Dowsing for Answers with Tangent-L

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Abstract. We present the math-aware search engine Tangent-L and report its results for the ARQMath Community Question Answering (CQA) task. Tangent-L, built on the text search platform Lucene, converts a math formula's Presentation MathML representation into a Symbol Layout Tree and then extracts tuples from the tree to serve as search terms. It applies BM25$^+$ ranking to all math tuples and natural language terms in a document during searching.

For the CQA task, we indexed all question-answer pairs in the Math Stack Exchange corpus. At query time, we first converted a query "topic" into a bag of formulas and keywords that serve as a formal query. We then executed the queries using Tangent-L to find the best matches. Finally, we re-ranked the matches by a regression model that trained on meta attributes from the corpus. Our primary run produces an nDCG$'$ value of 0.278 and MAP$'$ value of 0.063, where these are two common measures of quality for ranked retrieval. However, our best performance, an nDCG$'$ value of 0.345 and MAP$'$ value of 0.139, is achieved by an alternative run without re-ranking. Follow-up experiments help to explain which aspects of our approach led to our success.

Keywords: Community Question Answering (CQA) · Mathematical Information Retrieval (MathIR) · Symbol Layout Tree · Lucene · Mathematics Stack Exchange.

1 Research Objectives

Mathematical Information Retrieval (MathIR) focuses on using mathematical formulas and terminology to search and retrieve documents that include mathematical content [9, 18]. MathIR is important because content expressed in formal mathematics and formulas is often crucial and non-negligible in STEM papers [23]. In the recent decade, the MathIR research community has been growing and developing ever-improved math-aware search systems (e.g. [5, 8, 11, 12, 19–24, 26, 28]). Most of these efforts have been encouraged through a series of MathIR evaluation workshops at NTCIR-10 [1], NTCIR-11 [2], and NTCIR-12 [25]. The workshops have provided corpora from the arXiv and the Wikipedia for traditional ad-hoc retrieval tasks and formula search tasks, and the data and tasks have since served as benchmarks for the research community.

The ARQMath Lab (ARQMath) [14, 27] is the first answer retrieval task for questions involving math data, using collections from the Math Stack Exchange (MSE)[3], a mathematical question answering site. The training corpus contains approximately 1.1 million mathematical questions and 1.4 million answers, covering MSE threads from the year 2010 to 2018. Like the NTCIR workshops that precede it, it is centered around an evaluation exercise that aims to advance math-aware search and the semantic analysis of mathematical notation and texts. The main task of ARQMath considers the problem of Community Question Answering (CQA), in which participating systems need to find answers to a set of mathematical questions among previously posted answers on MSE. A secondary task considers matching relevant formulas drawn from the mathematical questions from the same collection. Participating teams were asked to submit up to five runs for either or both tasks, and selected results received relevance assessments from human evaluators.

Tasks related to the ARQMath Lab task have been held previously: a recent math question answering task was held as part of SemEval-2019 [10], following a series of CQA challenge series held at SemEval-2015 [16], SemEval-2016 [17], and SemEval-2017 [15]. The math question answering task at SemEval-2019 considered a question set that was derived from Math SAT practice exams. This task was different from the ARQMath CQA task, since it targeted identification of one uniquely correct answer, by multiple-choice selection or by numerical computation but not by identifying answers from threads in a forum. On the other hand, the earlier CQA challenge series at SemEval considered a collection of question-comment threads from the Qatar Living Forum, which is similar to the MSE site. The SemEval CQA tasks, however, differ from the ARQMath CQA task in that the questions are not necessarily mathematical, and the objective is to rank a given list of answer-candidates to the question without the need to retrieve the candidates from a corpus.

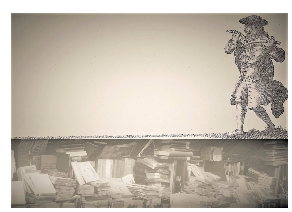

Fig. 1. Dowsing for math.

Our team of MathDowsers[4] participated in the ARQMath CQA task, with an approach based on the Tangent-L system, a MathIR system proposed by Fraser et al. [7]. Tangent-L was developed after NTCIR-12, using the data provided for all three NTCIR math search workshops, and appeared to be competitive with the systems participating in those workshops. We wished to determine whether it could, in fact, perform well against other traditional query systems and whether a traditional query system could compete with

[3] https://math.stackexchange.com

[4] *Water*loo researchers dowsing for math (Fig.1)

modern machine-learning approaches sure to be adopted by other workshop participants in a question-answering task.

Our experiments included five submitted runs that were designed to address the following research objectives:

RQ1 What is the best way to convert each topic (most of which is expressed in mathematical natural language) into a query composed of keywords and formulas?
RQ2 Should keywords or math formulas be assigned heavier weights in a query?
RQ3 What is the effect of a re-ranking algorithm that makes use of other meta attributes?

We present an overview of Tangent-L in Section 2. In Section 3, we describe our approach to CQA and provide details on how we retrieve and rank answer matches for a mathematical topic from the MSE corpus with the use of Tangent-L. The submitted runs and the results are discussed in Section 4. In Section 5, we present conclusions and propose future work.

2 Overview of Tangent-L

The Tangent-L system is the core component of our submission. It is built on the standard Lucene [3] text search platform, adding methods adapted from the Tangent-3 math search system [26]. Each math formula is converted into a "bag of terms" to be matched in the same way that natural language terms are handled by Lucene. More specifically, the presentation form of MathML[5] [4] is converted into a *symbol layout tree* [26] as depicted in Figure 2. Thereafter, the tree is traversed to extract a set of "features" of four types:

– symbol pairs: for each edge, the start and end symbols and the edge label
– terminal symbols: list of symbols with no outedges
– compound symbols: list of outedge labels for nodes with more than one outedge
– augmented locations: for each feature of the first three types, that feature together with the path to the feature's (first) symbol

For the formula and symbol layout tree in Figure 2, the extracted features, the "math tuples," are shown in Figure 3. In preparation for search, the math tuples replace the formula itself in the document and are then considered as if each were a term ("individual word") in the text to be matched. Therefore formulas and natural language text can be handled by a unified search engine.

The Tangent-L system applies BM25$^+$ ranking [13] to all the terms in a document. Specifically, given a collection of documents D containing $|D|$ documents and a query q consisting of a set of query terms, the score for a document $d \in D$ is given by

$$\text{BM25}^+(q,d) = \sum_{w \in q} \left(\frac{(k+1)\,tf_{w,d}}{k\left(1.0 - b + b\frac{|d|}{\bar{d}}\right) + tf_{w,d}} + \delta \right) \log \left(\frac{|D|+1}{|D_w|} \right) \qquad (1)$$

[5] For LaTeX input, LaTeXML (https://dlmf.nist.gov/LaTeXML/) produces MathML.

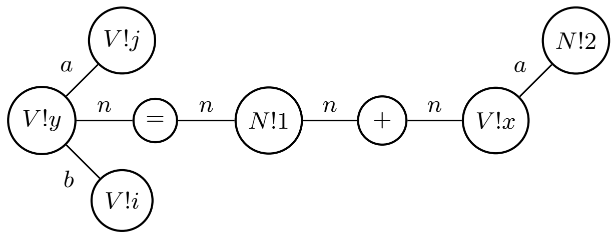

Fig. 2. Symbol Layout Tree for $y_i^j = 1 + x^2$, where a represents *above*, b *below*, and n *next* (to the right on the same line)

Feature Type	*Features*			
symbol pairs:	(V!y,V!j,a)	(V!y,=,n)	(V!y,V!i,b)	(=,N!1,n)
	(N!1,+,n)	(+,V!x,n)	(V!x,N!2,a)	
terminal symbols:	(V!j,0)	(V!i,0)	(N!2,0)	
compound symbols:	(V!y,abn)			
locations:	(V!y,V!j,a,−)	(V!y,=,n,−)	(V!y,V!i,b,−)	(=,N!1,n,n)
	(N!1,+,n,nn)	(+,V!x,n,nnn)	(V!x,N!2,a,$nnnn$)	(V!j,0,a)
	(V!i,0,b)	(N!2,0,$nnnna$)	(V!y,abn,−)	

Fig. 3. Extracted features ("math tuples") to represent the formula in Figure 2

where k, b, and δ are constants (following common practice, chosen to be 1.2, 0.75, and 1, respectively); $tf_{w,d}$ is the number of occurrences of term w in document d; $|d|$ is the total number of terms in document d; $\overline{d} = \sum_{d \in D} \frac{|d|}{|D|}$; and $|D_w|$ is the number of documents in D containing term w. This formula is easily applied to a bag of query terms: if a term is repeated in the query, the corresponding score for that term is naïvely accumulated multiple times. To allow for math tuples to be given a weight that differs from natural language terms, we assign weights to query terms as follows:

$$\mathrm{BM25}^{+}_{\mathrm{w}}(q_t \cup q_m, d) = \mathrm{BM25}^{+}(q_t, d) + \alpha \cdot BM25^{+}(q_m, d) \tag{2}$$

where q_t is the set of keywords in a query, q_m is the set of math feature tuples in that query, and α is a parameter to adjust the relative weight applied to math tuples.

For the NTCIR-12 arXiv Main task benchmark, the Tangent-L system gives a comparable performance to other MathIR systems [7]. We are interested to determine whether and how Tangent-L could be adapted to address the ARQLab CQA task.

3 Methodology

After indexing the corpus, we adopt a three-phase architecture to return answer matches for a mathematical topic:

1. *Conversion*: Transform the input topic (a question posed on MSE) into a well-formulated query consisting of a "bag" of formulas and keywords.
2. *Searching*: Use Tangent-L to execute the formal query to find the best matches against the indexed corpus (MSE question-answer pairs).
3. *Re-ranking*: Re-order the best matches by considering additional metadata (such as votes and tags) associated with question-answer pairs.

3.1 Conversion: Extracting Formulas and Keywords from Topics

For the CQA task, participants are given 98 real-world mathematical questions selected from MSE posts in 2019.[6] Each mathematical question is a *topic* that contains: (1) the *topic-ID*, (2) the *title* for this topic, (3) the *question* (body text) for this topic, and (4) the *tags* for this topic. Both the title and the question are free text, typically including formulas.

We adopt the following automated mechanism to extract a list of topic formulas and keywords:

Topic formulas. Formulas within the topic's title and the question are extracted. All formulas within the title are selected as topic formulas. Formulas within the question are selected only if they are not single variables (e.g., n or i) nor isolated numbers (e.g., 1 or 25).

Topic keywords. Keyword selection is summarized in Algorithm 1. Each of the topic's tags is selected as one of the topic keywords. For the free text in the title and the question, we first tokenize the text (Algorithm 2), followed by stemming to obtain a list of potential word tokens. An unstemmed word is then selected as a keyword if it contains a hyphen (such as "Euler-Totient" or "Cesáro-Stolz") (Algorithm 3) or it is not a stopword (Algorithm 4) and its stem appears on a pre-constructed list of *mathematical stems*.

The mathematical stem list is created in a pre-processing step by automatically extracting terms from two sources: (1) tags from the indexed MSE dataset and (2) titles from English Wikipedia articles comprising the corpus for the NTCIR-12 MathIR Wikipedia task [25]. For the former source, each tag present in the ARQMath MSE corpus is tokenized, stemmed, and added to the stem list. For example, two stems ("totient" and "function") are added to the list for the tag "totient-function." For the latter, we first collect the HTML filenames of all articles used in the NTCIR-12 MathIR Wikipedia corpus, where each filename reflects the corresponding Wikipedia article's title. The filenames are then transformed by removing file extensions and replacing underscores and parentheses with spaces. Each hyphenated term is expanded to include all components as well as the hyphenated term itself. The resulting cleaned text strings are tokenized, stemmed, and punctuation removed, and the resulting stems are added to the stem list. Thus, for example, four stems ("exponenti," "logarithm," "distribut," and "exponential-logarithm") are added for the filename Exponential-logarithmic_distribution.html. The resulting mathematical stem list comprises

[6] In addition, 3 extra questions are provided as samples with annotated answers.

about 1200 stems from MSE tags and about 21,000 stems from Wikipedia article titles.

When selecting keywords with this procedure, we use the Treebank tokenizer, the Porter stemmer, and a list of English stop-words provided by the Python NLTK library[7]. Using this approach, on average 8 topic formulas and 38 topic keywords were extracted for the CQA task (Table 1). The complete list of topic formulas and keywords can be found in the Appendix.

```
for tags in topic do
    Split input by ',' to obtain a list of tags;
    foreach tag do
        add tag to keyword list;
        if hyphen in tag then
            further split by '-' to obtain a list of parts;
            foreach part do
                add part to keyword list;
for question, title in topic do
    foreach token in Tokenize(input) do
        Stem(token) using PorterStemmer;
        if !IsStopword(token) && (IsPreserved(token) ||
          OnMathList(token)) then
            append to keyword list;
```

Algorithm 1: ExtractKeywords(topic)

```
Split the input by space into substrings;
foreach substring do
    if hyphen in substring then
        add substring to the token list;
Replace '-' with space throughout input;
foreach token in Treebank-Tokenization(input) do
    add token to the token list;
```

Algorithm 2: Tokenize(input)

```
return ('-' in token) || ('–' in token)
```

Algorithm 3: IsPreserved(token)

```
return (token in stopword set provided by the NLTK library) || (token
  contains only a single character) || (token is a numeric string)
```

Algorithm 4: IsStopword(token)

[7] https://www.nltk.org/

Table 1. Average, minimum and maximum number of topic formulas and keywords per topic. Example topics are included in the parentheses.

	Average	*Min*	*Max*
Formulas	8	1 (A.12†)	35 (A.81)
Keywords	38	5 (A.13)	155 (A.73)

(† among others)

3.2 Searching: Answer Retrieval from the Indexed Corpus

We use the Tangent-L system to retrieve answers that match the extracted keywords and formulas.

