

# Advertising on the Web

Advanced Search Techniques for Large Scale Data Analytics

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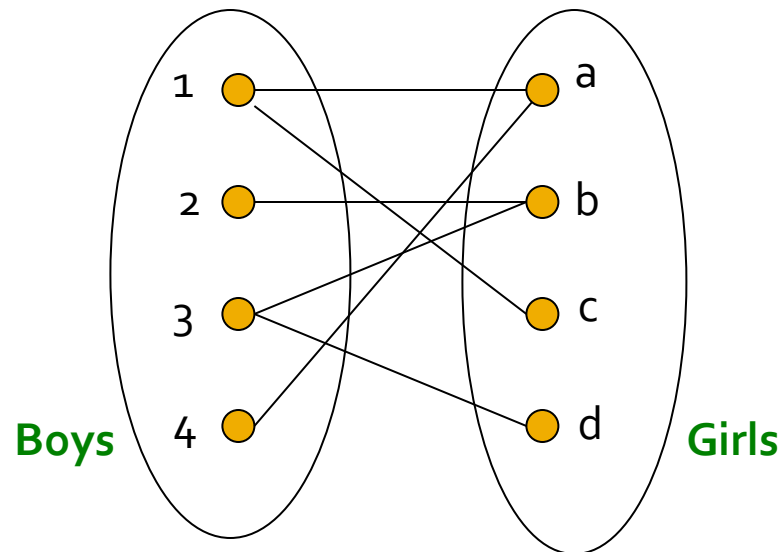
# Online Algorithms

- **Classic model of algorithms**
  - You get to see the entire input, then compute some function of it
  - In this context, “offline algorithm”
- **Online Algorithms**
  - You get to see the input one piece at a time, and need to make irrevocable decisions along the way
  - **Similar to the data stream model**

# Online Bipartite Matching

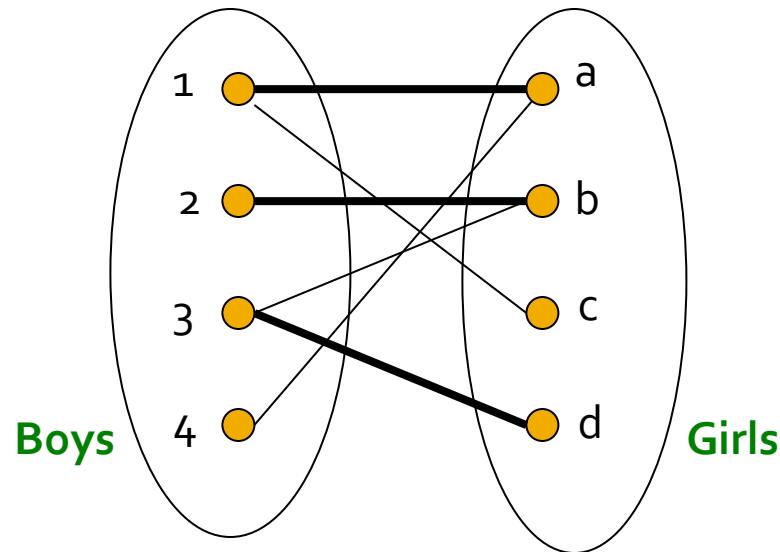
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# Example: Bipartite Matching



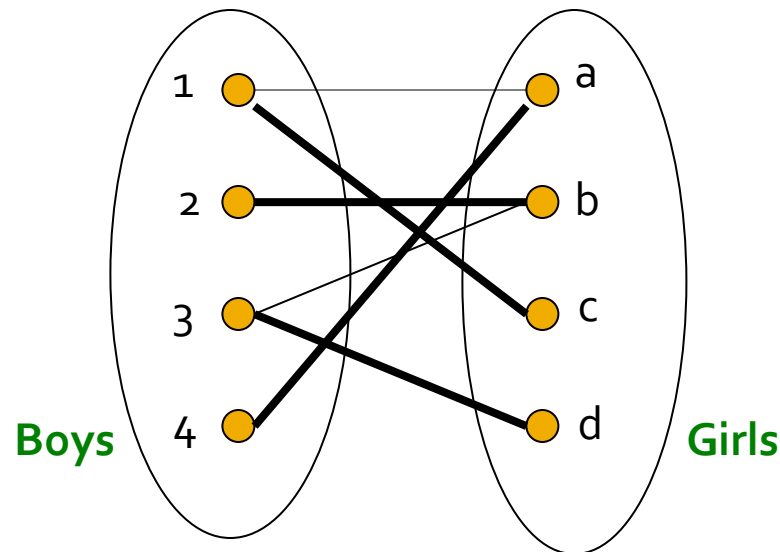
**Nodes: Boys and Girls; Edges: Preferences**  
**Goal: Match boys to girls so that maximum number of preferences is satisfied**

