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Introduction to Complex Networks, Basic Description

IV124

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Complexity, complex system

Complex system

- many units interacting with each other
- nontrivial interaction structure

Many shared properties

- discrete entities connected by (binary) connections (links)
- emergent properties which are more then just a summation of sub-parts
- important properties on many scales (multi-scale systems)

Examples of complex systems

Natural systems

- biological networks: gene expression, metabolic networks,…
- inter-species interaction: food network

Social systems

- social networks: friendship, communication,…
- science: citation network
- economic network: world trade

Technological systems

- internet, electrical network, transport network

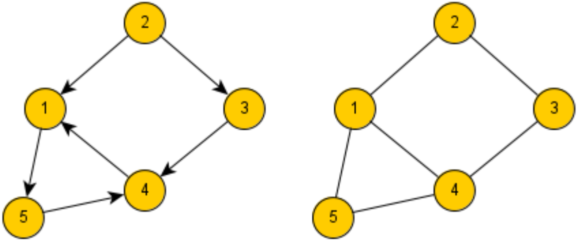
Complex Network Analysis

- How?
  Data collection $\rightarrow$ Analysis + Visualization
- Why?
  Insight + Prediction

Graphs and networks

Graph $G = (V, E)$:

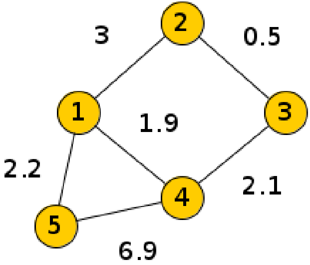
- V set of vertices
- $E \subseteq V \times V$ set of directed edges
- or $E \subseteq \{\{a, b\}; a, b \in V\}$ undirected edges



Graphs and networks

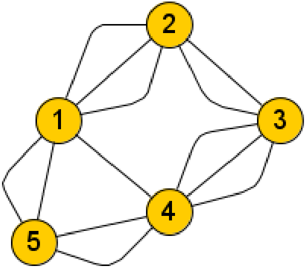
Weighted graph $G = (V, E, w)$

- weight function $w : E \to \mathbb{R}$



Multigraph $G = (V, E)$

- E is a multiset



Link strength

Strong links

- Structure
- Stability in time
- Resistant to change
- Short range – low prob. of new info
- Removal = high impact

Weak links

- Flexibility, adaptability
- High rate of fluctuance
- Susceptible to change
- Long range - high prob. of new info
- Removal = insignificant

Optimal link ratio

20 strong : 80 weak

- Ensures "dynamic stability"
- Adaptability
- Flexibility
- Resistance to stress
- Ability to relax
- Too little weak links: rigidity, fragility
- Too many weak links: instability, chaotic behavior

Graphs and networks – examples

World trade

- oriented, weighted (sum of transactions)

Social networks – friendship

- undirected, unweighted

Protein interaction

- undirected, unweighted, loops

Organization social network

- undirected, weighted multigraph (multilayer network)

Terminology

- edge vs. link / connection
- vertex vs. node / agent

A term *graph* is usually used in case of general mathematical apparatus, in models of specific systems we mostly use a term *network*.

Definition of a network describing a real system

What are the nodes? What are the edges?

- more possibilities of abstraction over the same dataset
- depends on a specific system, available data and research questions

For the same set of nodes we can create different networks

- frequency of email communication: cooperation network
- friendship network
- frequency of in-person meetings: epidemiology network

Network analysis – relevant questions

Network structure understanding

- which nodes are important in the network?
- do the nodes form clusters (communities)?
- is there any regularity in the structure of connections?

Study of network evolution

- how was the network created? How does it grow?
- Is there a suitable model for our network?

Dynamic processes on networks

- how does a diffusion of information or disease evolves?
- what is a temporary profile of a communication between nodes?

We will look into these topics in future lectures.

Data structures

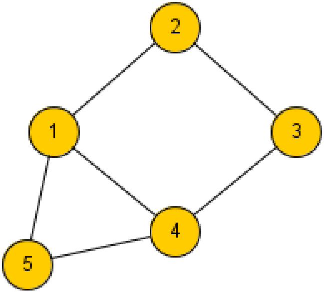
List of neighbors:

- list of neighbors for each node

Adjacency matrix

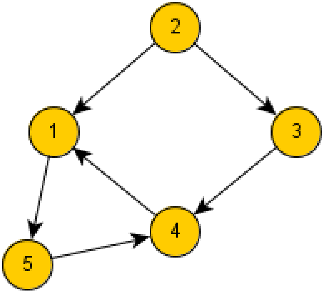
- rows – nodes (from), columns – nodes (to)
- 1 equals to existence of an edge, otherwise 0
- weighted graphs: an edge weight instead of 1
- diagonal: loops

Adjacency matrix – example



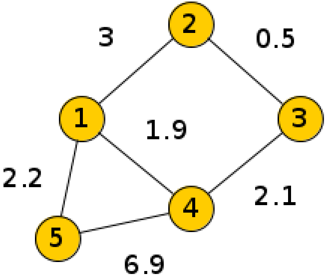
$$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Adjacency matrix – example



