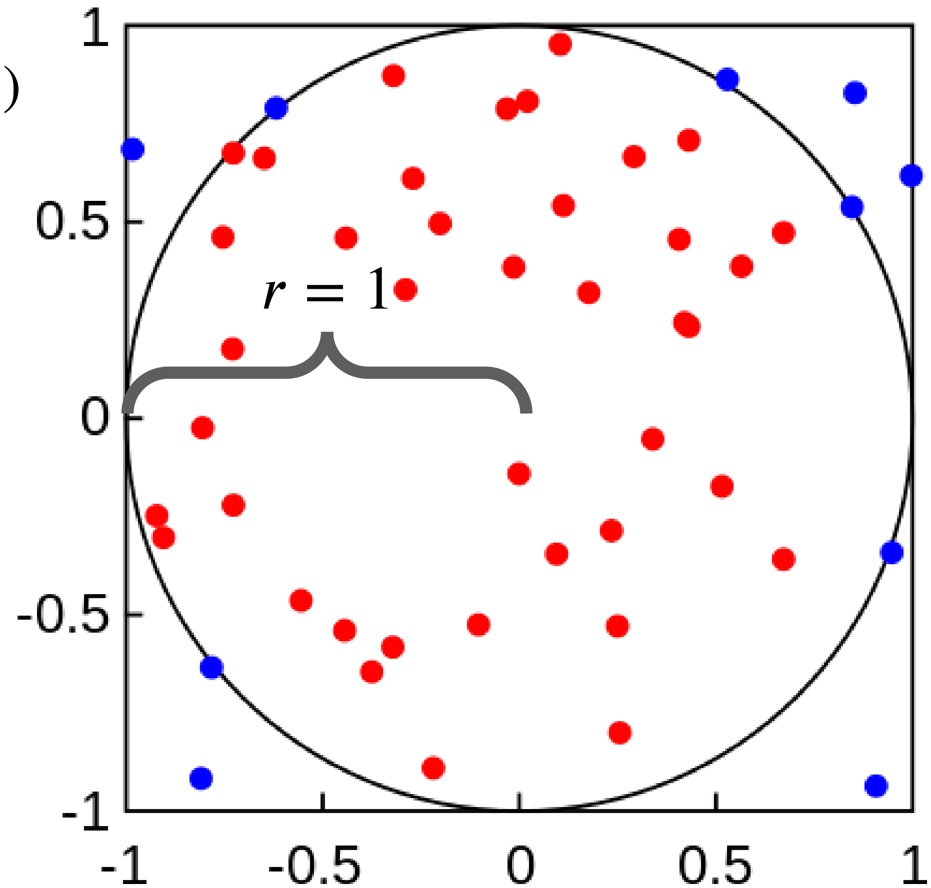
$x_i \sim \text{Uniform}(-1, 1) \quad y_i \sim \text{Uniform}(-1, 1)$

$$C = \pi \cdot r^2 = \pi$$
$$S = a^2 = (2r)^2 = 4$$

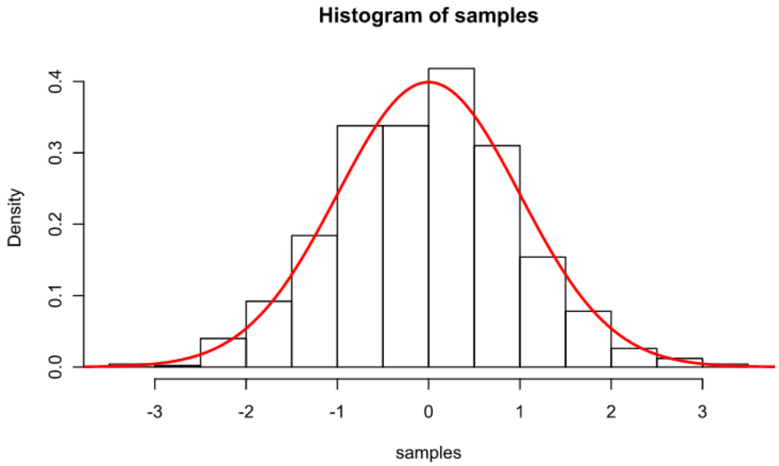
$$f(x_i, y_i) = \begin{cases} 1 & x_i^2 + y_i^2 < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{C}{S} \approx \frac{\sum_{i=1}^{n} f(x_i, y_i)}{n}$$

$$\hat{\pi} \approx \frac{4}{n} \sum_{i=1}^{n} f(x_i, y_i)$$



Naive approach



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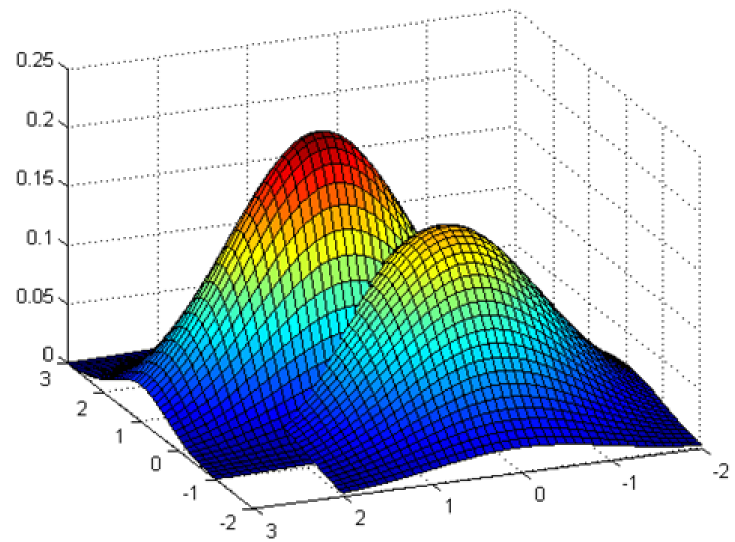
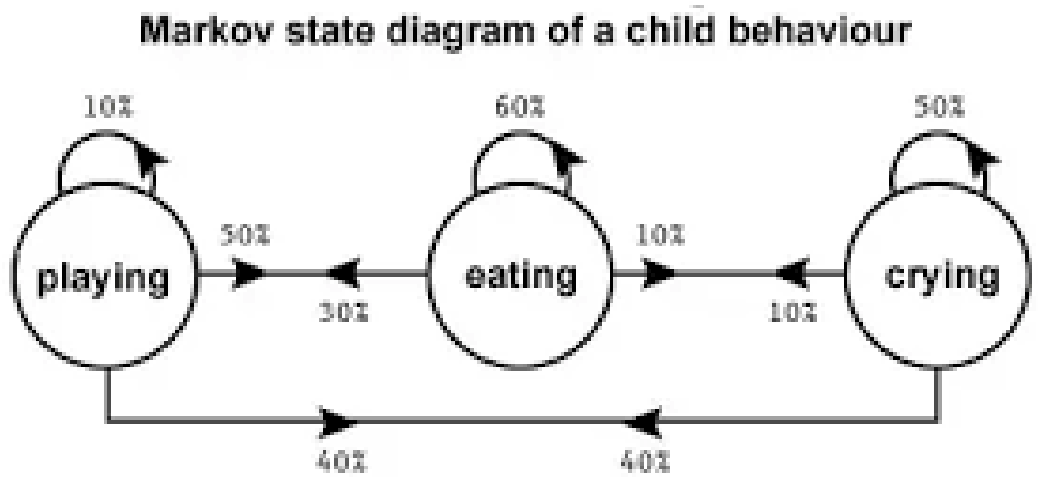
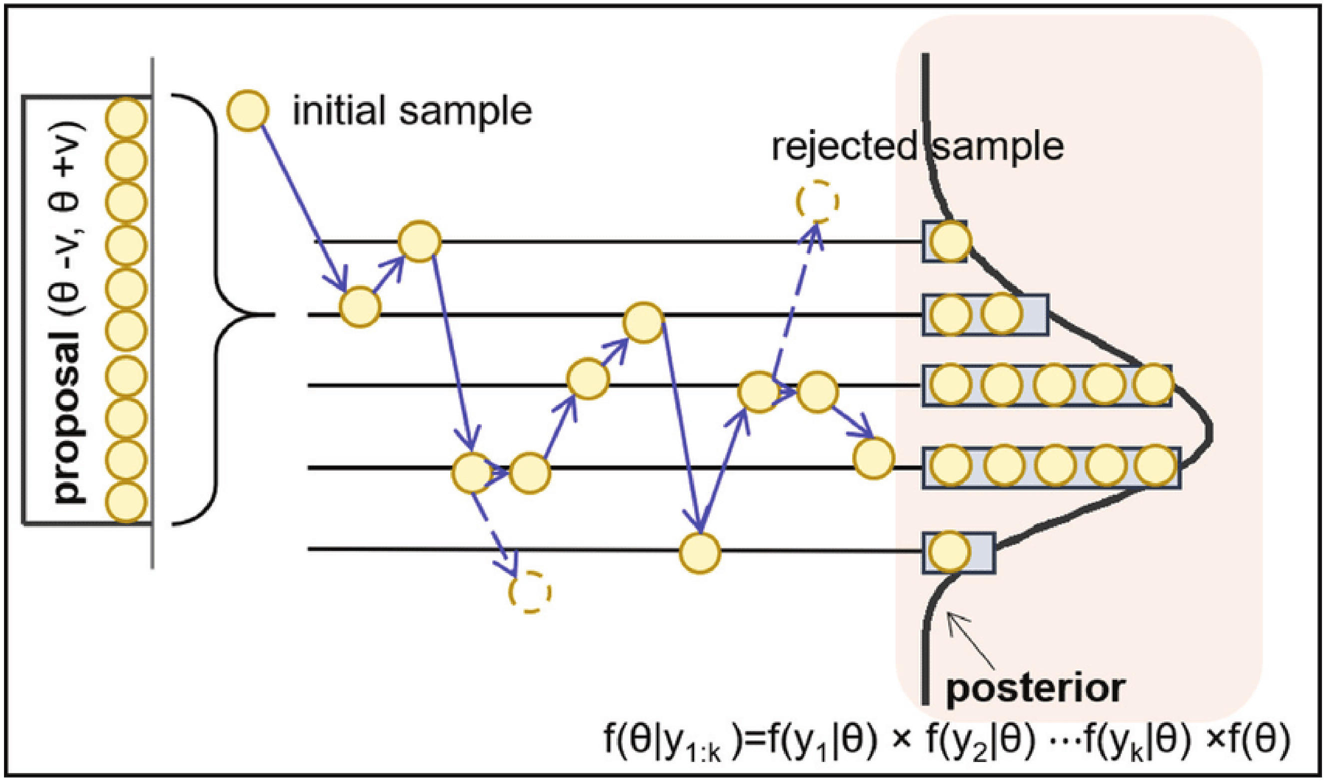