# Example: Bipartite Matching



$M = \{(1,a), (2,b), (3,d)\}$  is a **matching**  
Cardinality of matching =  $|M| = 3$

# Example: Bipartite Matching



$M = \{(1,c), (2,b), (3,d), (4,a)\}$  is a  
**perfect matching**

**Perfect matching** ... all vertices of the graph are matched

**Maximum matching** ... a matching that contains the largest possible number of matches

# Matching Algorithm

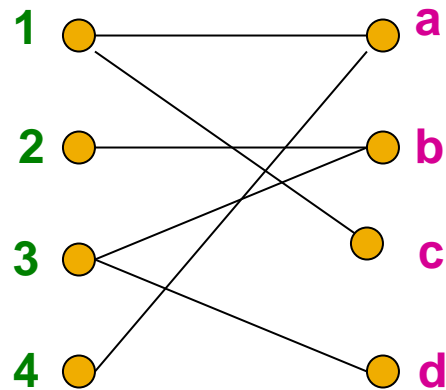
- **Problem:** Find a maximum matching for a given bipartite graph
  - A perfect one if it exists
- There is a polynomial-time offline algorithm based on augmenting paths (Hopcroft & Karp 1973, see [http://en.wikipedia.org/wiki/Hopcroft-Karp\\_algorithm](http://en.wikipedia.org/wiki/Hopcroft-Karp_algorithm))
- **But what if we do not know the entire graph upfront?**

# Online Graph Matching Problem

- Initially, we are given the set **boys**
- In each **round**, **one girl's choices are revealed**
  - That is, girl's **edges** are revealed
- **At that time, we have to decide to either:**
  - Pair the **girl** with a **boy**
  - Do not pair the **girl** with any **boy**
- **Example of application:**
  - Assigning tasks to servers



# Online Graph Matching: Example



**(1,a)**

**(2,b)**

**(3,d)**

# Greedy Algorithm

- **Greedy algorithm for the online graph matching problem:**
  - Pair the new girl with **any** eligible boy
    - If there is none, do not pair girl
- **How good is the algorithm?**

# Competitive Ratio

- For input  $I$ , suppose greedy produces matching  $M_{greedy}$  while an optimal matching is  $M_{opt}$

Competitive ratio =

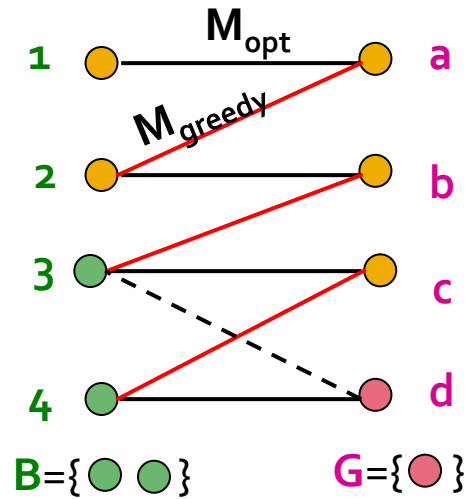
$$\min_{\text{all possible inputs } I} (|M_{greedy}| / |M_{opt}|)$$

(what is greedy's worst performance over all possible inputs  $I$ )

# Analyzing the Greedy Algorithm

- Consider a case:  $M_{greedy} \neq M_{opt}$
- Consider the set  $G$  of girls matched in  $M_{opt}$  but not in  $M_{greedy}$
- Then every boy  $B$  adjacent to girls in  $G$  is already matched in  $M_{greedy}$ :
  - If there would exist such non-matched (by  $M_{greedy}$ ) boy adjacent to a non-matched girl then greedy would have matched them
- Since boys  $B$  are already matched in  $M_{greedy}$  then
 