Indexing. We build the indexed corpus with question-answer pairs. Each indexed unit includes an MSE answer along with the content of its associated question[8]. For each question-answer pair, we extract the following content from the corpus XML files:

From the answer: the body text and the number of votes (from *Posts.xml*);
From the associated question: the title, body text, and tags of the question (from *Posts.xml*) plus comments associated with each question (from *Comments.xml*). Additionally, we also include the *titles* of all related and duplicated posts for this question (from *PostLinks.xml*), having first converted all one-way links between posts into two-way links.

All formulas within the text are replaced by their Presentation MathML form using the formula mapping in the TSV files provided with the corpus data. An HTML file containing the final extracted content is then assembled as an indexing unit (Figure 4.). The indexable version of the MSE corpus includes a total of 1,445,488 documents, indexed by Tangent-L in preparation for search.

Searching. Searching the corpus is a straightforward application of Tangent-L for the converted topics. The list of keywords and formulas (in MathML) are passed to the search engine. Tangent-L then converts the formulas to math tuples and uses corresponding postings lists in the index to compute BM25^+ scores (Equation 2), weighted by a value for α that depends on the experimental setup for the run.

3.3 Using Answers' Metadata for Re-ranking

The CQA task is made more challenging by the potential to use semantic information relating candidate answers to topic queries. For instance, an answer

[8] As many of us were admonished in school, "you should always include the question as part of your answer."

```
<body>
  <div class=”row” id=”question−header”>
    <h1> The cow in the field problem (intersecting circular areas) </h1>
  <div class=”question”>
    <div class=”question” data−questionid=”#QID#” id=”question”>
      <div class=”post−text”>
        <p>What length of rope should be used to tie a cow to an <strong>exterior fence
        post</strong> of a <em>circular</em> field so that the cow ....
      </div>
    </div>
    <div class=”post−taglist”>
        <span class=”post−tag”> geometry </span> <hr>
    </div>
  </div>
  <div class=”row” id=”answers”>
    <div class=”answer” data−answerid=”62”> <div class=”post−text”> <p>So, the area
    of the field is <span class=”math−container” id=795 fid=795><?xml version=”1.0”
    encoding=”UTF−8”?><math xmlns=”http://www.w3.org/1998/Math/MathML” alttext=
    ”\pi r^{2}” display=”block”> <mrow> <mi>π</mi> <mo>˜</mo> <msup> <mi>r</mi>
    <mn>2</mn> </msup> </mrow></math></span> and you want the cow to be able to
    graze an area equal to half of that.</p> <p>All you need to do is set up the equation ....
    </div>
  </div>
  <div class=”row” id=”question−comments”>
    <table>
        <tr><td> I’m guessing that most fence posts are at the edge of a field, which
        makes this a far more interesting problem. </td></tr><tr><td> ....
        </tr>
    </table>
  </div>
  <div class=”row” id=”duplicated”>
    <table>
        <tr><td> A confusing word problem related to geometry (circles) </td></tr><tr>
        <td> Is it possible to express the area of the intersection of 2 circles as a
        closed−form expression? </td></tr>
    </table>
  </div>
  <div class=”row” id=”related”>
    <table>
        <tr><td> Find the area where dog can roam </td></tr><tr><td> A goat tied to a
        corner of a rectangle </td></tr>
      </tbody>
  </div>
</body>
```

Fig. 4. A sample indexed unit.

should be ranked higher in position if it answers the topic question correctly. However, a correct answer does not necessarily have the highest score based merely on matching keywords and formulas by similarity.

As such, we are motivated to explore a re-ranking algorithm applied to the results returned from the search engine. To this end, we wish to use information from the complete MSE corpus to build a model that reflects how valuable a potential answer might be. We hypothesize, with the law of simplicity in mind, that a linear function of the following four variables might serve well:

1. **similarity:** The similarity of keywords and formulas between the target answer (including its associated question) and the topic query is clearly an important component. This is captured by the search score returned from the Tangent-L system.
2. **tags:** The number of overlapping tags between the question associated with the answer and the topic query reflects how well the answer matches the query's academic area(s).
3. **votes:** The fraction of votes received by the answer when posted with its associated question reflects the community's belief in the answer's value.
4. **reputation** The reputation of the author who wrote an answer implies the trustworthiness of that answer. Intuitively, a good answer comes from an author with a good reputation. This can be computed from the user reputation score, the number of user up-votes, and the number of user down-votes.

The remaining problem is to determine what coefficient values to use when linearly combining these inputs. For this we need a training set that includes relevance assessments, which are not available as part of the corpus.

Mock Relevance Assessments. As a substitute for assessed relevance, we realize that we can build a training set of queries from questions already in the corpus: we hypothesize that relevant answers include those that were actually provided for those questions as well as those provided for *duplicate* and *related* questions.

We use tags, related posts, and duplicate posts of a question, and the number of votes for each answer to calculate mock relevance assessments for answers to the training topics, based on the following two observations:

1. Considering the question associated with a target answer, the more "on-topic" that question is to the query, the more relevant the target answer is likely to be for that query. A corpus question and a query are related if they have overlapping tags, but are more related if one is marked as a related post of the other, and still more related if it is marked as a duplicate post of the other (or is, in fact, the same question).
2. If two potential answers are associated with the same question, the one with more votes should be preferred.

With these assumptions, an assessment value can be computed as follows:

- **Integral assessment value.**
  1. An answer gets an assessment value of **2**, if the question associated with that answer is a duplicate post to the query question, or if the answer comes from the same question.
  2. An answer gets an assessment value of **1**, if the question associated with that answer is a related post to the query question;
  3. Otherwise, the answer gets an integral assessment value of **0**
- **Fractional assessment value.**
  1. Assuming that all votes for answers are non-negative, an answer gets a normalized fractional assessment value **between 0 and 1**, calculated as the fraction of its votes to the sum of votes of all answers for the associated question, if the associated question is the same or a duplicate or related post to the query question or if the associated question contains any overlapping tags with the query question.
  2. Otherwise, the answer gets a fractional assessment value of **0**.

The final mock relevance assessment is the sum of the integral and fractional assessment values and, when all answers have non-negative votes, ranges from 0 to 3 (matching the assessment range expected in the CQA task):

Score	*Meaning*
0	irrelevant
0 .. 1	tags overlap
1 .. 2	posts are related
2 .. 3	posts are identical or duplicates

In fact, some answers in the corpus have negative votes (down-votes). In this case, the fractional assessment value is adjusted as follows: for a thread containing any answers with negative votes, the fraction's denominator is the absolute value of the total negative votes plus the sum of votes within a thread. If an answer has positive votes, the fraction's numerator is the absolute value of the total negative votes plus the number of votes for the answer, otherwise, the fraction's numerator is the (negative) value of the votes. For example, suppose a thread contains three answers, and their votes are -2, 2, 6, respectively. The fractional assessment values for these answers are -0.2, 0.4, 0.8, respectively. Thus the mock relevance assessment penalizes answers with negative votes.

Linear Regression Formula. To determine the coefficients for the linear regression model, we first generate 1300 training topics from the MSE collection. We use the Tangent-L system with $\alpha = 1$ (in Equation 2) to retrieve the top 10,000 answers for each valid topic, resulting in a total of 12,970,000 answers. Next, we generate the mock relevance assessments for these answers and associate these assessments with the values for similarity, tags, votes, and reputation for those answers, as discussed above. These 12,970,000 tuples then serve as training data for the linear regression model. Example items of the training data are shown in Table 2, and the trained coefficients are shown in Table 3.

Table 2. Example items of generated training data for the linear regression model, in decreasing order of the search score returned from the Tangent-L system.

Regressand	*Regressors*						
mock relevance	similarity	tags	vote*	votes-d†	u-reputation	u-upvotes	u-downvotes
3	2617.5	3	2	2	45496	26331	66
3	1226.3	1	5	5	305891	20695	647
1	1197.6	2	3	3	6227	130	2
1	1012.6	1	0	0	884	26	9
1	932.8	2	5	5	4268	244	27
1	930.0	1	2	2	1743	128	2
0.5	911.4	2	1	2	77437	2507	17
1	895.3	2	1	1	52161	1796	252
1	864.5	1	1	1	53311	3710	504
1	852.5	2	4	4	48172	9298	1270
0	811.7	0	1	1	294098	5057	218

* the original answer vote
† the denominator of the fractional assessment value

Table 3. Parameters of the trained linear regression model, trained on un-normalized data.

Parameter	*Coefficient Value*
(intercept)	0.021
similarity	1.68×1e-6
tags	0.398
vote	1.59×1e-4
vote-d (denominator)	-6.17×1e-5
user reputation	9×1e-8
user upvotes	1.5×1e-7
user downvotes	1.20×1e-6

We validate the trained linear regression model by first applying the model to predict a relevance score for the top 1000 retrieved answers of a separate set of another 100 topics, and then re-ranking the answers according to the predicted mock relevance score. Finally, we adopt the normalized Discounted Cumulative Gain (nDCG), which measures the gain of a document based on its position in the result list and its graded relevance. We compare the nDCG value of the results for those 100 topics according to the mock relevance assessment before and after re-ranking. This simple model gives a slight improvement in nDCG after the re-ranking (from 0.3192 to 0.3318).

Table 4. A summary of the setup for MathDowser submissions. **P** indicates a primary run, **A** indicates an alternative run, and **M** indicates a manual run.

	Formula Weight	*Topic Conversion*	*Re-ranking*
alpha05 (P)	$\alpha = 0.5$	auto-extracted	yes
alpha02 (A)	$\alpha = 0.2$	auto-extracted	yes
alpha10 (A)	$\alpha = 1.0$	auto-extracted	yes
alpha05-translated (A, M)	$\alpha = 0.5$	manual	yes
alpha05-noReRank (A)	$\alpha = 0.5$	auto-extracted	no

4 Experiments

4.1 Description of Runs

The MathDowsers submitted a primary run and four alternative runs, each run returning at most 1000 answer matches for all 98 questions. The primary run is designed as a combination of our hypotheses for the best configuration for all research objectives. The alternative runs are designed to test these hypotheses: the setup for each of them is the same as the primary run, except for a single aspect that is associated with a testing hypothesis, as described here and summarized in Table 4.

RQ1 *Topic Conversion*
In the primary run, we use the auto-extracted topic keywords and formulas described in Section 3.1 and listed in the Appendix as the query input to the search engine. To compare the effectiveness of our extraction algorithm with human understanding of the questions, an alternative run takes as input a list of topic keywords and formulas that had been made available by the lab organizers. This list is a set of keywords (maximum of 5 terms) and formulas (maximum of two) that were chosen ("*translated*") manually after reading each topic. This alternative run is a manual run, in contrast to all other runs that are "automatic."

RQ2 *Formula Weight* We test the weights of math formulas in a query by tuning Tangent-L's α parameter (Equation 2). In the primary run, $\alpha = 0.5$, which means formula terms are given half the weight of keywords in a query. The reasoning behind this choice is that each formula generates many terms, one for each feature extracted, and previous experiments showed some benefit in reducing the weight accordingly [7]. Two alternative runs operate with $\alpha = 0.2$ (even less weight on formulas) and $\alpha = 1.0$ (Tangent-L's default setting), respectively.

RQ3 *Re-ranking* In the primary run, we re-ranked the results from the search engine, using the model described in Section 3.3. To evaluate the effectiveness of the model, an alternative run has with no re-ranking.

4.2 Machine configuration

Indexing. Indexing is done on a Ubuntu 16.04.6 LTS server, with two Intel® Xeon E5-2699 V4 Processors (22 cores 44 threads, 2.20 GHz for each), 1024GB RAM and 8TB disk space (on an USB3 external hard disk). The size of the document corpus is 24.3GB.

Tangent-L requires 5.0GB of storage on the hard drive and approximately 6 hours to index all documents with parallel processing.

Searching and Re-ranking. Training and testing the model for re-ranking is done on a Linux Mint 19.1 machine, with an Intel® Core™ i5-8250U Processor (4 cores 8 threads, up to 3.40 GHz), 24GB RAM and 512GB disk space[9].

It takes less than 30 seconds to complete the model training using the Python scikit-learn library[10], and around 3 seconds to apply re-ranking for all 98 topics in a run.

Searching is executed on this same Mint machine, and retrieval time statistics for Tangent-L are reported in Table 5.

4.3 Results and Observations

The retrieval performance for the MathDowsers' submissions is summarized in Table 6. We also include the performance of *Linked MSE posts*, a baseline system "built from duplicate post links from 2019 in the MSE collection (which were not available to participants). This baseline returns all answer posts from 2018 or earlier that were in threads from 2019 or earlier that MSE moderators had marked as duplicating the question post in a topic. The posts are sorted in descending order by their vote scores."[27] Since six test topics (namely, A.9, A.15, A.24, A.26, A.51, and A.58) do not have duplicate posts, the result of this baseline system is reported over 71 topics.

[9] A NVIDIA GeForce MX150 graphics card with 2GB on-card RAM is available on the machine, but it was not used for the experiments.

[10] https://scikit-learn.org

Table 5. Retrieval times per topic, in seconds, using single-threaded processing.

	Avg. (sec)	*Top-2 Min (sec)*	*Top-2 Max (sec)*
alpha05 †	13.3	0.669 (A.67) / 0.775 (A.94)	63.4 (A.76) / 48.7 (A.28)
alpha02	13.3	0.661 (A.67) / 0.850 (A.94)	59.1 (A.76) / 49.5 (A.28)
alpha10	13.1	0.616 (A.67) / 0.784 (A.83)	54.0 (A.76) / 48.7 (A.28)
alpha05-translated	5.3	0.247 (A.99) / 0.291 (A.94)	32.8 (A.11) / 25.0 (A.67)
alpha05-noReRank	13.3	0.669 (A.67) / 0.775 (A.94)	63.4 (A.76) / 48.7 (A.28)

† The run *alpha05* does not, in fact, take any additional retrieval time, since it merely re-ranks the retrieved items from the *alpha05-noReRank* run.

The primary measure of the task is the Normalized Discounted Cumulative Gain (nDCG) with unjudged documents removed (thus nDCG′), but we also compute Mean Average Precision (MAP) with unjudged documents removed (MAP′), Precision at top-10 matches (P@10), and the bpref measure. Additionally, we include the count for *Unjudged Answers* within the top-k retrievals of each topic.

Table 6. CQA (main task) results, averaged over 77 topics (71 for the baseline system). For post-experiment runs, top-2 values of each column are highlighted.

	Evaluation Measures				*Unjudged Answers*		
	nDCG′	*MAP′*†	*P@10*†	*bpref*†	*top-10*	*top-20*	*top-50*
Baseline							
Linked MSE posts	**(0.303)**	**(0.210)**	**(0.417)**	**(0.233)**	15	33	45
Submissions							
alpha05 (P)	0.278	0.063	0.073	0.041	0	1	3
alpha02 (A)	0.301	0.069	0.075	0.044	0	1	530
alpha10 (A)	0.267	0.063	0.079	0.042	0	2	236
alpha05-translated (A)	0.298	0.074	0.079	0.050	0	1	3
alpha05-noReRank (A)	**0.345**	**0.139**	**0.161**	**0.126**	2	3	1796
Post-experiment							
alpha10-noReRank	(0.327)	(0.134)	(0.158)	(0.123)	4	79	1865
alpha10-trans-noR	(0.357)	(0.149)	(0.164)	(0.137)	211	557	2253
alpha05-trans-noR	(0.365)	(0.152)	(0.162)	(0.140)	207	545	2227
alpha02-noReRank	(0.368)	(0.146)	**(0.178)**	(0.134)	35	157	1784
alpha02-trans-noR	(0.372)	(0.153)	(0.168)	(0.138)	193	535	2205
alpha01-noReRank	**(0.388)**	(0.153)	**(0.182)**	**(0.142)**	105	362	1930
alpha01-trans-noR	**(0.387)**	**(0.158)**	(0.169)	**(0.142)**	198	560	2203
alpha001-noReRank	(0.247)	(0.091)	(0.048)	(0.100)	616	1271	3296
alpha001-trans-noR	(0.348)	**(0.157)**	(0.155)	**(0.155)**	371	884	2734

(† using H+M binarization)

From the top half of Table 6, the first observation is that the primary run, with our presumed best configuration, does not outperform the other runs. Instead, the alternative run without re-ranking achieves the best performance in all evaluation measures. It also outperforms the baseline system in the official measure nDCG′, although not for other measures. The second observation is that lowering the weight placed on math terms ($\alpha = 0.2$) improves the performance, and using the default weight ($\alpha = 1.0$) hurts the performance. Thirdly, the alternative run using manually extracted formulas and keywords outperforms the primary run. In retrospect, it appears as if all aspects of our primary run leave room for improvement.

In order to explore these observations more closely, we execute several additional runs designed after the conclusion of the formal experiment (*post-experiment*).

As detailed in Table 7, these runs examine the performance of our system without re-ranking for additional values of α and with automatic or manual choice of keywords and formulas. We can now summarize our insights from all of Table 6 with respect to our research objectives.

The effect of re-ranking. Comparing the result from the submissions and the post-experiment runs, we see that our re-ranking design was detrimental to the performance (**RQ3**). A consistent drop of at least 0.06 in nDCG′ can be observed for runs after re-ranking (e.g *alpha02* vs. *alpha02-noReRank*). A similar deterioration can be observed in other evaluation measures as well.

The effect of α. Considering runs without re-ranking, we confirm that the performance gradually improves with the decrease of weight being set for math formulas in a query. When we decrease α from 1.0 to 0.1, a gain of 0.06 in nDCG′ is achieved for runs with auto-extracted topic conversion (*alpha10-noReRank* vs. *alpha01-noReRank*). Similarly, a gain of 0.03 in nDCG′ is achieved for runs with manual topic conversion (*alpha10-trans-noR* vs. *alpha01-trans-noR*). Gains are also observed using other evaluation measures.

Unfortunately, when α is set to a much smaller value, namely 0.01, the overall performance is questionable since the result of different evaluation measures is contradictory. We observe that for the run with auto-extracted topic conversion (*alpha001-noReRank*), all measures are at their lowest compared to runs with other choices of α. However, for the run with manual topic conversion (*alpha001-trans-noR*), nDCG′ and P@10 are still at their lowest, but MAP′ is the second-best, and bpref is at its highest among all other choices of α.

Table 8 shows that with $\alpha = 0.01$ unjudged answer coverage at top-k for *alpha001-noReRank* is extraordinary high (at least 80%). Similarly, among the manual runs, *alpha001-trans-noR* also has by far the highest unjudged answer coverage for various values of k across all choices of α. The large percentage of unjudged answers implies that the evaluation of these two runs might not actually be informative.

Table 7. A summary of the setup of post-experimental runs.

	Formula Weights	*Topic Conversion*	*Re-ranking*
alpha10-noReRank	$\alpha = 1.0$	auto-extracted	no
alpha10-trans-noR	$\alpha = 1.0$	manual	no
alpha05-trans-noR	$\alpha = 0.5$	manual	no
alpha02-noReRank	$\alpha = 0.2$	auto-extracted	no
alpha02-trans-noR	$\alpha = 0.2$	manual	no
alpha01-noReRank	$\alpha = 0.1$	auto-extracted	no
alpha01-trans-noR	$\alpha = 0.1$	manual	no
alpha001-noReRank	$\alpha = 0.01$	auto-extracted	no
alpha001-trans-noR	$\alpha = 0.01$	manual	no

Table 8. *Unjudged answers* coverage for runs without re-ranking for 77 topics.

Topic Conversion	*Top-10* (%)	*Top-20* (%)	*Top-50* (%)
Auto-extracted			
alpha10-noReRank	0.5	5.1	48.4
alpha05-noReRank	0.3	0.2	46.6
alpha02-noReRank	4.5	10.2	46.3
alpha01-noReRank	13.6	23.5	50.1
alpha001-noReRank	**80.0**	**82.5**	**85.6**
Manual			
alpha10-trans-noR	27.4	36.2	58.5
alpha05-trans-noR	26.9	35.4	57.8
alpha02-trans-noR	25.1	34.7	57.3
alpha01-trans-noR	25.7	36.4	57.2
alpha001-trans-noR	**48.3**	**57.4**	**71.0**

Table 9. Topic counts by category.

Dependency	*Count*	*Topic Type*	*Count*	*Difficulty*	*Count*
Text (T)	13	Computation (CP)	26	Easy (E)	32
Formula (F)	32	Concept (CC)	10	Medium (M)	21
Both (B)	32	Proof (P)	41	Hard (H)	24

We conclude that keywords should be assigned heavier weights in a query (**RQ2**). Good performance is achieved when math terms are given one-tenth of the weight of keywords in a query ($\alpha = 0.1$). We are unable to determine the effect when the weight of math formulas is set even smaller.

The effect of topic conversion. Table 6 shows that with high α values (0.5 and 1.0), runs with manual topic conversion consistently outperform runs with auto-extracted topic conversion in all evaluation measures. However, with lower α values (0.1 and 0.2), runs with auto-extracted topic conversion performs better for P@10. When $\alpha = 0.1$, which we have just concluded to be the best setting for α, the performance of the two runs with respect to nDCG$'$ and bpref measures becomes essentially indistinguishable. We conclude that the proposed way to convert a topic into a query composed of keywords and formulas is competitive with human ability to select search terms (**RQ1**). We discuss this observation further in Section 5.

4.4 Strength and Weakness

To gain a deeper insight into the behaviour of our best-performing submission run (*alpha05-noReRank*), we examine its performance within each topic category, as determined by the organizers over three aspects. *Dependency* shows to what extent a topic depends on the text, formula, or both. *Topic Type* gives a broad categorization of whether a topic asks about a computation, a concept, or a proof. *Difficulty* approximates how hard a topic is to answer, using three levels of difficulty: easy, medium, and hard. The breakdown of the 77 judged topics by category is shown in Table 9.

Table 10. Comparison of *alpha05-noReRank* to *Linked MSE posts*, with the better performance measure for each category highlighted in bold.

	alpha05-noReRank				*Linked MSE posts*			
	nDCG′	*MAP′*	*emphP@10*	*bpref*	*nDCG′*	*MAP′*	*P@10*	*bpref*
Overall	**0.345**	0.139	0.161	0.126	(0.303)	**(0.210)**	**(0.417)**	**(0.233)**
Dependency								
Text	0.273	0.084	0.115	0.085	**(0.286)**	**(0.157)**	**(0.454)**	**(0.180)**
Formula	**0.446**	0.192	0.203	0.164	(0.309)	**(0.222)**	**(0.415)**	**(0.252)**
Both	0.272	0.109	0.138	0.105	**(0.305)**	**(0.222)**	**(0.403)**	**(0.237)**
Topic Type								
Computation	**0.325**	0.116	0.146	0.106	(0.249)	**(0.164)**	**(0.350)**	**(0.187)**
Concept	0.189	0.025	0.040	0.023	**(0.277)**	**(0.198)**	**(0.390)**	**(0.206)**
Proof	**0.395**	0.182	0.200	0.164	(0.339)	**(0.239)**	**(0.462)**	**(0.265)**
Difficulty								
Easy	**0.361**	0.147	0.178	0.132	(0.313)	**(0.232)**	**(0.453)**	**(0.257)**
Medium	0.306	0.114	0.129	0.098	**(0.323)**	**(0.209)**	**(0.411)**	**(0.224)**
Hard	**0.357**	0.151	0.167	0.143	(0.271)	**(0.180)**	**(0.373)**	**(0.206)**

Table 10 shows that our system has weaker performance than the baseline with respect to MAP′, P@10, and bpref measures in all categories. However, with respect to the nDCG′ measure, it outperforms the baseline in several categories. In terms of dependency, our system has strong performance for topics that depend on formulas. Table 11 shows that 7 out of top-10 topics with the highest nDCG′ in our system are categorized as *Formula*(F)-dependent, and Table 12 shows that none of the bottom-10 topics with respect to nDCG′ are similarly categorized. We attribute this strong performance to Tangent-L, the underlying MathIR system that is competitive with other state-of-the-art systems for formula retrieval [7]. We further conclude that setting $\alpha = 0$ would be a poor design decision.

In terms of topic type, we observe that our system is particularly weak at *Concept*-type (CC) topics when compared with the other two types of topics. Tables 11 and 12 show that none of these are among the top-10 topics with respect to nDCG′, but three of them are listed among the bottom-10 topics with the lowest nDCG′ in our system. With further inspection, we see that none of the *Concept*-type topics have a *Formula*-dependency, which might be the reason why our system does not perform as well.

In terms of difficulty, our system excels at topics categorized as *Easy*(E) and as *Hard*(H). Apparently, whereas it may be hard for a human to answer a query, it may be easier for our system to recognize a good answer when it sees it!

Finally, we observe that our system can be tuned to boost performance to accommodate its weaknesses if the type of category were known in advance. For instance, by comparing the best-performing submission run with the primary run, we observe that re-ranking can boost nDCG$'$ for *Concept*-type topics (see Table 13). Similarly, decreasing the weight for math formulas in a query helps boost nDCG$'$ for *Text*-dependent topics and thus *Concept*-type topics, as can be seen from Table 14. This leads us to speculate that a system that adapts to the type of query being posed might well out-perform a one-size-fits-all solution.

5 Conclusions and Future Work

We conclude that a traditional approach to MathIR remains a viable option for addressing the CQA task. In particular, Tangent-L is well-suited to retrieve answers to many computational and proof-like questions in the presence of mathematical formulas. We hypothesize that part of the success for all five MathDowser runs results from having indexed question-answer pairs, thereby providing a suitable context for evaluating the suitability of each answer in serving as an answer to newly posed topic questions.

Nevertheless, several experimental design decisions turn out to be somewhat disappointing. In the remainder of this section, we share our thoughts regarding room for improvement.

Improvements for re-ranking. The first improvement is with respect to the design and training of the re-ranking model described in Section 3.3. Several avenues are open:

1. We realize in retrospect that we should normalize the scores returned by Tangent-L within each topic: as for many search engines, the scores for one query are not comparable to the scores for another query. However, even with proper normalization, the linear regression model does not produce valuable re-ranking in post-experiment. Perhaps other normalization is also appropriate.
2. The mock relevance scores used for training the model might not be indicative of assessed relevance, therefore giving a poor model for re-ranking. For future work, now that there are actual assessments from the ARQMath Lab, we could use those for training and avoid the need for mock scores. This approach could be tested by conducting some cross-validation studies.
3. Perhaps using linear regression is not appropriate for this application. An alternative approach is to first transform (mock or actual) assessments into

Table 11. Top-10 topics with the highest nDCG′ per topic, from the best-performing submission run (*alpha05-noReRank*)

Topic	*nDCG′*	*Topic Title* (Tags)	*Dependency*	*Topic Type*	*Difficulty*
A.60	0.861	Limiting value of a sequence when n tends to infinity (calculus, sequences-and-series, limits, products)	B	P	H
A.50	0.822	Divergent series $\sum \frac{1}{n^{2+\cos n}}$ (real-analysis, integration, sequences-and-series, analysis	F	P	E
A.45	0.764	How to prove that $\sin(x), \sin(2x), \sin(3x), ..., \sin(nx)$ is independent in $\mathbb{R}$? (ordinary-differential-equations)	B	P	H
A.55	0.764	$\frac{1}{\sqrt{-1}} = \sqrt{-1}$? (complex-numbers, definition)	F	P	E
A.13	0.754	How to simplify expression $\int_a^b f(x)dx + \int_{f(a)}^{f(b)} f^{-1}(x)dx$? (calculus)	F	CP	E
A.69	0.702	Induction with two variable parameters (combinatorics)	B	P	E
A.79	0.696	Inequality with complex exponential (inequality, exponential-function, fourier-transform)	F	P	E
A.43	0.695	Prove $\sum_{n\geq 1} \frac{1}{n^2+1} = \frac{\pi \coth \pi - 1}{2}$ (real-analysis, sequences-and-series)	F	P	H
A.4	0.666	How to compute this combinatoric sum? (combinatorics, number-theory, summation, proof-explanation)	F	CP	E
A.65	0.653	How can we show that $e^{-2\lambda t}\lambda^2 \leq \frac{1}{e^2 t^2}$ for all $\lambda, t \geq 0$? (calculus, inequality, exponential-function)	F	P	M

Table 12. Bottom-10 topics with the lowest nDCG′ per topic, from the best-performing submission run (*alpha05-noReRank*)

Topic	*nDCG′*	*Topic Title*	*Dependency*	*Topic Type*	*Difficulty*
A.33	0.070	Physical meaning and significance of third derivative of a function (PHYSICS)	B	CC	E
A.29	0.068	Dividing Complex Numbers by Infinity (ALGEBRA-PRECALCULUS, LIMITS, COMPLEX-NUMBERS, INFINITY)	T	CP	H
A.83	0.067	Is the sequence of sums of inverse of natural numbers bounded? (CALCULUS, SEQUENCES-AND-SERIES, HARMONIC-NUMBERS)	T	P	E
A.36	0.036	Proof by contradiction, status of initial assumption after the proof is complete. (LOGIC, PROOF-WRITING)	B	CC	H
A.37	0.025	Non trivial examples of $f \circ g = g \circ f$ but $f^{-1} \neq g$ and $f \neq \text{id} \neq g$.((REAL-ANALYSIS, FUNCTIONAL-ANALYSIS, FUNCTIONS)	B	CC	M
A.68	0.020	Prove $a^n + 1$ is divisible by $a + 1$ if n is odd (POLYNOMIALS, INDUCTION, DIVISIBILITY)	B	P	M
A.11	0.000	What's the cross product in 2 dimensions? (MULTIVARIABLE-CALCULUS, VECTORS)	B	CP	H
A.44	0.000	For $A, B \in \mathscr{M}_{2\times 2}(\mathbb{Q})$ of finite order, show that AB has infinite order (MATRICES, GROUP-THEORY, CYCLIC-GROUPS)	B	P	M
A.54	0.000	By using a diagonal argument, show that the powerset $P(N) = (S \vert S \subseteq N)$ is uncountable. (DISCRETE-MATHEMATICS, ELEMENTARY-SET-THEORY)	B	P	E
A.74	0.000	Show that the image of the function $f : (0, \infty) \to \mathbb{R} f(x) = x + \frac{1}{x}$ is the interval $[2, \infty)$. (FUNCTIONS, ELEMENTARY-SET-THEORY)	B	P	E

Table 13. All *Concept*-type topics, with nDCG′ from the best-performing submission run (*alpha05-noReRank*), in decreasing order, and the primary run (*alpha05*, indicated by nDCG′-P.

Topic	nDCG′	nDCG′-P	*Topic Title*	*Dependency*	*Difficulty*
A.42	**0.378**	0.306	What is a simple, physical situation where complex numbers emerge naturally? (COMPLEX-NUMBERS, PHYSICS, APPLICATIONS)	T	H
A.35	**0.360**	0.240	When does a function NOT have an antiderivative? (INTEGRATION)	T	M
A.32	0.313	**0.509**	Are definitions axioms? (TERMINOLOGY, DEFINITION, FIRST-ORDER-LOGIC, AXIOMS)	B	M
A.41	0.263	**0.340**	Confusion in how to find number of onto functions if two sets are given (COMBINATORICS, FUNCTIONS, COMBINATIONS)	T	E
A.39	0.206	**0.208**	How to know which value is bigger? (ALGEBRA-PRECALCULUS, LOGARITHMS)	B	E
A.38	0.126	**0.148**	Uses of Axiom of Choice (SET-THEORY)	T	H
A.40	**0.115**	0.101	What is the meaning of the term "linear" (LINEAR-ALGEBRA)	B	H
A.33	0.070	**0.114**	Physical meaning and significance of third derivative of a function (PHYSICS)	B	E
A.36	**0.036**	0.034	Proof by contradiction, status of initial assumption after the proof is complete. (LOGIC, PROOF-WRITING)	B	H
A.37	0.025	**0.026**	Non trivial examples of $f \circ g = g \circ f$ but $f^{-1} \neq g$ and $f \neq \mathrm{id} \neq g$. (REAL-ANALYSIS, FUNCTIONAL-ANALYSIS, FUNCTIONS)	B	M
(*Average*)	0.189	**0.203**			

Table 14. A topic-dependency breakdown of the post-experiment run *alpha05-trans-noR* and the corresponding runs with a decreasing formula weight.

Dependency	*nDCG′*	*Topic Type*	*nDCG′*
alpha05-trans-noReRank			
Text	(0.279)	Computation	(0.378)
Formula	(0.431)	Concept	(0.172)
Both	(0.333)	Proof	(0.403)
alpha02-trans-noReRank			
Text	(0.297)	Computation	(0.384)
Formula	(0.439)	Concept	(0.196)
Both	(0.336)	Proof	(0.407)
alpha01-trans-noReRank			
Text	(0.355)	Computation	(0.397)
Formula	(0.444)	Concept	(0.224)
Both	(0.343)	Proof	(0.421)
alpha001-trans-noReRank			
Text	**(0.450)**	Computation	(0.300)
Formula	(0.317)	Concept	**(0.367)**
Both	(0.337)	Proof	(0.374)

a set of discrete scores, and then treat these scores as categories to be predicted by a classification model, such as a Support Vector Machine. Other approaches to model CQA-type features should also be investigated, including those that have been shown to be successful in the SemEval CQA Challenge series [15–17].

Improvements for topic conversion. Another area for improvement is with respect to the design of the auto extraction algorithm described in Section 3.1. Although the use of our automatic extraction algorithm performs comparably to manual topic conversion, we have not considered constraining the number of keywords and formulas in the algorithm. With a widely fluctuating formula-to-keyword ratio across topics, the performance of our extraction algorithm might be hindered by using any fixed α value for all queries. Preliminary investigations have been unable to establish a correlation between the best value for α and the ratio of the number of keyword terms to the number of terms extracted from math formulas [6], but this new benchmark might provide additional insights into how to choose a value for α "on the fly" that depends on the number of keyword terms and math terms.

Alternatively, constraining the maximum number of keywords and formulas and the sizes of formulas extracted from a topic description might be a better approach, since it would also constrain the number of keyword and math terms. However this poses a new question on determining the best maximum limit. We believe that research related to Automatic Term Extraction (ATE) in technical domains, or in mathematical domains, might provide valuable insights into our problem.

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Appendix

Below is the list of formulas and keywords converted from the given topics using the method described in Section 3.1. Among the 98 topics submitted for each run, 77 were (partially) assessed for the CQA task. The topics with no assessments are A.2, A.6, A.22, A.25, A.34, A.46, A.57, A.64, A.70, A.71, A.73, A.76, A.81, A.82, A.84, A.91, A.92, A.94, A.95, A.97 and A.100, marked with a dagger (†) below.

Topic	List of formulas and keywords
A.1	[c, $f(x) = \frac{x^2+x+c}{x^2+2x+c}$, $[-1, -\frac{1}{3}]$, $f(x) = \frac{x^2+x+c}{x^2+2x+c}$, $f(x)$, $[-1, -\frac{1}{3}]$, Finding, value, range, rational, function, does, contain, calculate, range, rational, function, reverse, across, this, find, value, range, does, contain, functions]
A.2†	[$f'(x) = f(x+1)$, $\frac{df}{dx} = f(x+1)$, Solving, differential, equations, form, solve, differential, equations, form, ordinary, differential, equations, ordinary-differential-equations]
A.3	[$\sqrt{5}$, 10^{-10}, $\sqrt{5}$, 10^{-10}, $\sqrt{5}$, $f(x)$, $[a, b]$, $f(a)$, $f(b)$, Approximation, correct, resolve, problem, Find, approximation, correct, using, bisection, has, placed, function, sure, go, function, Mathematica, given, calculations, function, interval, opposite, signs, tolerance, number, iterations, numerical, methods, algorithms, bisection, numerical-methods]
A.4	[$\sum_{k=0}^{n} \binom{n}{k} k$, $n2^{n-1}$, compute, this, combinatoric, sum, sum, know, result, know, does, one, even, sum, like, this, has, binomial, coefficients, combinatorics, number, theory, summation, proof, explanation, number-theory, proof-explanation]
A.5	[$P((2)\|(1)) = \frac{P((2)\cap(1))}{P((1))} = \frac{P((2))P((1))}{P((1))} = P((2))$, family, has, two, Given, one, boy, probability, boys, family, has, two, Given, one, boy, probability, boys, was, this, question, using, conditional, probability, event, first, boy, event, second, probability, second, boy, given, first, boys, formula, since, second, boy, does, depend, first, detailed, solution, correct, probability, proof, verification, conditional, probability, proof-verification, conditional-probability]
A.6†	[$5^{133} \mod 8.$, $5^n \mod 8 = 5$, $5^{133} \mod 8 = 5$, calculate, mod, number, big, exponent, find, noticed, lead, say, know, prove, prove, this, case, find, solution, algebra, precalculus, arithmetic, algebra-precalculus]

A.7 [$\frac{1110-1}{100}$, $1110-1$, $1110-1 = x \pmod{100}$, $112 \equiv 21 \pmod{100}$, $(112)2 \equiv (21)2 \pmod{100}$, $114 \equiv 441 \pmod{100}$, $114 \equiv 41 \pmod{100}$, $(114)2 \equiv (41)2 \pmod{100}$, $118 \equiv 1681 \pmod{100}$, $118 \equiv 81 \pmod{100}$, $118 \times 112 \equiv (81 \times 21) \pmod{100}$, $1110 \equiv 1701 \pmod{100} \implies 1110 \equiv 1 \pmod{100}$, $1110 - 1 \equiv (1-1) \pmod{100} \implies 1110-1 \equiv 0 \pmod{100}$, $x = 0, 1110-1$, Finding, remainder, using, modulus, divided, solve, term, mod, tried, mod, mod, mod, mod, mod, mod, mod, mod, mod, mod, mod, mod, mod, value, divisible, this, approach, long, time, competitive, exam, math, contest, without, using, process, determine, remainder, above, problem, very, helpful, advance, elementary, number, theory, modular, arithmetic, divisibility, alternative, proof, elementary-number-theory, modular-arithmetic, alternative-proof]

A.8 [$\lim_{n\to\infty} \sqrt[n]{\frac{(27)^n(n!)^3}{(3n)!}}$, $\lim_{n\to\infty} \sqrt[n]{\frac{(27)^n(n!)^3}{(3n)!}}$, finding, value, Finding, value, try, solve, help, limits]

A.9 [$\sum_{n=0}^{N} nx^n$, $\sum_{i=0}^{n} i^2 = \frac{(n^2+n)(2n+1)}{6}$, this, series, need, write, series, form, does, involve, summation, notation, example, Does, anyone, idea, this, multiple, ways, including, using, generating, functions, sequences, series, sequences-and-series]

A.10 [$\int_0^\infty \frac{\sin x}{x^a}$, $\int_0^\infty \frac{\sin x}{x^a}$, Find, values, improper, integral, sin, converges, Find, values, improper, integral, sin, converges, expand, using, series, expansion, improper, integrals, improper-integrals]

A.11 [R^3, $u = (a, b, c)$, $v = (d, e, f)$, R^2, R^3, $u = (a, b)$, $v = (d, e)$, R^2, R^3, $\Phi(u,\ v) = (\ f(u),\ \ g(v)\ )$, R^2, R^3, R^2, $\Phi(u, v) = (2u\cos v,\ \ u\sin v)$, cross, product, dimensions, math, book, using, states, cross, product, two, vectors, defined, direction, resultant, determined, curling, fingers, vector, pointing, direction, cross, product, cross, product, defined, Is, degenerate, case, cross, product, like, this, type, determinant, instance, parameterization, needed, calculate, examples, book, calculating, determinate, cos, sin, multivariable, calculus, vectors, multivariable-calculus]

A.12 [$(1 + i\sqrt{3})^{1/2}$, Finding, roots, complex, number, was, solving, practice, problems, this, question, It, find, roots, sketch, linear, algebra, complex, numbers, polar, coordinates, linear-algebra, complex-numbers, polar-coordinates]

A.13 [$\int_a^b f(x)dx + \int_{f(a)}^{f(b)} f^{-1}(x)dx$?, $\int_a^b f(x)dx + \int_{f(a)}^{f(b)} f^{-1}(x)dx$?, $bf(b) - af(a)$, expression, expression, answer, answer, calculus]

A.14 [$y = xy' + \frac{1}{2}(y')^2$, $\frac{1}{2}y'(2x + y') = y$, $x^2 + y = t$, Help, solving, first, order, differential, equation, first-order, first, order, differential, equation, this, find, way, solve, use, derivate, idea, first-order, ordinary, differential, equations, ordinary-differential-equations]

A.15 [$\sum_{i=1}^{n} ix^{i-1}$, $1 + 2x + 3x^2 + 4x^3 + 5x^4 + ... + nx^{n-1} + ...$, $x \neq 1, |x| < 1$, S_n, $S_2 = 1 + x + x^2 + x^3 + x^4 + ...$, $\frac{d(S_2)}{dx} = S_1$, $|x| < 1$, S_2, $\frac{1-x^n}{1-x} = \frac{1}{1-x}$, $S_1 = \frac{d(S_2)}{dx} = \frac{d(\frac{1}{1-x})}{dx} = \frac{1}{(1-x)^2}$, Derive, sum, series, need, find, partial, sums, finally, sum, tried, series, source, series, sum, geometric, progression, this, answer, sequences, series, convergence, summation, power, series, sequences-and-series, power-series]

A.16 [$\int_0^1 \frac{\ln(1+x)\ln(1-x)}{1+x}dx$, $\int_0^1 \frac{\ln(1+x)\ln(1-x)}{1+x}\,dx$, $I(a,b) = \int_0^1 \frac{\ln(1-ax)\ln(1+bx)}{1+x}\,dx$, $\frac{d^2 I(a,b)}{dadb}$, Finding, Calculate, My, try, Let, compute, happy, see, ideas, order, kill, this, integral, integration, sequences, series, definite, integrals, closed, form, sequences-and-series, definite-integrals, closed-form]

A.17 [$\int_{x=0}^{\infty} \frac{\sin(x)}{x}$, $\frac{e^{iz}}{z}$, $\int_{x=0}^{\infty} \frac{\sin(x)}{x}$, $f(z) = \frac{e^{iz}}{z}$, $\Gamma = \gamma_1 + \gamma_R + \gamma_2 + \gamma_\epsilon$, $\gamma_1(t) = t, t \in [i\epsilon, iR]$, $\gamma_R(t) = Re^{it}, t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\gamma_2(t) = t, t \in [-iR, -i\epsilon]$, $\gamma_\epsilon(t) = \epsilon e^{it}, t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\frac{\sin(x)}{x}$, $\int_{\gamma_\epsilon} f = -i\pi$, $\epsilon \to 0$, $\int_{\gamma_R} f = 0$, $R \to \infty$, Calculate, sin, function, calculate, sin, function, using, closed, path, use, sin, even, function, has, anti, derivative, integral, closed, path, managed, show, show, Help, complex, analysis, improper, integrals, complex-analysis, improper-integrals]

A.18 [$\lim_{n\to\infty} \frac{[(n+1)(n+2)\cdots(n+n)]^{1/n}}{n}$, $\lim_{n\to\infty} \frac{[(n+1)(n+2)\cdots(n+n)]^{\frac{1}{n}}}{n}$, $\log 2 + n \log \frac{n}{n+1}$, $\frac{2}{e}$, $\frac{4}{e}$, $2/e$, Evaluate, Evaluate, using, Stolz, know, many, question, like, this, solve, using, Stolz, method, log, applied, Stolz, log, log, answer, answer, help, Edit, On, log, log, log, log, log, log, log, log, log, log, Cesáro-Stolz, Cesáro-Stolz, Cesáro-Stolz,, sequences, series, sequences-and-series]

A.19 [p^4-1, p^4-1, 7^4-1, $2^4*3*5*2$, 11^4-1, $2^4*3*5*61$, 2^4*3*5, p^4-1, $(p^2+1)(p-1)(p+1)$, 2^4, $16n+x$, Greatest, common, factor, was, find, greatest, common, factor, primes, First, value, has, divisors, has, divisors, has, prove, divisible, even, integers, know, prove, divisibility, since, check, numbers, prove, divisibility, assigning, divisibility, greatest, common, divisor, greatest-common-divisor]

A.20 [$n \in \mathbb{N}\backslash\{41\}$, $\phi(n) = 40$, $n \in \mathbb{N}$, $\phi(n) = 40$, ϕ, $n = 41$, $n's$, Calculate, looking, Euler, Totient, Function, found, one, namely, calculate, Euler-Totient, totient, function, totient-function]

A.21 [$9^{9^{9^{\cdots 9}}}$, $9^{9^{9^{\cdots 9}}}$, $9^{9^{9^{\cdots 9}}} \equiv x(\text{mod}100)$, $0 \le x \le 100$, $9^{9^{9^{\cdots 9}}}$ (nine9s) $= 9^a$, $9^a(\text{mod}100)$, $a(\text{mod}\phi(100))$, $\phi(100) = 40$, $a = b(\text{mod}40)$, $9^{9^{9^{\cdots 9}}}$ (eight9s) $= 9^b$, $9^b(\text{mod}40)$, $b(\text{mod}\phi(40))$, $\phi(40) = 16$, $b = c(\text{mod}16)$, $9^{9^{9^{\cdots 9}}}$ (seven9s) $= 9^c$, $9^c(mod16)$, $c(\text{mod}phi(16))$, $\phi(16) = 8$, $c(\text{mod}8)$, $9 = 1(\text{mod}8)$, $c = 1(\text{mod}8)$, Finding, last, two, digits, nine, continuing, learning, modular, arithmetic, confused, this, question, Find, last, two, digits, nine, phi, function, used, this, problem, far, this, In, order, know, need, know, In, order, know, need, know, In, order, know, need, know, need, find, like, made, along, way, lot, back, order, value, last, two, anyone, help, this, number, theory, modular, arithmetic, number-theory, modular-arithmetic]

A.22† [$d, d+1, d+2... = N$, N, $d, d+1, d+2...$, $N = 30$, $Ans = 3$, $d_1 = 4; d_2 = 6; d_3 = 8$, $d_1 : 4+5+6+7+8 = 30$, $(\sum(d+n) - \sum(d-1))$, Find, number, satisfies, challenge, cat, trip, walk, way, sum, given, many, ways, Example, Edit, way, see, many, subsets, way, sequences, series, number, theory, sequences-and-series, number-theory]

A.23 [$2^7 \cdot 3^8 \cdot 5^2 \cdot 7^{11}$, $2^3 \cdot 3^4 \cdot 5$, $\cdot x = 2^3 \cdot 3^4 \cdot 5x$, find, product, two, integers, greatest, common, divisor, least, common, multiple, this, question, help, solve, tried, assuming, Gcd, product, factors, prime, numbers, greatest, common, divisor, least, common, multiple, prime-numbers, greatest-common-divisor, least-common-multiple]

A.24 [$\sqrt{2i-1}$?, $\sqrt{2i-1}$?, $2i-1=(a+bi)^2$, $a^2+2abi-b^2$, $a^2-b^2=-1$, $2ab=2$, $a^2=b^{-2}$, $b^{-2}-b^2=-1$, $-b^4+1=-1$, $b^4=2$, $b=\sqrt[4]{2}$, $\sqrt{2i-1}$, Is, this, only, way, evaluate, work, solve, way, algebra, precalculus, algebra-precalculus]

A.25† [$P(x^2+1)=(P(x))^2+1$, $P(0)$, $P(x^2+1)=(P(x))^2+1$, $P(x)$, $P(0)=0$, $P(1)=1$, $P(2)=2$, $P(5)=5$, $P(26)=26$, $P(677)=677$, $P(x)=x$, $y=P(x)$, $y=x$, $P(0)=2$, $P(x)=(x^2+1)^2+1$, $P(0)=3$, $\lim_{x\to\infty}\log_x P(x)$, $P(x)$, $P(0)$, $P(x)$, polynomial, polynomial, Let, points, log, does, converge, So, values, makes, polynomial, polynomials]

A.26 [$\int_0^\infty \frac{\sin x}{x}dx.$, $\int_0^x \frac{sinx}{x}dx$, solve, indefinite, integral, using, Taylor, series, trying, show, integral, convergent, sin, My, first, taylor, series, sin, next, step, real, analysis, calculus, integration, taylor, expansion, riemann, integration, real-analysis, taylor-expansion, riemann-integration]

A.27 [$e^{3i\pi/2}$, $e^{\pi i}=-1$, $e^{3\pi i/2}$, $(e^{\pi i})^{3/2}$, $(\sqrt{-1})^3=i^3=-i$, $\sqrt{(-1)^3}=\sqrt{-1}=i$, value, solving, value, know, confused, right, answer, evaluate, two, possible, answers, this, one, correct, answer, going, complex, numbers, exponentiation, complex-numbers]

A.28 [$\sin(18^\circ)=\frac{a+\sqrt{b}}{c}$, $a+b+c$, $\sin(18)=\frac{a+\sqrt{b}}{c}$, $a+b+c$, $\sin(18)$, $\frac{x}{z}$, $x=a+\sqrt{b}, z=c, y=\sqrt{c^2-(a+\sqrt{b})^2}$, $\cos(18)=\frac{y}{z}=\frac{\sqrt{c^2-(a+\sqrt{b})^2}}{c}$, $b=(c\sin(18)-a)^2=c^2\sin^2(18)-2ac\sin(18)+a^2$, $\sin(18)=\frac{-1+\sqrt{5}}{4}$, A, B, C, $A\sin(18)^2+B\sin(18)+C=0$, $\sin(18)$, Ax^2+Bx+C, $a=-B, b=B^2-4AC, c=2A$, $sin(18)=(-1+\sqrt{5})/4$, sin, sin, form, sin, right, triangle, sides, front, corner, angle, degrees, hypotenuse, find, cos, found, sin, sin, sin, solution, says, sin, intuition, find, sin, sin, sin, root, Totally, This, question, prove, part, solution, algebra, precalculus, trigonometry, euclidean, geometry, contest, math, algebra-precalculus, euclidean-geometry, contest-math]

A.29 [$3+2i$, $-4i$, $i=\sqrt{-1}$, $\frac{5i}{\infty}=0$, $\frac{5}{\infty}\cdot\frac{\sqrt{-1}}{\infty}=0\cdot 0=0$,, $\frac{5x}{\infty}=0$, $\frac{5}{\infty}\cdot\frac{x}{\infty}=0\cdot 0=0.$, Dividing, Complex, Numbers, Infinity, My, PreCalculus, properties, limits, before, test, stated, real, number, divided, infinity, equals, This, whether, complex, number, divided, infinity, equal, This, completely, was, theoretical, calculation, knowing, calculated, complex, number, since, using, properties, utilized, real, numbers, state, since, Is, this, theoretical, calculation, correct, concept, this, algebra, precalculus, limits, complex, numbers, infinity, algebra-precalculus, complex-numbers]

A.30 [$a^3+b^3+c^3-3abc$, $a^3+b^3+c^3-3abc$, $(a)\ 1$, $(b)\ 0$, $(c)-1$, $(d)-2$, a, b, e^x, Find, binomial, theorem, Find, help, solve, this, added, It, expansion, know, use, binomial, theorem, binomial-theorem]

A.31 `training data`

A.32 [$Empty(x) \iff \nexists y(y\in x)$, $Empty(x) \iff \nexists y(y\in x)$, definitions, axioms, very, elementary, definition, first, order, logical, example, say, Define, Is, definition, axiom, call, definitional, this, one, place, predicate, symbol, Empty, new, among, listed, primitives, say, Zermelo, has, only, identity, membership, primitive, So, stating, definitions, effect, stating, axioms, characterizing, primitive, definitional, axioms, complete, reference, specified, set, symbols, Is, correct, case, why, call, axiom, state, mean, why, say, example, Definitional, axiom, terminology, definition, first, order, logic, axioms, first-order-logic]

A.33 [$f(t,x)$, $\frac{\partial^3 f}{\partial t^3}$, $\frac{\partial^3 f}{\partial x^3}$, Physical, meaning, significance, third, derivative, function, Given, physical, quantity, represented, function, meaning, third, derivative, physics]

A.34† [$a, b > 0$, $a \uparrow b$, $a \downarrow\downarrow\downarrow b$, $a\ b$, $a \downarrow^n b$, $n \geq 3$, $a \downarrow\downarrow b = a + 1$, $b + 1$, $a \uparrow^n b$, $n \leq 0$, $a \downarrow\downarrow b = b + 1$, $a \downarrow b = a + b - 1$, $a > b$, $a \downarrow b$, $a \downarrow\downarrow\downarrow b$, $a > b$, $a \downarrow\downarrow b = b + 1 + \left\lfloor \frac{a}{b} \right\rfloor$,, $a \downarrow\downarrow\downarrow b$, Extending, Knuth, non, positive, values, up-arrow/hyperoperations, non-positive, So, idea, extend, Knuth, arrow, notation, included, zero, negative, It, normally, defined, basically, hyperoperation, sequence, only, try, go, backwards, trivial, extension, letting, arrows, represent, negative, arrows, why, heck, coming, expression, does, Alternatively, way, defining, zero, arrows, does, So, question, Is, extension, Knuth, arrow, notation, exists, Edit, this, question, initially, was, correct, So, example, extension, modified, question, An, extension, define, satisfies, recursive, definition, Edit, turns, correct, this, imply, example, close, need, exception, case, does, show, evaluating, need, defined, try, extend, instance, abuse, case, allowed, let, finding, intractable, result, up-arrow, up-arrow, hyperoperation, ackermann, function, ackermann-function]

A.35 [$\int e^{x^2} dx$, $\int e^{2x} dx$, does, function, antiderivative, know, this, question, sound, why, ca, write, does, antiderivative, In, light, this, question, sufficient, conditions, function, need, careful, examination, function, say, does, antiderivative, way, see, function, right, say, does, antiderivative, integration]

A.36 [$\neg P$, $true$, $\neg P \to A_1 \to \ \dots \ \to A_n \to P$, $\neg P \to P \iff \neg(\neg P) \vee P \iff P$, $\neg P$, $\neg P$, $\neg P$, Proof, contradiction, status, initial, assumption, proof, complete, First, like, say, looked, answers, specific, question, found, existing, question, Say, need, prove, statement, method, Assuming, holds, using, list, statements, proven, hold, derived, proof, arrive, list, proven, statements, was, proofs, contain, lines, contradiction, proves, initial, assumption, was, holds, initial, assumption, proven, FALSE, why, sure, derived, holds, particular, holds, On, hand, derived, assumption, explain, why, this, type, argument, used, logic, proof, writing, proof-writing]

A.37 [$f \circ g = g \circ f$, $f^{-1} \neq g$, $f \neq \text{id} \neq g$, $f \circ g = g \circ f?$, $f(x) = 2x$, $g(x) = 3x$, $f\colon \mathbb{R} \setminus \{1\} \to \mathbb{R}$, $f(x) = 2x/(1-x)$, $g(x) = f^{-1} \circ g \circ f = \frac{g(\frac{2x}{1-x})}{2+g(\frac{2x}{1-x})}$,, g_0, $g_{n+1} \mapsto f^{-1} \circ g_n \circ f$, g_0, Non, trivial, examples, real, valued, functions, inverses, identity, linear, Examples, trivial, this, given, function, one, go, example, function, commutes, this, similar, fixed, point, iteration, defined, need, function, function, finding, fixed, point, possible, strategy, ca, find, real-valued, real, analysis, functional, analysis, functions, real-analysis, functional-analysis]

A.38 [$a, b, q, r : a = bq + r$, $Choose$, $q : qb <= a$, Uses, Axiom, Choice, first, year, maths, student, drift, years, ZFC, axioms, first, time, college, stuff, far, nowhere, near, ZFC, terms, happened, use, axiom, choice, time, module, even, know, name, example, proof, non, negative, integers, exist, integers, known, restrictions, proof, like, this, largest, integer, Is, axiom, choice, allows, this, simple, important, step, couple, questions, name, simple, proofs, theorems, results, etc, axiom, choice, essential, read, has, long, topic, dispute, mathematicians, even, people, accept, alternative, axiomatic, systems, work, equally, well, without, needing, first-year, non-negative, set, theory, set-theory]

