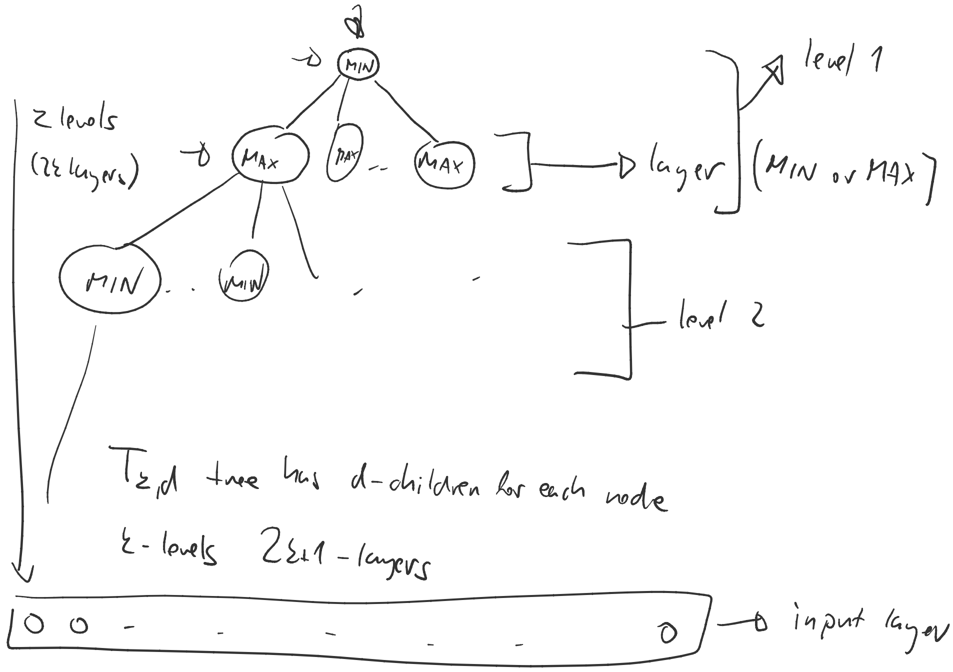
Tutorial 4

10 March 2022 12:04

Game tree evaluation

Min-Max trees (definition)



$T_{z,d}$ tree has d-children for each node

z-levels $2z+1$ - layers

leaves of the tree (input layer) are labeled with numbers $\in \{0, \dots, d-1\}$

How many leaves does a $T_{z,d}$ tree have? d^{2z} - leaves

Input is a string of length d^{2z} numbers $\{0, \dots, d-1\}$.

→ Each $\boxed{MIN}$ node contains the smallest of it's children values.

→ Each $\boxed{MAX}$ node contains the largest of it's children values.

→ Goal is to calculate the value of the root.

$T_{z,2}$ - binary trees with input $2^{2z} = 4^z$ bits.

min → $\wedge$

max → $\vee$

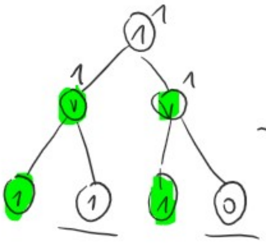
min

$\boxed{0}\ 1 \to 0$ $\qquad$ $0\ \boxed{1} \to 1$

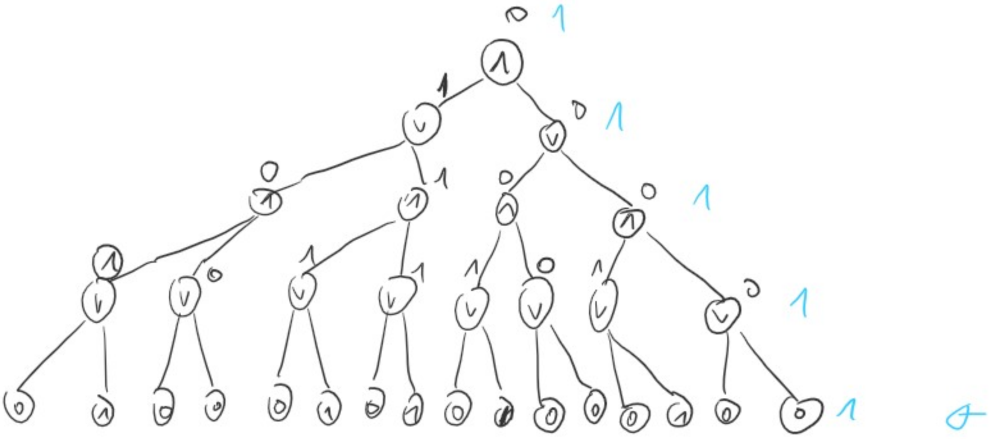
$1\ \boxed{0} \to 0$ $\qquad$ $\boxed{1}\ 0 \to 1$

$\boxed{0}\ \boxed{0} \to 0$ $\qquad$ $\boxed{1}\ \boxed{1} \to 1$

Deterministic evaluation: 'depth first', 'left child first'



→ Only 2 out of 4 leaves were accessed.