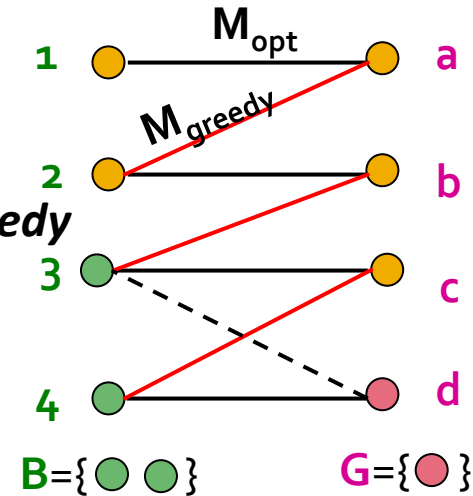
(1)  $|M_{greedy}| \geq |B|$



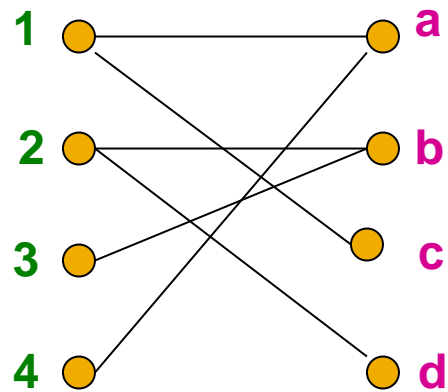
# Analyzing the Greedy Algorithm

- **Summary so far:**

- Girls  $G$  matched in  $M_{opt}$  but not in  $M_{greedy}$
- (1)  $|M_{greedy}| \geq |B|$
- There are at least  $|G|$  such boys ( $|G| \leq |B|$ ) otherwise the optimal algorithm couldn't have matched all girls in  $G$ 
  - So:  $|G| \leq |B| \leq |M_{greedy}|$
- By definition of  $G$  also:  $|M_{opt}| \leq |M_{greedy}| + |G|$ 
  - Worst case is when  $|G| = |B| = |M_{greedy}|$
- $|M_{opt}| \leq 2|M_{greedy}|$  then  $|M_{greedy}|/|M_{opt}| \geq 1/2$



# Worst-case Scenario



(1,a)

(2,b)

# Web Advertising

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# History of Web Advertising

## ■ **Banner ads (1995-2001)**

- Initial form of web advertising

- Popular websites charged X\$ for every 1,000

“impressions” of the ad

- Called “**CPM**” rate  
(Cost per thousand impressions)
- Modeled similar to TV, magazine ads

- From **untargeted** to **demographically targeted**

- **Low click-through rates**

- Low ROI for advertisers

The screenshot shows the homepage of The New York Times. At the top right, there is a red-bordered box containing the text "SHOP NOW AT MARCJACOBS.COM". Below the main navigation bar, there is a blue banner for "Get a Full Times Experience." with a red-bordered box around the "NO DIRECT" button. The main content area features several articles, including "U.S. Sergeant Is Said to Kill 16 Civilians in Afghanistan" and "In Assessing the Damage, Fears of an Emboldened Taliban". On the right side, there is a "MARKETS" section and a "SPECIAL OFFER" for 4 weeks for 99¢. At the bottom right, there is a large green-bordered box for an Audible.com advertisement that says "a-list collection HEAR GREAT BOOKS PERFORMED BY HOLLYWOOD'S FINEST GET ONE OF THESE BOOKS FREE".

**CPM...cost per mille**  
**Mille...thousand in Latin**



# Performance-based Advertising

- **Introduced by Overture around 2000**
  - Advertisers **bid on search keywords**
  - When someone searches for that keyword, the **highest bidder's ad is shown**
  - Advertiser is charged only if the ad is clicked on
- Similar model adopted by Google with some changes around 2002
  - Called **Adwords**

# Ads vs. Search Results

## Web

Results 1 - 10 of about 2,230,000 for **geico**. (0.04 sec)

### [GEICO Car Insurance. Get an auto insurance quote and save today ...](#)

GEICO auto insurance, online car insurance quote, motorcycle insurance quote, online insurance sales and service from a leading insurance company.

[www.geico.com/](#) - 21k - Sep 22, 2005 - [Cached](#) - [Similar pages](#)

[Auto Insurance](#) - [Buy Auto Insurance](#)

[Contact Us](#) - [Make a Payment](#)

[More results from www.geico.com »](#)

### [Geico, Google Settle Trademark Dispute](#)

The case was resolved out of court, so advertisers are still left without legal guidance on use of trademarks within ads or as keywords.

[www.clickz.com/news/article.php/3547356](#) - 44k - [Cached](#) - [Similar pages](#)

### [Google and GEICO settle AdWords dispute | The Register](#)

Google and car insurance firm GEICO have settled a trade mark dispute over ... Car insurance firm GEICO sued both Google and Yahoo! subsidiary Overture in ...

[www.theregister.co.uk/2005/09/09/google\\_geico\\_settlement/](#) - 21k - [Cached](#) - [Similar pages](#)

### [GEICO v. Google](#)

... involving a lawsuit filed by Government Employees Insurance Company (GEICO). GEICO has filed suit against two major Internet search engine operators, ...

[www.consumeraffairs.com/news04/geico\\_google.html](#) - 19k - [Cached](#) - [Similar pages](#)

## Sponsored Links

### [Great Car Insurance Rates](#)

Simplify Buying Insurance at Safeco  
See Your Rate with an Instant Quote  
[www.Safeco.com](#)

### [Free Insurance Quotes](#)

Fill out one simple form to get multiple quotes from local agents.  
[www.HometownQuotes.com](#)

### [5 Free Quotes. 1 Form.](#)

Get 5 Free Quotes In Minutes!  
You Have Nothing To Lose. It's Free  
[sayyessoftware.com/Insurance](#)  
Missouri

# Web 2.0

- **Performance-based advertising works!**
  - Multi-billion-dollar industry
  
- **Interesting problem:**  
**What ads to show for a given query?**

# Adwords Problem

## ■ Given:

- 1. A set of bids by advertisers for search queries
- 2. A click-through rate for each advertiser-query pair
- 3. A budget for each advertiser (say for 1 month)
- 4. A limit on the number of ads to be displayed with each search query

## ■ Respond to each search query with a set of advertisers such that:

- 1. The size of the set is no larger than the limit on the number of ads per query
- 2. Each advertiser has bid on the search query
- 3. Each advertiser has enough budget left to pay for the ad if it is clicked upon

# Adwords Problem

- A stream of queries arrives at the search engine:  $q_1, q_2, \dots$
- Several advertisers bid on each query
- When query  $q_i$  arrives, search engine must pick a subset of advertisers whose ads are shown
- **Goal: Maximize search engine's revenues**
  - **Simple solution:** Instead of raw bids, use the “expected revenue per click” (i.e.,  $\text{Bid} \cdot \text{CTR}$ )
- **Clearly we need an online algorithm!**

# The Adwords Innovation

Advertiser	Bid	CTR	Bid * CTR
A	\$1.00	1%	1 cent
B	\$0.75	2%	1.5 cents
C	\$0.50	2.5%	1.125 cents

Click through  
rate

Expected  
revenue

# Complications: Budget

- **Two complications:**
  - Budget
  - CTR of an ad is unknown
- **Each advertiser has a limited budget**
  - Search engine guarantees that the advertiser will not be charged more than their daily budget

# Complications: CTR

- **CTR: Each ad has a different likelihood of being clicked**
  - **Advertiser 1** bids \$2, click probability = 0.1
  - **Advertiser 2** bids \$1, click probability = 0.5
  - **Clickthrough rate (CTR)** is measured **historically**
    - **Very hard problem: Exploration vs. exploitation**  
**Exploit:** Should we keep showing an ad for which we have good estimates of click-through rate  
**or**  
**Explore:** Shall we show a brand new ad to get a better sense of its click-through rate



# Greedy Algorithm

- **Our setting: Simplified environment**
  - There is **1** ad shown for each query
  - All advertisers have the same budget ***B***
  - All ads are equally likely to be clicked
  - Value of each ad is the same (**=1**)
- **Simplest algorithm is greedy:**
  - For a query pick any advertiser who has bid **1** for that query
  - **Competitive ratio of greedy is 1/2**

# Bad Scenario for Greedy

- **Two advertisers  $A_1$  and  $A_2$** 
  - $A_1$  bids on query  $x$ ,  $A_2$  bids on  $x$  and  $y$
  - Both have budgets of \$4
- **Query stream:  $x x x x y y y y$** 
  - Worst case greedy choice:  $A_2 A_2 A_2 A_2 \_ \_ \_ \_$
  - Optimal:  $A_1 A_1 A_1 A_1 A_2 A_2 A_2 A_2$
  - **Competitive ratio =  $\frac{1}{2}$**
- **This is the worst case!**
  - **Note:** Greedy algorithm is deterministic – it always resolves draws in the same way

# BALANCE Algorithm [MSVV]

- **BALANCE** Algorithm by Mehta, Saberi, Vazirani, and Vazirani
  - For each query, pick the advertiser with the largest unspent budget
    - Break ties arbitrarily (but in a deterministic way)

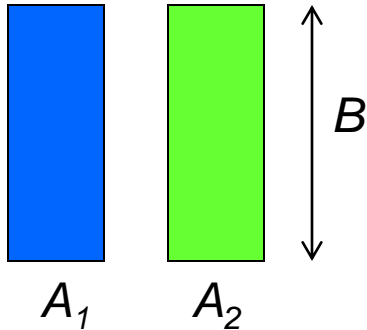
# Example: BALANCE

- **Two advertisers  $A_1$  and  $A_2$** 
  - $A_1$  bids on query  $x$ ,  $A_2$  bids on  $x$  and  $y$
  - Both have budgets of \$4
- **Query stream:  $x x x x y y y y$**
- **BALANCE choice:  $A_1 A_2 A_1 A_2 A_2 A_2 \_ \_$** 
  - Optimal:  $A_1 A_1 A_1 A_1 A_2 A_2 A_2 A_2$
- **In general: For BALANCE on 2 advertisers**  
**Competitive ratio =  $\frac{3}{4}$**

# Analyzing BALANCE

- **Consider simple case (w.l.o.g.):**
  - 2 advertisers,  $A_1$  and  $A_2$ , each with budget  $B$  ( $\geq 1$ )
  - Optimal solution exhausts both advertisers' budgets
- **BALANCE must exhaust at least one advertiser's budget:**
  - **If not, we can allocate more queries**
    - Whenever BALANCE makes a mistake (both advertisers bid on the query), advertiser's unspent budget only decreases
    - Since optimal exhausts both budgets, one will for sure get exhausted
  - Assume BALANCE exhausts  $A_2$ 's budget, but allocates  $x$  queries fewer than the optimal
  - **Revenue:  $BAL = 2B - x$**

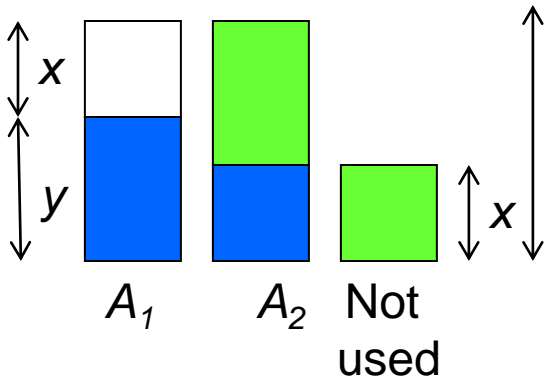
# Analyzing Balance



- Queries allocated to  $A_1$  in the optimal solution
- Queries allocated to  $A_2$  in the optimal solution

Optimal revenue =  $2B$

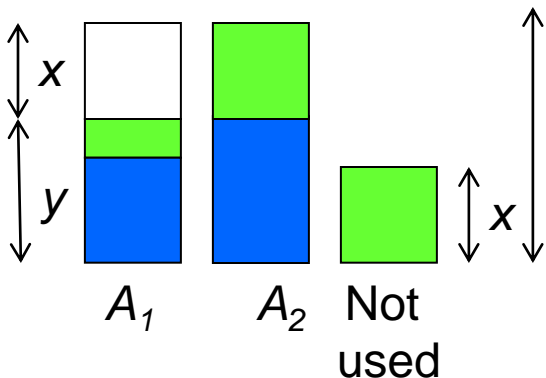
Assume Balance gives revenue =  $2B - x = B + y$



**Unassigned queries should be assigned to  $A_2$**   
(if we could assign to  $A_1$  we would since we still have the budget)

**Goal: Show we have  $y \geq x$**

**Case 1)  $\leq 1/2$  of  $A_1$ 's queries got assigned to  $A_2$**   
then  $y \geq B/2$



**Case 2)  $> 1/2$  of  $A_1$ 's queries got assigned to  $A_2$**   
then  $x \leq B/2$  and  $x + y = B$

**Balance revenue is minimum for  $x = y = B/2$**

Minimum Balance revenue =  $3B/2$

**Competitive Ratio =  $3/4$**

**BALANCE exhausts  $A_2$ 's budget**

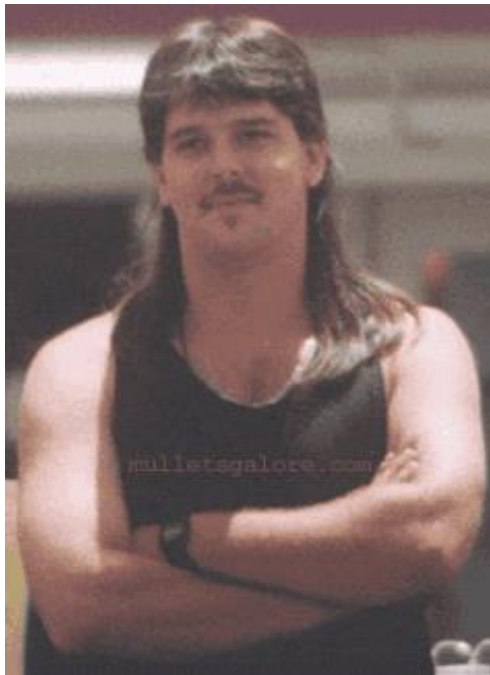
# BALANCE: General Result

- In the general case with  $N$  advertisers, worst competitive ratio of BALANCE is  $1 - 1/e =$  approx. 0.63
  - Interestingly, no online algorithm has a better competitive ratio!

# Recommender Systems: Content-based Systems & Collaborative Filtering



# Example: Recommender Systems



## ■ Customer X

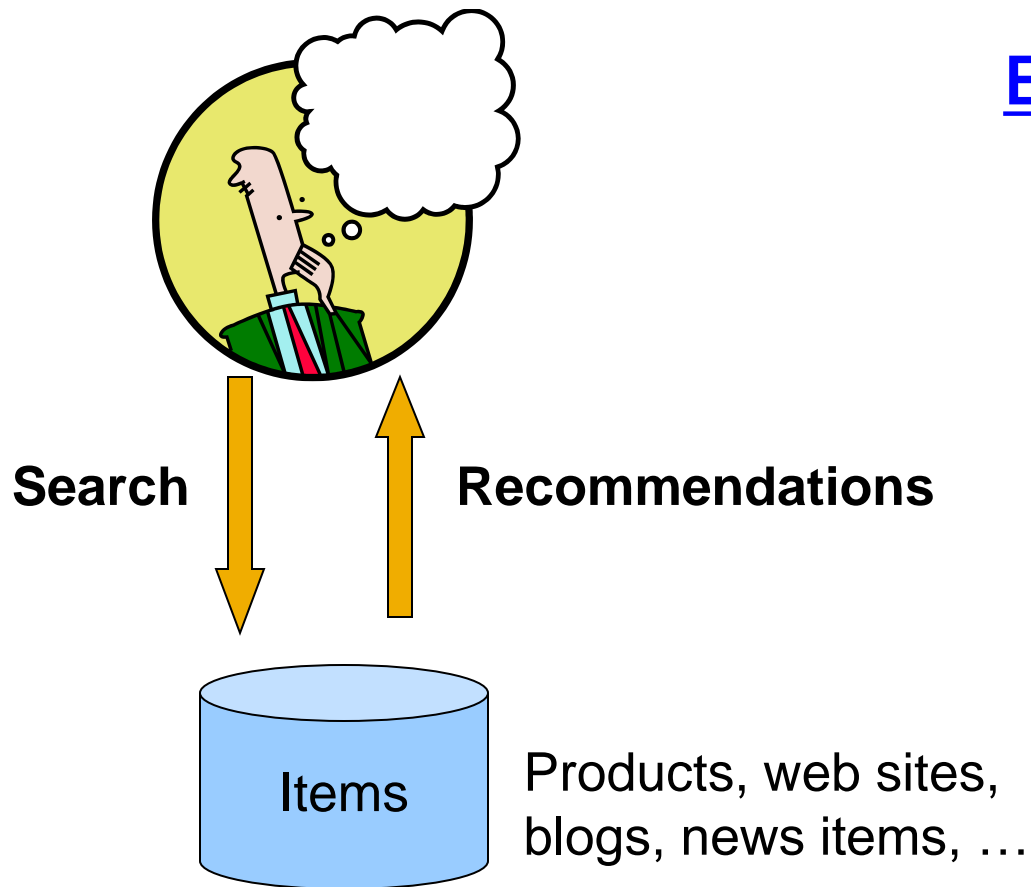
- Buys Metallica CD
- Buys Megadeth CD



## ■ Customer Y

- Does search on Metallica
- Recommender system suggests Megadeth from data collected about customer X

# Recommendations



## Examples:

amazon.com.



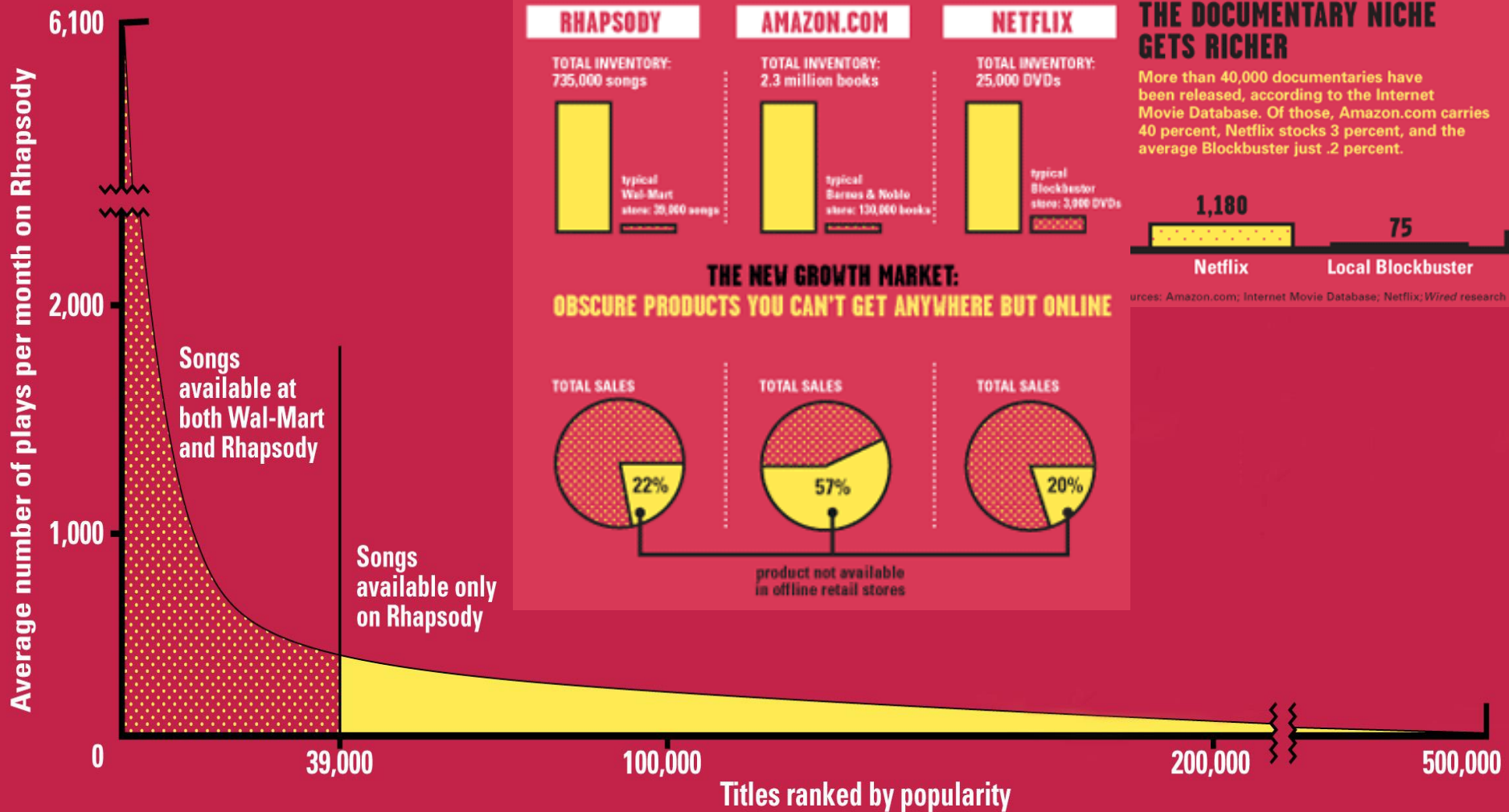
movie lens  
helping you find the *right* movies



# From Scarcity to Abundance

- **Shelf space is a scarce commodity for traditional retailers**
  - Also: TV networks, movie theaters,...
- **Web enables near-zero-cost dissemination of information about products**
  - From scarcity to abundance
- **More choice necessitates better filters**
  - Recommendation engines
  - How **Into Thin Air** made **Touching the Void** a bestseller: <http://www.wired.com/wired/archive/12.10/tail.html>

# Sidenote: The Long Tail



Sources: Erik Brynjolfsson and Jeffrey Hu, MIT, and Michael Smith, Carnegie Mellon; Barnes & Noble; Netflix; RealNetworks  
Source: Chris Anderson (2004)

# Types of Recommendations

- **Editorial and hand curated**
  - List of favorites
  - Lists of “essential” items
- **Simple aggregates**
  - Top 10, Most Popular, Recent Uploads
- **Tailored to individual users**
  - Amazon, Netflix, ...

# Formal Model

- $X$  = set of **Customers**
- $S$  = set of **Items**
- **Utility function**  $u: X \times S \rightarrow R$ 
  - $R$  = set of ratings
  - $R$  is a totally ordered set
  - e.g., **0-5 stars**, real number in **[0,1]**

# Utility Matrix

	Avatar	LOTR	Matrix	Pirates
Alice	1		0.2	
Bob		0.5		0.3
Carol	0.2		1	
David				0.4

# Key Problems

- **(1) Gathering “known” ratings for matrix**
  - How to collect the data in the utility matrix
- **(2) Extrapolate unknown ratings from the known ones**
  - Mainly interested in high unknown ratings
    - We are not interested in knowing what you don't like but what you like
- **(3) Evaluating extrapolation methods**
  - How to measure success/performance of recommendation methods



# (1) Gathering Ratings

## ■ Explicit

- Ask people to rate items
- Doesn't work well in practice – people can't be bothered

## ■ Implicit

- Learn ratings from user actions
  - E.g., purchase implies high rating
- What about low ratings?

# (2) Extrapolating Utilities

- **Key problem:** Utility matrix  $U$  is **sparse**
  - Most people have not rated most items
  - **Cold start:**
    - New items have no ratings
    - New users have no history
- **Three approaches to recommender systems:**
  - 1) Content-based
  - 2) Collaborative
  - 3) Latent factor based

# Content-based Recommender Systems

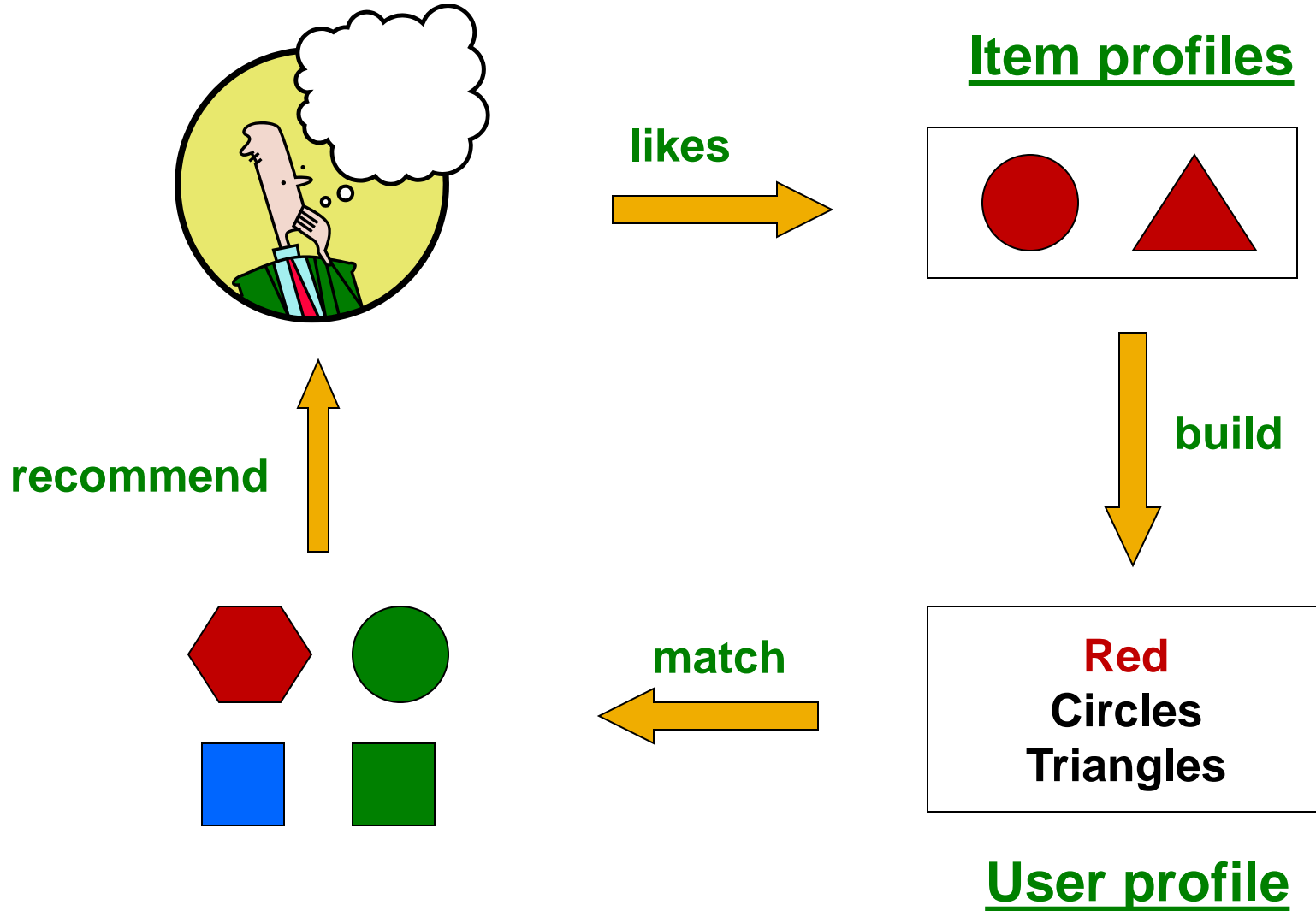
# Content-based Recommendations

- **Main idea:** Recommend items to customer  $x$  similar to previous items rated highly by  $x$

## *Example:*

- **Movie recommendations**
  - Recommend movies with same actor(s), director, genre, ...
- **Websites, blogs, news**
  - Recommend other sites with “similar” content

# Plan of Action



# User Profiles and Prediction

- **User profile possibilities:**

- Weighted average of rated item profiles
- **Variation:** weight by difference from average rating for item
- ...

- **Prediction heuristic:**

- Given user profile  $\mathbf{x}$  and item profile  $\mathbf{i}$ , estimate

$$u(\mathbf{x}, \mathbf{i}) = \cos(\mathbf{x}, \mathbf{i}) = \frac{\mathbf{x} \cdot \mathbf{i}}{\|\mathbf{x}\| \cdot \|\mathbf{i}\|}$$

# Collaborative Filtering

Harnessing quality judgments of other users

# Collaborative Filtering

- Consider user  $x$
- Find set  $N$  of other users whose ratings are “similar” to  $x$ ’s ratings
- Estimate  $x$ ’s ratings based on ratings of users in  $N$