A.39 [2018^{2019}, 2019^{2018} , $2019 \log(2018)$, $2018 \log(2019)$, $\log 2019 > \log 2018$, 2019^{2018}, know, value, logs, sides, log, log, know, log, log, does, this, mean, one, algebra, precalculus, logarithms, algebra-precalculus]

A.40 [$a_1x_1 + a_2x_2 + a_3x_3 + ... + a_nx_n =$, $f(x+y) = f(x) + f(y)$, $f(cx) = cf(x)$, meaning, term, linear, called, linear, equation, represents, equation, line, dimensional, So, linear, comes, word, line, higher, power, graph, function, straight, called, linear, differential, equation, derivatives, power, equal, similar, above, definition, linear, function, called, linear, In, this, definition, linearity, function, does, word, linear, means, does, relate, straight, line, Finally, does, term, linear, means, case, linear, vector, spaces, reference, straight, line, So, whether, linear, word, used, different, contexts, Does, different, meaning, different, situation, Or, linearity, refers, relation, straight, line, At, Least, explain, linearity, function, linear, vector, space, relate, equation, line, linear, algebra, linear-algebra]

A.41 [$n \leq m$, $\sum_{r=1}^{n}(-1)^{(n-r)}\binom{n}{r}(r)^m$, Confusion, find, number, onto, functions, two, sets, given, In, book, given, two, finite, sets, containing, elements, number, onto, functions, well, ca, combinations, combinatorics, functions, combinations]

A.42 [$x^2 + 1 = 0$, $\sqrt{-1}$, simple, physical, situation, complex, numbers, emerge, naturally, trying, teach, middle, emergence, complex, numbers, motivate, this, this, mean, sort, real, world, problem, people, trying, solve, led, realize, needed, extend, real, numbers, instance, Greeks, forced, irrational, numbers, pure, mathematical, reasons, length, diagonal, square, unit, length, irrational, this, kind, geometrical, situation, dealing, similar, situation, lead, complex, numbers, terms, say, try, solve, equation, physical, abstract, sort, answer, like, object, define, certain, properties, turn, consistent, important, entirely, satisfying, complex, numbers, physics, applications, complex-numbers]

A.43 [$\sum_{n\geq 1}\frac{1}{n^2+1} = \frac{\pi\coth\pi - 1}{2}$, $\sum_{n\geq 1}\frac{1}{n^2+1} = \frac{\pi\coth\pi - 1}{2}$, $S = \sum_{n\geq 1}\frac{1}{n^2+1}$, $x = \pi$, $e^\pi = \frac{\sinh\pi}{\pi} + \frac{2\sinh\pi}{\pi}S$, $S = \frac{\pi e^\pi}{2\sinh\pi} - \frac{1}{2}$, Prove, trying, prove, Letting, Fourier, series, exponential, function, sinh, sinh, cos, sin, Plugging, sinh, sinh, cos, sin, sinh, sinh, sinh, sinh, sinh, nowhere, near, prove, identity, real, analysis, sequences, series, real-analysis, sequences-and-series]

A.44 [$A, B \in \mathscr{M}_{2\times 2}(\mathbb{Q})$, AB, $(\mathscr{M}_{2\times 2}(\mathbb{Q}), \times)$, AB, finite, order, show, has, infinite, order, Let, group, Let, order, Let, order, Show, has, infinite, only, reasoning, possible, contradiction, tried, before, concrete, matrices, group, theory, cyclic, groups, group-theory, cyclic-groups]

A.45 [$\sin(x), \sin(2x), \sin(3x), ..., \sin(nx)$, $\mathbb{R}$, $\sin(x), \sin(2x), \sin(3x), ..., \sin(nx)$, $\mathbb{R}$, $W \neq 0$, $W \neq 0$, prove, sin, sin, sin, sin, independent, Prove, sin, sin, sin, sin, independent, trial, know, trivial, solution, trivial, show, sin, sin, sin, sin, cos, cos, cos, cos, sin, sin, sin, sin, cos, cos, cos, cos, looks, like, Vandermonde, matrix, cant, prove, ordinary, differential, equations, ordinary-differential-equations]

A.46† [f, $[0,1]$, $\int x^k f(x)dx = 0$, $f = 0$a.e., $[0,1]$, $\int_0^1 x^k f(x)\,dx = 0$, $k = \{0,1,2,3,...\}$, $f = 0$, $\int_0^1 f^2(x)\,dx = 0$, $\epsilon > 0$, $M = \sup_{x\in[0,1]} |f(x)|$, $[0,1]$, $\int_0^1 x^k f(x)\ dx = 0$, $k = \{0,1,2,3,...\}$, $f = 0$, $[0,1]$, $\int_0^1 x^k f(x)\ dx = 0$, $k = \{0,1,2,3,...\}$, $f = 0$, $f = 0$a.e., Lebesgue, integrable, function, satisfies, prove, indefinitely, differentiable, real, valued, function, satisfies, prove, My, prove, this, assertion, prove, approximation, theorem, find, polynomial, sup, It, need, condition, indefinitely, differentiable, continuous, hold, My, question, Lebesgue, integrable, real, valued, function, satisfies, prove, except, set, measure, TRUE, Riemann, integrable, real, valued, function, satisfies, prove, except, set, measure, EDIT, prove, prove, fourier, coefficient, equal, cos, cos, proof, complete, real, analysis, real-analysis]

A.47 [p, $0 < r < p-1$, q, $rq \equiv 1 \bmod p$, $0 < r < p-1$, $rq \equiv 1 \bmod p$, $rq - 1 = kp$, $qr - kp = 1$, $\sum_{r=1}^{p-2} r = \frac{(p-2)(p-1)}{2} = p\frac{p-3}{2} + 1 \equiv 1 \bmod p$, $(p-1)! \equiv -1 \bmod p$, $(p-1)!+2 \equiv 1 \bmod p$, $(p+1)$, $(p+1)(p-1)! = -(p+1) \bmod p$, $(p+1)(p-1)! = 1 \bmod p$, Prove, given, prime, exists, mod, Prove, given, prime, exists, mod, only, taken, one, number, theory, course, years, this, popped, computer, science, class, was, assuming, this, proof, elementary, since, current, class, algorithm, cours, basic, tried, look, couple, approaches, reverse, engineer, arrive, conclusion, need, little, manipulation, looks, ca, see, sum, mod, looks, good, know, final, Wilson, Lagrange, vaguely, this, theorem, was, looking, old, book, was, see, arrived, prime, mod, multiplier, built, factorial, expression, was, adding, side, mod, dead, end, sure, was, multiplying, mod, results, mod, multiple, sure, valid, elementary, number, theory, proof, verification, elementary-number-theory, proof-verification]

A.48 [$x, y \geq 0$, $(x+y)^k \geq x^k + y^k$, $k \geq 1$, $x, y \geq 0$, $(x+y)^k \geq x^k + y^k$, $k \in \mathbb{R}_{\geq 1}$, $x \leq x + y$, $x^k \leq (x+y)^k$, $k \geq 1$, $y^k \leq (x+y)^k$, $x^k + y^k \leq 2(x+y)^k$, $x, y \geq 0$, $f(k) = (x+y)^k$, $g(k) = x^k + y^k$, $(x+y)^r \geq x^r + y^r$, $r \in \mathbb{Z}^+$, $k = 1$, $(x+y)^r > x^r + y^r$, $r \geq 2$, $m \notin \mathbb{Z}$, $f(m) = g(m)$, $k = 1$, $(x+y)^r > x^r + y^r$, $r \geq 2$, $k = 1$, showing, trying, show, So, far, tried, things, since, sides, inequality, positive, inequality, hold, Adding, inequalities, this, close, course, Alternatively, was, fix, let, let, show, using, binomial, theorem, show, intersect, only, better, proving, statement, since, positive, integers, real, number, property, exist, since, violate, only, continuity, positive, integers, course, this, dependent, showing, intersect, only, running, low, real, analysis, functions, inequality, real-analysis]

A.49 [$\binom{2n}{n} = \sum_{k=0}^n \binom{n}{k}^2$, $(1+x)^{2n} = [(1+x)^n]^2$, x^n, $\binom{2n}{n}$, $\sum_{k=0}^n \binom{n}{k}^2$, Is, simple, combinatoric, interpretation, this, identity, across, exercise, prove, identity, exercise, It, use, prove, identity, expressions, identity, coefficients, expansions, expressions, course, number, was, whether, equivalent, counting, interpretation, It, clear, number, ways, half, elements, set, this, possible, interpret, equivalently, equivalent-counting, combinatorics, binomial, coefficients, binomial-coefficients]

A.50 [$\sum \frac{1}{n^{2+\cos n}}$, $\sum \frac{1}{n^{2+\cos n}}$, ϵ, A_ϵ, n, $|2+\cos n| \leq 1+\epsilon$, $(n \in A_\epsilon \iff -1 \leq \cos n \leq -1+\epsilon)$, A_ϵ, Divergent, series, cos, Show, cos, divergent, My, main, problem, infinitely, small, positive, real, number, define, set, cos, cos, divergence, come, sum, idea, handle, this, real, analysis, integration, sequences, series, analysis, real-analysis, sequences-and-series]

A.51 [$\sum_{r=0}^{n}\binom{n+r}{r}\frac{1}{2^r}=2^n$, x^n, Sum, series, binomial, coefficients, Prove, try, coefficient, solve, binomial, coefficients, binomial-coefficients]

A.52 [$\forall n \in \mathbb{N}$, $\exists m \in \mathbb{N}$, $m > n$, m, $n_1, n_2, ..., n_k \in \mathbb{N}$, $n = n_1 n_2 ... n_k + 1$, $n_1, n_2, ..., n_k$, $a, b \in \mathbb{N}$, $\mathbb{Z}^+$, $a = qb + r$, $0 \leq r < q$, $k \in \mathbb{N}$, $n_1, n_2, ..., n_k \geq 1$, $\forall i$, $n_i \nmid n = n_1 n_2 ... n_k + 1$, $\exists n \in \mathbb{N}, \forall m \in \mathbb{N}, m \leq n$, Prove, prime, two, parts, Prove, least, divisible, numbers, Prove, truth, negation, leads, Use, theorem, exist, unique, quotient, remainder, part, given, show, set, sure, go, part, know, negation, prime, sure, proof, theorem, proof, writing, prime, numbers, proof-writing, prime-numbers]

A.53 [$g \in G$, $gh = 1$, $hg = 1$, $AB = 1 \not\Rightarrow BA = 1$, $AB = 1 \Rightarrow BA = 1$, Show, one, sided, inverse, square, matrix, TRUE, inverse, one-sided, know, group, element, does, mean, In, case, matrices, linear, maps, vector, spaces, TRUE, This, happens, square, matrices, case, even, form, group, multiplication, restrict, square, matrices, simple, proof, this, avoids, chasing, entries, makes, use, simply, vector, space, structure, linear, transformations, In, prove, this, this, imply, group, one, sided, two, sided, inverses, has, infinite, since, finite, group, finite, dimensional, representation, one-sided(but, two-sided), linear, algebra, group, theory, linear-algebra, group-theory]

A.54 [$P(N) = (S|S \subseteq N)$, $P(N) = (S|S \subseteq N)$, using, diagonal, argument, show, uncountable, tips, solutions, this, one, using, diagonal, argument, show, uncountable, discrete, mathematics, elementary, set, theory, discrete-mathematics, elementary-set-theory]

A.55 [$\frac{1}{\sqrt{-1}} = \sqrt{-1}$, $\sqrt{-1}$, $(\sqrt{-1})^2 = -1$, $\frac{1}{\sqrt{-1}} = \sqrt{\frac{1}{-1}} = \sqrt{-1}.$, calculation, set, new, number, property, write, know, this, correct, result, missing, complex, numbers, definition, complex-numbers]

A.56 [$\exists p\ \big(\text{pisprime} \to \forall x(\text{xisprime})\big)$, curious, logical, formula, involving, prime, numbers, Let, set, natural, Is, formula, TRUE, FALSE, know, answer, this, question, shortest, way, arrive, conclusion, using, deduction, system, logic, first, order, logic, first-order-logic]

A.57† [$\mathbb{R}^n$, $f : B \to \mathbb{R}^m$, f^{-1}, $f(B)$, $f(X)$, $\mathbb{R}^n$, $f : X \to \mathbb{R}^m$, $f^{-1} : f(X) \mapsto X$, $f(X)$, Preimage, continuous, one, one, function, connected, domain, continuous, one-to-one, know, given, compact, subset, continuous, injective, one, one, function, continuous, This, TRUE, know, image, connected, subset, connected, continuous, let, connected, non, compact, subset, continuous, injective, one, one, trying, counterexample, mapping, continuous, parametrized, example, advance, (one-to-one), (non-compact), (one-to-one), real, analysis, general, topology, analysis, continuity, metric, spaces, real-analysis, general-topology, metric-spaces]

A.58 [$3\arcsin\frac{1}{4} + \arccos\frac{11}{16} = \frac{\pi}{2}$, $3\arcsin\frac{1}{4} + \arccos\frac{11}{16} = \frac{\pi}{2}$, Prove, arcsin, help, this, exercise, know, prove, answer, fully, Exercise, Prove, arcsin, trigonometry]

A.59 [$\sum_{d|n}\phi(d) = n$, $\phi(n)$, $\sum_{d|n}\phi(d) = n$, $n = \prod_{k=1}^{m} p_k^{\alpha_k}$, $d = \prod_{k=1}^{m} p_k^{\beta_k}$, $0 \leq \beta_k \leq \alpha_k$, Multiple, proofs, looking, multiple, proofs, statement, denotes, Euler, totient, one, unique, factorisation, theorem, align, group, theory, number, theory, alternative, proof, big, list, group-theory, number-theory, alternative-proof, big-list]

A.60 [$a_n = \left(1-\frac{1}{\sqrt{2}}\right) ... \left(1-\frac{1}{\sqrt{n+1}}\right)$, $n \geq 1$, $\lim_{n\to\infty} a_n$, $\frac{1}{\sqrt{\pi}}$, a_n, $(0.293)*(0.423)*(0.5)*(0.553)*(0.622)*(0.647)*(0.667)*....$, (D), a_n, Limiting, value, sequence, tends, infinity, Let, equals, does, exist, equals, equals, My, Approach, particular, direction, procedure, find, value, tends, So, tried, like, this, simple, way, substitute, values, trying, find, limiting, value, So, value, tending, option, tried, like, this, find, value, converges, tends, infinity, help, procedure, solve, this, question, calculus, sequences, series, limits, products, sequences-and-series]

A.61 [$i, j \in \mathbb{N}$, $n = 3i+5j$, $n \geq 8$, $i, j \in \mathbb{N}$, $n = 3i+5j$, $n \geq 8$, $n = 8 \implies 8 = 3\cdot 1+5\cdot 1$, $n = 9 \implies 9 = 3{\cdot}3+5{\cdot}0$, $n = 10 \implies 10 = 3{\cdot}0+5{\cdot}2$, $n = h$, $n = h+1$, $k+1 = 3i+5j$, exists, Prove, exists, hard, time, this, exercise, trying, prove, induction, Basis, step, Induction, step, TRUE, TRUE, So, know, proving, elementary, number, theory, discrete, mathematics, induction, diophantine, equations, elementary-number-theory, discrete-mathematics, diophantine-equations]

A.62 [$\mathbb{Q}$, $\mathbb{Z}$, $|\mathbb{Q}| = |\mathbb{Z}|$, $\mathbb{Q}$, $\mathbb{Z}\times\mathbb{Z}$, $|\mathbb{Z}\times\mathbb{Z}| = |\mathbb{Z}|$, Prove, cardinality, set, rational, numbers, set, integers, equal, learned, cardinality, discrete, class, days, this, This, confusing, entirely, sure, even, full, question, Let, denote, set, rational, numbers, denote, set, Prove, saying, element, element, know, prove, bijection, even, prove, help, discrete, mathematics, discrete-mathematics]

A.63 [gcd, lcm, 2, n_1, n_2, $\mathrm{lcm}(n_1, n_2) = \frac{n_1 n_2}{\mathrm{gcd}(n_1,n_2)}$, $n_1, n_2, n_3, ..., n_r$, gcd, positive, integers, two, positive, integers, relationship, greatest, common, divisor, least, common, multiple, given, gcd, set, positive, integers, does, relationship, hold, Is, TRUE, like, this, prove, handle, proof, explanation, greatest, common, divisor, least, common, multiple, proof-explanation, greatest-common-divisor, least-common-multiple]

A.64† [$f : [a, b] \to \mathbb{R}$, $f([a, b]) \subset [a, b]$, $c \in [a, b]$, $f(c) = c$, $f(a) = a$, $f(b) = b$, $f(a) = a$, $f(b) = b$, $[a, b]$, $f(a) > a$, $f(b) < b$, $f(a) > a$, $f(b) < b$, $x, y \in [a, b]$, $f(a) = x$, $f(b) = y$, $f[a, b] = [x, y]$, $[x, y] \subset [a, b]$, $[x, y]$, $c \in [a, b]$, $f(c) = c$, continuous, Prove, exists, point, satisfying, continuous, Prove, exists, point, satisfying, left, show, well, assume, Since, values, this, assuming, So, far, Assume, Let, means, Notice, Since, continuous, exists, equal, equal, This, assume, Intermediate, Value, Theorem, real, analysis, continuity, proof, explanation, real-analysis, proof-explanation]

A.65 [$e^{-2\lambda t}\lambda^2 \leq \frac{1}{e^2t^2}$, $\lambda, t \geq 0$, $e^{-2\lambda t}\lambda^2 \leq \frac{1}{e^2t^2}1$, $\lambda, t \geq 0$, ln, (1), $t\lambda \leq e^{t\lambda-1}2.$, $x \leq e^{x-1}$, $x \geq 0$, show, show, Applying, sides, yields, equivalent, So, show, this, calculus, inequality, exponential, function, exponential-function]

A.66 [$x, h \in \mathbb{R}^d$, $A \in \mathbb{R}^{d\times d}$, $(x^TAh)^T = h^TA^Tx$, $x, h \in \mathbb{R}^d$, $A \in \mathbb{R}^{d\times d}$, $(x^TAh)^T = h^TA^Tx$, possible, possible, linear, algebra, transpose, linear-algebra]

A.67 [$k\times k$, $k\times l$, $l\times l$, Combination, matrixes, matrix, matrix, matrix, prove, matrix, elements, equal, know, rules, calculating, determinants, know, this, question, calculus, determinant]

A.68 [$a^n + 1$, $a + 1$, n, $a^n + 1$, $a + 1$, -1, $n \in \mathbb{N}$, $2k + 1$, $a^{2(k+1)+1} + 1$, $= a^{2k+3} + 1$, $= a^3 \cdot a^{2k} + 1$, $= (a^3 + 1)\cdot a^{2k} - a^{2k} + 1$, a^{2k}, $a^n + 1$, $a + 1$, Prove, divisible, odd, Prove, divisible, odd, know, Since, odd, rewrite, assume, holds, prove, holds, next, sure, Since, means, exponential, term, even, cant, use, divisible, odd, polynomials, induction, divisibility]

A.69 [$\binom{s}{s} + \binom{s+1}{s} + ... + \binom{n}{s} = \binom{n+1}{s+1}$, $n \geq s$, $0 \leq s \leq n$, Induction, two, variable, parameters, So, was, assigned, this, problem, tried, professor, explanations, My, professor, saying, statement, need, prove, formula, correct, need, use, induction, variables, sure, help, combinatorics]

A.70† [$\sum_{j=0}^{N-1} \cos \frac{(2j+1)\pi}{2N} = 0$, $l \in \mathbb{Z}$, $N \in \mathbb{N}$, $\sum_{j=0}^{N-1} \cos \left(l \frac{(2j+1)\pi}{2N} \right) = 0$, Proving, cos, Let, need, prove, cos, tried, use, Euler, formula, sum, first, terms, geometric, serie, ideas, trigonometry, summation]

A.71† [$1^2+2^2+....+n^2 = \frac{n(n+1)(2n+1)}{6}$, $1^2+2^2+....+n^2 = \frac{n(n+1)(2n+1)}{6}$, $LHS = 1^2$, $RHS = \frac{(1+1)(2+1)}{6} = \frac{2*3}{6} = 1$, $LHS_p = 1^2 + 2^2 + ... + p^2$, $RHS_p = \frac{p(p+1)(2p+1)}{6}$, $LHS_{p+1} = 1^2 + 2^2 + ... + p^2 + (p+1)^2$, $RHS_{p+1} = \frac{(p+1)((p+1)+1)(2(p+1)+1)}{6}$, $RHS_{p+1} = RHS_p + (p+1)^2$, $RHS_{p+1} = \frac{p(p+1)(2p+1)}{6} + (p+1)^2$, $RHS_{p+1} = \frac{(p+1)((p+1)+1)(2(p+1)+1)}{6}$, $\frac{p(p+1)(2p+1)}{6} + (p+1)^2$, Show, induction, Show, induction, My, Case, Case, Case, show, this, induction, need, show, So, need, rewrite, equal, Anyone, see, Or, solution, induction]

A.72 [$\mathcal{X} = \mathcal{Y}$, $\mathcal{X} \in \mathcal{Y}$, $\mathcal{X} = \mathcal{Y}$, $\mathcal{X} \in \mathcal{Y}$, Is, possible, Is, possible, set, equal, set, set, element, set, set, theory, axioms, set-theory]

A.73† [$\binom{n}{0}^2 + \binom{n}{1}^2 + ... + \binom{n}{n}^2 = \binom{2n}{n}$, $\binom{n}{0}^2 + \binom{n}{1}^2 + ... + \binom{n}{n}^2 = \binom{2n}{n}$, $(1+x)^n(1+x)^n = (1+x)^{2n}$, x^n, $\binom{n}{0}1 + \binom{n}{1}x + ... + \binom{n}{r}x^r + ... + \binom{n}{n}x^n$, x^n, x^i, x^{n-i}, x^n, $\binom{n}{n-r} = \binom{n}{r}$, $\binom{n}{0}^2 + \binom{n}{1}^2 + ... + \binom{n}{n}^2$, x^n, $\binom{2n}{n}$, x^n, $\binom{2n}{n}$, $\binom{n}{0}^2 + \binom{n}{1}^2 + ... + \binom{n}{n}^2 = \binom{2n}{n}$, x^n, x^n, x^n, Help, proof, proof, required, made, binomial, expose, was, forward, questions, exposing, see, question, marks, like, this, one, points, quite, numeration, This, Prove, use, equality, call, result, proved, finding, coefficient, terms, this, equality, binomial, theorem, left, hand, side, this, equation, product, two, factors, equal, factors, multiply, term, term, first, factor, has, term, second, factor, has, coefficients, Since, summation, equal, So, left, hand, side, equation, coefficient, expand, right, hand, side, equation, find, coefficient, left, hand, side, equation, prove, equal, In, This, was, My, one, heck, does, this, equation, come, know, equation, come, requested, prove, different, equality, two, why, solution, first, equation, finding, coefficients, one, made, see, one, three, why, coefficient, left, hand, side, equation, made, coefficient, right, hand, side, this, equation, well, prove, original, equation, one, was, prove, first, place, know, many, guys, help, long, post, long, (?-n), (?-1), (?-2)., left-hand, right-hand, (?-3), left-hand, (?-1), (?-2), (?-3), right-hand, discrete, mathematics, binomial, coefficients, binomial, theorem, discrete-mathematics, binomial-coefficients, binomial-theorem]

A.74 [$f : (0,\infty) \to \mathbb{R}$, $f(x) = x + \dfrac{1}{x}$, $[2,\infty)$, $f : (0,\infty) \to \mathbb{R}$, $f(x) = x + \dfrac{1}{x}$, $[2,\infty)$, $x = 1$, $f(1) = 2$, $[2,\infty)$, Show, image, function, interval, Show, image, function, interval, So, show, function, interval, functions, elementary, set, theory, elementary-set-theory]

A.75 [m, $\lim_{u\to\infty} \frac{u^m}{e^u} = 0$, $\lim_{u\to\infty} \frac{u^m}{e^u} = 0$, e^u, $>$, $\frac{u^{m+1}}{(m+1)!}$, Prove, integer, show, integer, Looking, solutions, sure, this, logical, step, real, analysis, calculus, limits, real-analysis]

A.76† [$\mathbb{Z}$, $a+b\mathbb{Z} = \{z \in \mathbb{Z} \mid z = a+bk \text{forsome} k \in \mathbb{Z}\}$, $a \in \mathbb{Z}$, $b \in \mathbb{Z}-\{0\}$, $\{a_i+b_i\mathbb{Z} \mid i \in \mathbb{N}\}$, $\bigcup_{i\in\mathbb{N}}(a_i + b_i\mathbb{Z}) = \mathbb{Z}$, $I \subset \mathbb{N}$, $\bigcup_{i\in I}(a_i + b_i\mathbb{Z}) = \mathbb{Z}$, $\{p_k\} = \{2,3,5,\dots\}$, ℓ_1, ℓ_2, $\ell_1 = \ell_2 = 5$, $\{0 + p_{k_j}\}$, $j = 1,\dots,n$, $p > \max\{p_{k_1},\dots,p_{k_n}\}$, $p \notin \bigcup_{1\le j\le n}(0 + p_{k_j}\mathbb{Z})$, $p \notin (-1 + 5\mathbb{Z}) \cup (1 + 5\mathbb{Z})$, $5 \nmid p - 1$, $5 \nmid p + 1$, Covering, arithmetic, progressions, solving, problems, old, exam, topology, translated, problem, algebraic, terms, problem, Let, collection, sets, satisfying, Show, whether, always, possible, extract, finite, lot, elementary, algebra, Let, set, construct, non, negative, integers, instance, pick, finite, sub, collection, assume, prime, max, run, This, course, possible, prime, last, this, approach, means, prove, infinitely, many, primes, ending, like, thing, prove, simple, problem, like, Surely, way, solving, this, EDIT, interested, solution, topology, whether, solution, like, solution, works, non-negative, sub-collection, general, topology, elementary, number, theory, general-topology, elementary-number-theory]

A.77 [$(-1)(-1) = 1$, $(-1)(-1) = 1$, $1 \cdot 1 = 1$, $1 \cdot x = x$, Show, relation, consequence, distributive, law, Show, relation, consequence, distributive, This, question, first, problem, Theory, Andre, point, tried, using, help, work, elementary, number, theory, elementary-number-theory]

A.78 `training data`

A.79 [$\left|\dfrac{e^{-ixu} - 1}{u}\right| \le |x|$, $u \ne 0$, Inequality, complex, exponential, Rudin, Real, Complex, Analysis, uses, this, proof, near, chapter, real, Why, this, TRUE, Edit, real, inequality, exponential, function, fourier, transform, exponential-function, fourier-transform]

A.80 [$\mathbb{N}$, $\emptyset,\{1\},\{2\},\{1,2\},\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{4\},\dots$, $\{1\}$, $\{2\}$, $[n] = \{1,2,\dots,n\}$, $[n]$, 2^n, Why, does, this, proof, set, finite, subsets, countable, set, work, set, subsets, found, this, proof, thread, found, simple, answers, sort, formula, like, trying, way, see, set, finite, subsets, countable, probably, list, elements, set, one, coming, first, next, shows, set, pattern, see, least, In, words, first, comes, comes, time, new, integer, list, subsets, contain, ones, contain, showed, subsets, show, first, elements, this, applies, finite, subsets, cant, why, apply, set, subsets, continue, this, scheme, assume, way, infinity, quite, help, analysis, elementary, set, theory, proof, explanation, elementary-set-theory, proof-explanation]

A.81† [$n \in \mathbb{N}$, $\{1,\cdots,n\}$, $M \ne \emptyset$, $x \in M$, $f(1)$, $f(1) := x$, $\{1\}$, $\{1,\cdots,n\}$, $M - \{f(1),\cdots,f(n)\}$, $x \in M - \{f(1),\cdots,f(n)\}$, $g(1),\cdots,g(n+1)$, $g(1) := f(1),\cdots,g(n) := f(n)$, $g(n+1) := x$, $\{1,\cdots,n+1\}$, n_1, $h(n_1)$, $\{1,\cdots,n_1\}$, $g(n_1)$, $h(n_1)$, $(1,g(1)),\cdots,(n_1,g(n_1))$, $h(n_2)$, $n_2 \le n_1$, $g(n_2)$, $g(n_2)$, $h(n_2)$, $h(n_3)$, $n_3 > n_1$, $(n_1+1,g(n_1+1)),\cdots,(n_3,g(n_3))$, $g(n_3)$, $h(n_3)$, $h(n)$, $n \in \mathbb{N}$, $h : \mathbb{N} \to M$, $h : \mathbb{N} \to M$, $h : \mathbb{N} \to M$, infinite, set, contains, countable, Why, proof, Axiom, Choice, Let, infinite, Proposition, exists, injection, Since, exists, Define, injection, exists, injection, Since, infinite, set, empty, So, exists, Define, injection, Let, arbitrary, natural, calculate, injection, Proposition, return, value, pairs, calculate, search, database, value, database, return, value, calculate, pairs, database, Proposition, return, value, calculate, above, injection, Why, proof, man, know, injection, man, know, injection, elementary, set, theory, axiom, choice, elementary-set-theory, axiom-of-choice]

A.82† [$f(x)$, $[0, 2\pi]$, $f(x)$, $[0, 2\pi]$, $A = \int_0^{2\pi} f(x)\mathrm{d}x$, $g(x)$, $f(x) = g(x) \cdot \cos(x)$, $f(x)$, $A = \int_0^{2\pi} g(x) \cdot \cos(x)\mathrm{d}x$, $t = \sin(x)$, $\mathrm{d}t = \cos(x)\mathrm{d}x$, $x = \arcsin(t)$, $t = \sin(x)$, $\sin(0) = 0$, 2π, $\sin(2\pi) = 0$, $A = \int_0^{0} g(\arcsin(t))\mathrm{d}t$, $A = 0$, definite, integrals, evaluate, using, periodic, functions, know, reasoning, know, went, discuss, this, Maths, even, find, Let, assuming, function, continuous, has, antiderivative, interval, Let, area, curve, interval, exist, function, cos, Substituting, value, cos, Using, substitution, Let, sin, cos, arcsin, Changing, limits, sin, sin, sin, Substituting, definite, integral, arcsin, Definite, Integral, lower, upper, bounds, So, help, calculus, integration, trigonometry, definite, integrals, inverse, function, definite-integrals, inverse-function]

A.83 [$a_n \leq M$, Is, sequence, sums, inverse, natural, numbers, bounded, reading, Infinite, Sequences, right, trivial, matter, determine, boundedness, mind, sequence, before, learn, sequence, know, sequence, bounded, above, number, calculus, sequences, series, harmonic, numbers, sequences-and-series, harmonic-numbers]

A.84† [$4, x$, $Z[x]$, $I =<; p, x >$, $Z[x]$, $I =< p, x >$, $Z[x]$, $4, x$, $Z[x]$, Is, ideal, generated, principal, ideal, principal, ideal, My, question, Is, principal, ideal, prime, ideal, generated, principal, ideal, abstract, algebra, ring, theory, ideals, principal, ideal, domains, abstract-algebra, ring-theory, principal-ideal-domains]

A.85 [N, $(+1)$, $1/2$, $1/2$, $p_n = \frac{1}{2}p_{n-1}$, p_n, $p_N = 1$, $p_{N-1} = 2$, $p_0 = 2^N$, Expected, number, steps, bug, reach, position, bug, time, position, At, step, bug, moves, right, step, probability, returns, origin, probability, expected, number, steps, this, bug, reach, position, tried, first, find, possibility, this, bug, reaches, number, steps, recurrence, equation, find, possibility, bug, position, reach, boundary, condition, see, does, make, sense, sort, value, probability, first, number, expected, steps, sure, recurrence, equation, markov, chains, random, walk, markov-chains, random-walk]

A.86 [$\sum_{k=0}^{n} k \cdot \begin{pmatrix} n \\ \\ k \end{pmatrix} = O\left(2^{n \log_3 n}\right)$?, Is, TRUE, log, Problem, Is, TRUE, log, My, solution, this, upper, bound, way, large, ca, find, solution, combinatorics, elementary, number, theory, discrete, mathematics, elementary-number-theory, discrete-mathematics]

A.87 [$\forall n \in \mathbb{N} : (\sum_{i=1}^{n} a_i)(\sum_{i=1}^{n} \frac{1}{a_i}) \geq n\%$, a_i, $\forall i \in \mathbb{N} : a_i \in \mathbb{R}^+$, $n = 1$, $n = 2$, $n = 3$, $a, b \in \mathbb{R}^+$, $ab = 1$, $a + b \geq 2$, $n = 3$, $a, b, c \in \mathbb{R}^+$, $\left(\frac{a}{b} + \frac{b}{a}\right) \geq 2$, $\left(\frac{a}{c} + \frac{c}{a}\right) \geq 2$, $\left(\frac{b}{c} + \frac{c}{b}\right) \geq 2$, $P(1)$, $P(n)$, Is, TRUE, positive, TRUE, prove, this, holds, using, lemma, Lemma, Let, example, case, proven, like, this, Let, lemma, sure, generalized, version, natural, ca, come, counterexample, try, prove, induction, Let, Base, case, Inductive, hypothesis, assume, Inductive, step, know, Is, this, inequality, TRUE, prove, anyone, show, counterexample, algebra, precalculus, inequality, algebra-precalculus]

A.88 [$x^4 + 10x^2 + 1$, $\mathbb{Z}[x]$, $x^4 + 10x^2 + 1$, $\mathbb{Z}[x]$, Is, polynomial, reducible, Is, polynomial, reducible, abstract, algebra, ring, theory, field, theory, irreducible, polynomials, abstract-algebra, ring-theory, field-theory, irreducible-polynomials]

A.89 [$A^2 + B^2 = C^2 + D^2$, A, B, C, D, Parametrization, pythagorean, like, equation, pythagorean-like, Is, known, complete, parametrization, Diophantine, equation, positive, rational, numbers, equivalently, integers, number, theory, diophantine, equations, number-theory, diophantine-equations]

A.90 [$n \times n$, $n \times n$, A^{-1}, $A^{-1}A = \mathbb{I}_n \quad \wedge \quad AA^{-1} = \mathbb{I}_n \qquad (1)$, $\mathbb{I}_n$, $n \times n$, $A^{-1}A = \mathbb{I}_n$, $AA^{-1} \neq \mathbb{I}_n$, Question, definition, Inverse, matrix, definition, matrix, inverse, matrix, property, identity, cases, way, around, making, FALSE, statement, linear, algebra, matrices, linear-algebra]

A.91† [$\mathbb{R} \to \mathbb{R}$, x^2, Continuous, function, reaches, value, range, times, Is, continuous, function, reaches, possible, values, value, range, times, example, perfect, was, question, almost, certain, functions, knows, bunch, real, analysis, calculus, real-analysis]

A.92† [$\mathbb{F}_q[X,Y]$, $K[X,Y]$, $\mathbb{F}_q[X,Y]$, $I = (P(X,Y))$, $Q(X)$, $\mathbb{F}_q[X,Y]/(Q(X)) \cong (\mathbb{F}_q[X]/(Q(X)))[Y]$, $P(X,Y) = \sum_{i=0}^{n} P_i(X)Y^i$, $d \geq 1$, P_i, P_n, $\mathbb{F}_q$, P_n, $\prod_{\alpha \in \mathbb{F}_q}(X - \alpha)$, $\alpha \in \mathbb{F}_q$, $P_n(\alpha) \neq 0$, $I := (P, X - \alpha)$, (P), (P), $\mathbb{F}_q[X,Y]$, $1 \in I$, $X = \alpha$, $n = \deg_Y(P) = 0$, $P(X,Y) = (X^{p-1}-1)XY-1$, $P(X,Y) = (X^{p-1}-1)XY-X-1$, $\mathbb{F}_p[X,Y]$, Is, principal, maximal, ideal, Given, infinite, field, one, prove, maximal, ideal, ca, In, non, principal, prime, ideal, maximal, ideal, generated, two, whether, result, holds, find, principal, ideal, irreducible, polynomial, maximal, ideal, polynomial, positive, degrees, given, irreducible, polynomial, only, one, variable, quotient, ring, polynomials, field, polynomial, form, polynomials, polynomial, vanishes, identically, divisible, ideal, contain, maximal, whole, Writing, evaluating, leads, conclusion, This, infer, above, tried, looking, prime, number, determining, whether, quotient, field, know, answer, problem, proof, counter, example, very, advance, non-principal, counter-example, polynomials, commutative, algebra, field, theory, finite, fields, commutative-algebra, field-theory, finite-fields]

A.93 [$AB =$, BA, A, B, $\mathbb{R}$, nxn, $det(xI - AB) = det(xI - BA)$, Characteristic, Polynomial, characteristic, polynomial, Let, matrix, size, prove, singular, matrix, linear, algebra, polynomials, linear-algebra]

A.94† [$2^{\aleph_0}$, $2^{\aleph_0}$, sets, natural, numbers, two, finite, intersection, Question, sets, natural, numbers, two, finite, read, infinite, families, need, ignore, Property, family, finitely, many, sets, family, sets, infinite, probably, find, ones, property, Property, family, almost, tower, pick, two, sets, family, one, almost, contained, probably, meaning, find, intersection, finite, contained, find, rational, numbers, natural, numbers, rational, numbers, paired, natural, ones, solving, this, problem, families, rational, numbers, solving, families, natural, need, various, ways, defining, real, help, "almost-tower":, almost-contained, cardinals, rational, numbers, natural, numbers, rational-numbers, natural-numbers]

A.95† [$\sin x$, $\sin x$, $\angle B = x$, $\lim_{x\to 0} \frac{\sin x}{x} = 1$, $\sin x$, $\sin x < x < \tan x$, $\sin x < x < \sin x + 1 - \cos x$, $\sin x$, $\mathbb{R}^2$, $F \subset G$, $|\partial F| < |\partial G|$, $|\partial F|$, $\int_s^t \sqrt{1+y'(u)^2}du$, y'', $y(s) = y(t)$, y', $y = \sqrt{1-x^2}$, $-1 \le x \le 1$, $\sin x$, Length, convex, paths, bounding, sin, problem, Defining, sin, leg, right, triangle, radians, hypotenuse, prove, sin, motivation, find, derivative, sin, elementary, pre, Taylor, pre, series, context, many, times, proof, based, length, arc, satisfies, sin, sin, sin, cos, lower, bound, clear, sin, length, straight, segment, length, curved, segment, find, upper, bound, based, this, intuitive, two, convex, subsets, length, boundary, proof, tried, trying, bound, example, negative, realize, ca, bounded, note, example, Question, Is, possible, derivative, sin, inequality, lengths, bounds, Is, calculus, text, this, approach, similar, define, functions, $pre - Taylor$, $pre - series$, real, analysis, derivatives, trigonometry, reference, request, arc, length, real-analysis, reference-request, arc-length]

A.96 [$\sum_i a_i$, a_i, $\sum_i \frac{a_i}{a_i+a_{i+1}+a_{i+2}+\cdots}$, $\sum_i a_i$, a_i, $s_n = \sum_{i=n}^{\infty} a_i$, $\sum_i \frac{a_i}{s_i}$, $a_i = r^i$, $a_i = 1/i^2$, Let, convergent, sum, positive, Does, always, diverge, Let, convergent, sum, Let, Does, always, diverge, tried, examples, geometric, series, always, diverge, sequences, series, analysis, sequences-and-series]

A.97† [$\mathbb{R}$, $\mathbb{Q}$, $\mathbb{C}$, $\mathbb{R}$, $\mathbb{C} = span\{1, i\}$, $\mathbb{R}$, $\mathbb{Q}$, $m := n+1$, $x_1, ...x_m$, $\frac{a_1}{b_1}x_1 + ... + \frac{a_m}{b_m}x_m = 0$, $lcm(b_1, ..., b_m)$, dimension, vector, space, field, look, vector, space, dimension, question, dimension, vector, space, like, answer, infinity, dimension, was, finite, say, real, numbers, was, linear, combination, rational, coefficients, Multiplying, real, numbers, linear, combination, natural, coefficients, FALSE, prove, linear, algebra, vector, spaces, linear-algebra, vector-spaces]

A.98 [$R : S^1 \to S^1$, $\{R^n([x])\}$, S^1, α, $R : S^1 \to S^1$, $[x] \to [x+\alpha]$, $[x] \in S^1$, $\{R^n([x])\}$, S^1, $R(e^{2\pi ix}) = e^{2\pi i(x+\alpha)}.$, $R^n(x)$, $n \in \mathbb{Z}$, $R^n(e^{2\pi ix}) = e^{2\pi i(x+n\alpha)}$, $e^{2\pi ix} \in S^1$, $[y] = e^{2\pi iy} \in S^1$, $[y]$, $[z_y] = e^{2\pi iz}$, $[z_y] = R^n([x])$, $n \in \mathbb{Z}$, irrational, rotation, dense, points, Let, irrational, number, irrational, rotation, need, prove, set, dense, First, write, So, write, need, prove, neighborhood, contains, point, Is, way, correct, does, does, need, this, question, showing, proof, different, proofs, general, topology, circles, rotations, irrational, numbers, general-topology, irrational-numbers]

A.99 [$f : \mathbb{R} \to \mathbb{R}$, $\mathbb{Q}$, Rationals, set, continuity, function, functions, discontinuities, rationals, continuities, irrationals, function, continuities, rationals, Or, words, set, continuities, function, real, analysis, calculus, continuity, real-analysis]

A.100† [$E = (0, 1]$, $E \subseteq R$, Is, this, set, closed, Is, correct, say, this, set, set, real, numbers, closed, real, analysis, real-analysis]

A.101 `training data`