$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Adjacency matrix – example



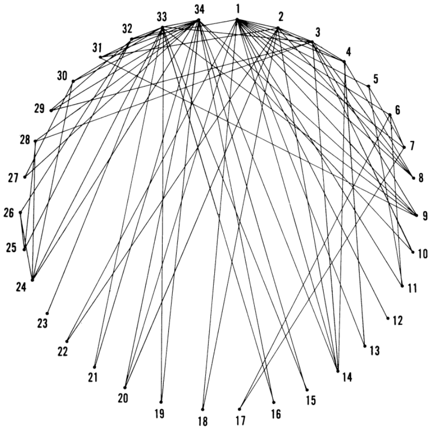
$$\begin{pmatrix} 0 & 3 & 0 & 1.9 & 2.2 \\ 3 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 2.1 & 0 \\ 1.9 & 0 & 2.1 & 0 & 6.9 \\ 2.2 & 0 & 0 & 6.9 & 0 \end{pmatrix}$$

Zachary Karate Club

- Domain: Anthropology
- Paper: *Wayne W. Zachary: An Information Flow Model for Conflict and Fission in Small Groups*
- Background story:
  - New instructor introduced; after some time, he proposes increased training fee
  - Club President refuses, seeing this as his intent to make more money
  - Both figures have their own supporters
  - Tension escalates; the club splits into two

Karate Club Network

Social Network Model of Relationships in the Karate Club



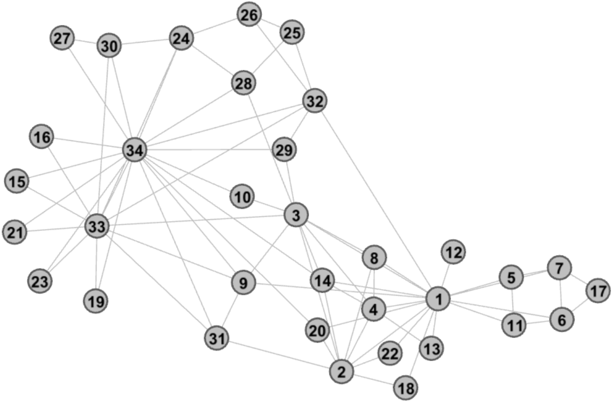
Links present if members interact outside karate lessons

Karate Club Adjacency Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
1	0	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0
2	1	0	1	1	0	0	0	1	0	0	0	0	0	1	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	1	0	0	0
3	1	1	0	1	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0
4	1	1	1	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1
10	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
11	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
17	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
20	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
22	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	1	1
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
28	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1
29	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	1
31	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
32	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	1	1
33	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	0	1	0	1	1	0	0	0	0	0	1	1	1	0	1
34	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	1	1	1	1	1	1	0

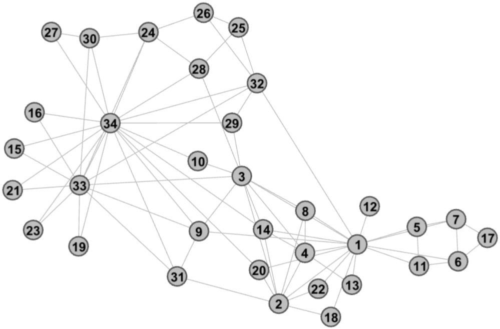
Karate Club Adjacency Matrix

Gephi - Force Atlas Layout

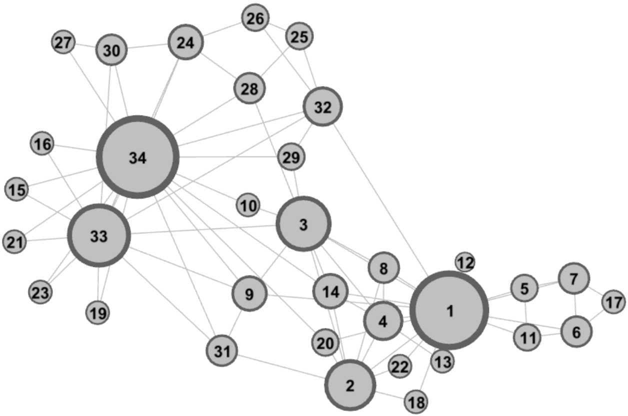


General observations

- Giant component, no submodules
- Unweighted, undirected, unreflexive network
- $N = 34, L = 78$

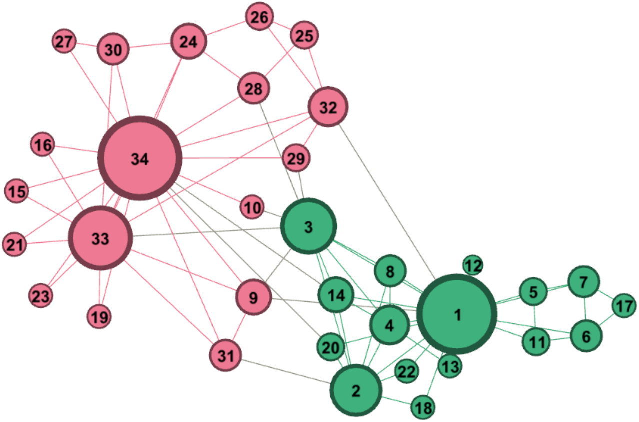


Karate Club - Node Degree Centrality



avg. degree = 4.588

Karate Club - