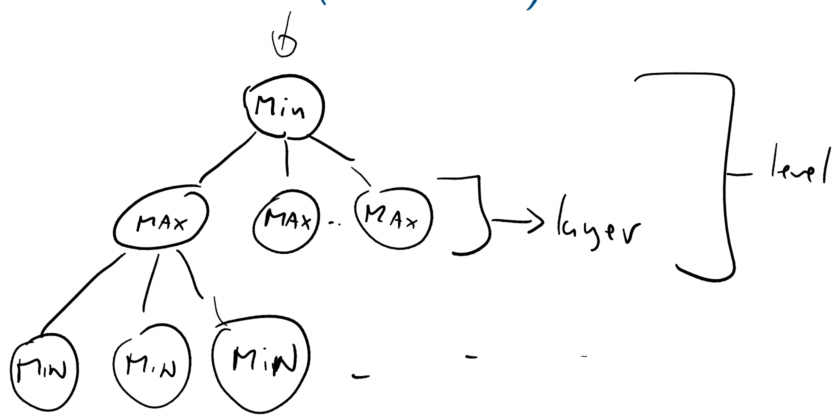
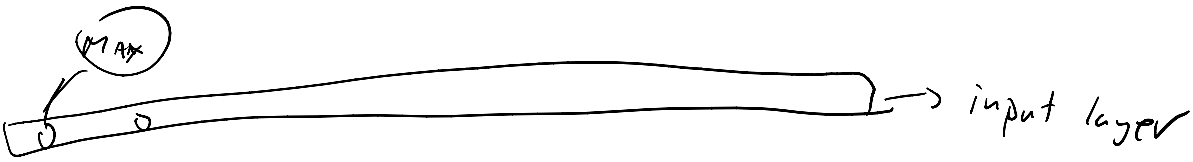
Tutorial 4

15 March 2023 14:05

Game tree evaluation

Min-Max trees (definition)





$T_{k,d}$ - tree with d- children of each non-leaf node

k- levels $2k+1$ layers

Leaves of the tree (input layer) are labeled with numbers $\in \{0, .., d-1\}$

How many leaves does a $T_{k,d}$ tree have? d^{2k}

Input layer is a string of length d^{2k} numbers $\{0, .., d-1\}$

- → Each MIN node contains the value of the smallest of its children
- → Each MAX node contains the value of the largest of its children
- → the goal is to evaluate the root.

T_{k} - binary trees with $2^{2k} = 4^k$ bits

$T_{k,2}$ - binary trees with $2^{2k} = 4^k$ bits

min → $\wedge$

max → $\vee$

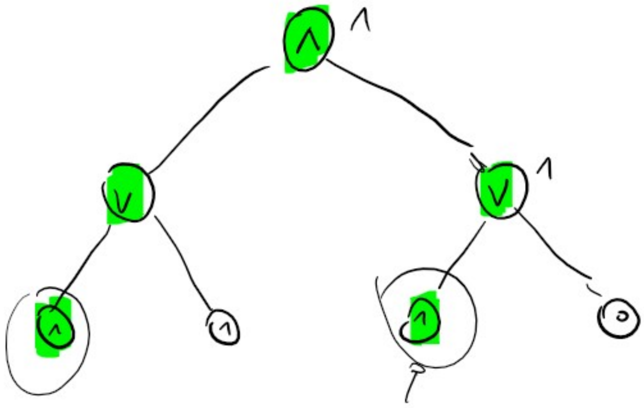
$\wedge$

1	1	1
1	0	0
0	1	0
0	0	0

$\vee$

1	1	1
0	1	1
1	0	1
0	0	0

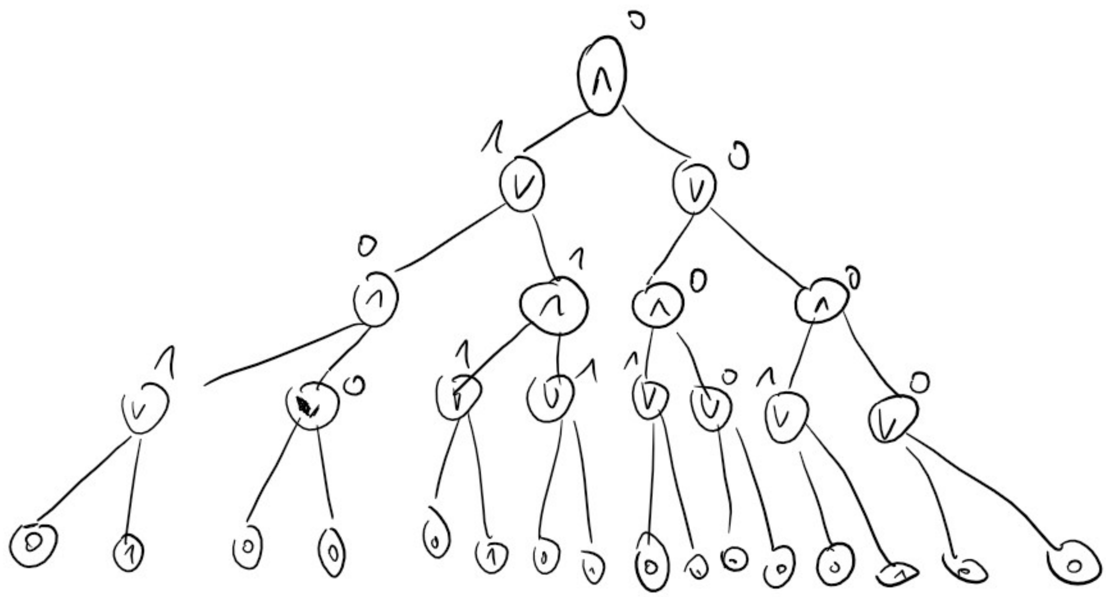
Deterministic evaluation: 'depth first', 'left child first'



1 layer 2

k layers 2^k

$2(k+1)$ $2^k + 2^k = 2 \cdot 2^k = 2^{k+1}$



All 32 leaves need to be accessed.

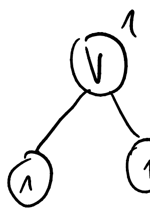
$\Rightarrow$ the difficulty of any algorithm is $O(4^k)$

$\boxed{n = 4^k}$

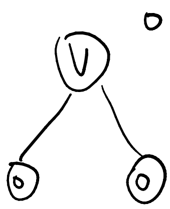
Randomized algorithm - chose a child at random

Expected complexity $O(3^k)$ $\left(n^{0{,}79\ldots}\right)$

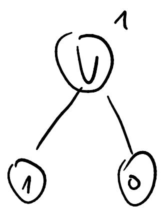
$\boxed{\text{Induction basis}}$



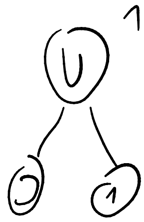
1 evaluation



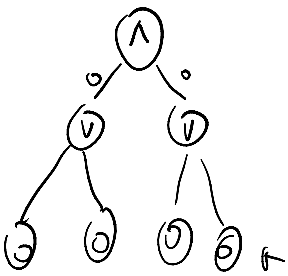
2 evaluations



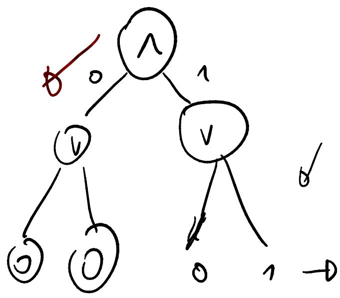
$\frac{1}{2} \cdot 1 + \frac{1}{2} 2 = 3/2$



$= 3/2$



2 evaluations



$0 \quad 1 \to 3/2$

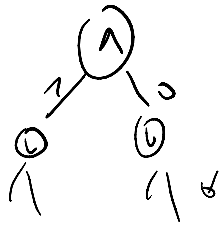
$1 \quad 0 \to 3/2$

$1 \quad 1 \to 1 < 3/2$

$\frac{1}{2} \cdot 2 + \frac{1}{2} \cdot (3/2 + 2)$

$\parallel$

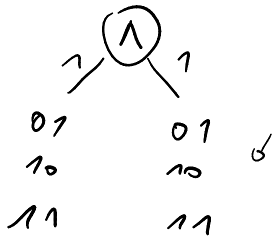
$\left(1 + \frac{7}{4}\right) < 3$



Symmetric

$\longleftarrow$

< 3



$\frac{1}{2}\left(3/2 + 3/2\right) + \frac{1}{2}\left(\frac{3}{2} + \frac{3}{2}\right)$

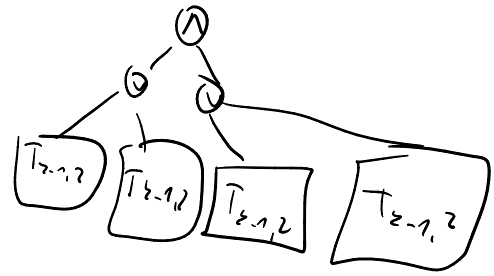
$\shortparallel$

3

$\boxed{\text{AVG} \leq 3}$

IH: $T_{k-1,2}$ tree can be evaluated with our randomized algorithm by accessing less than 3^{k-1} leaves on avg.

IS: $T_{k,2}$



By I.H. each of the subtrees needs on avg 3^{k-1} leaf evaluations

By 1.B. we need to evaluate on average 3 of these subtrees

$3 \cdot 3^{k-1} = 3^{k}$ leaves.