All the 32 leaves need to be accessed.

⇒ the difficulty of any deterministic algorithm is $O(4^k)$ (worst case)

$$n = \underbrace{\boxed{4^k}}_{\varphi}$$

Randomized algorithm – choose a child to evaluate at random.

claim: Expected complexity is $\underset{\varphi}{O(3^k)}$ $\quad \boxed{n^{0,79\ldots}}$

Induction basis: Tree with 1 layer needs on average 3 leaf evaluations

1 evaluation

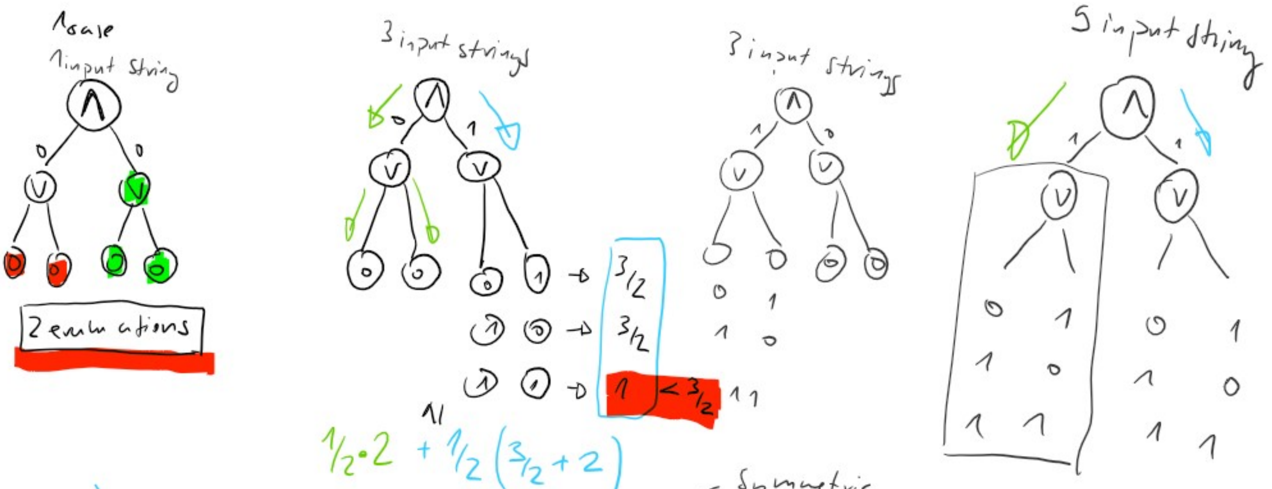
2 evaluation

Red first – 1 evaluation

Green first – 2 evaluation

Average $3/2$

— " —



$\frac{1}{2}\cdot 2 + \frac{1}{2}\left(\frac{3}{2}+2\right)$

$= 1 + \frac{7}{4} < 3$

symmetric

< 3

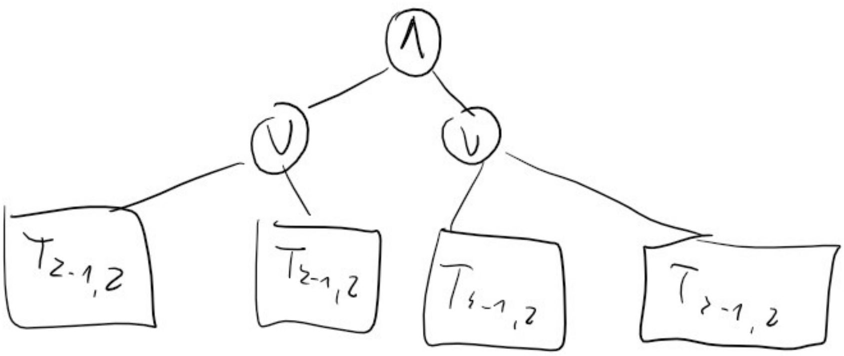
$\frac{1}{2}\left(\frac{3}{2}+\frac{3}{2}\right) + \frac{1}{2}\left(\frac{3}{2}+\frac{3}{2}\right)$

$= 3$

AVG evaluations ≤ 3

Induction hypothesis: $T_{z-1,2}$ tree can be evaluated with our randomized algorithm by accessing less than 3^{z-1} leaves on average.

Induction step $T_{z,2}$



By I.H. each of the $T_{k-1,2}$ subtrees needs on average 3^{k-1} leaf evaluations. By I.B. we need to evaluate on average 3 of the $T_{k-1,2}$ subtrees $\Rightarrow$ $T_{k,2}$ needs at most $3 \cdot 3^{k-1}$ leaf evaluations.

$\square$