Markov chain

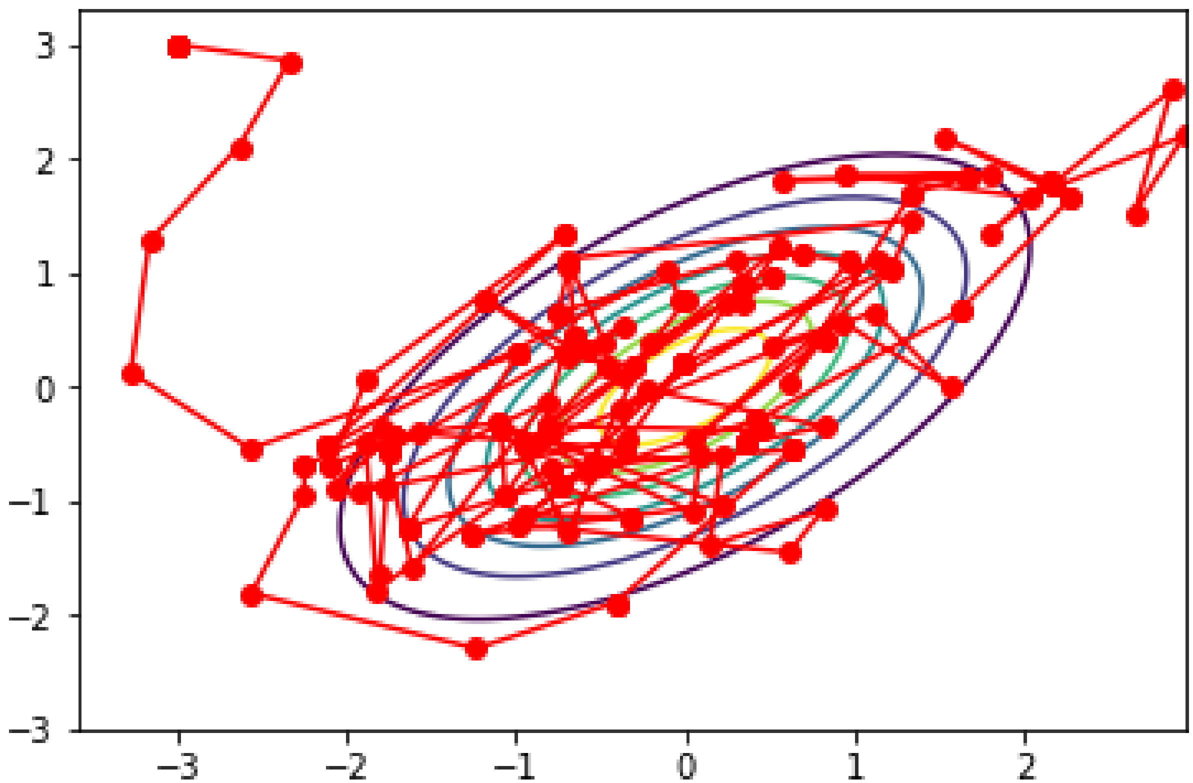


$$P(x_{n+1} \mid x_n, x_{n-1}, \ldots, x_1) = P(x_{n+1} \mid x_n)$$

Markov Chain Monte Carlo a.k.a. **MCMC**



https://www.researchgate.net/publication/331494053_Prognostics_102_Efficient_Bayesian-Based_Prognostics_Algorithm_in_MATLAB



https://relguzman.blogspot.com/2018/04/sampling-metropolis-hastings.html

Bibliography

Cameron Davidson-Pilon. *Bayesian Methods for Hackers: Probabilistic Programming and Bayesian Inference*. 2015. ISBN: 978-0133902839.

Avi Pfeffer. *Practical Probabilistic Programming*. Manning Publications Co. 456 pp. ISBN: 9781617292330.

And also *https://docs.pymc.io/*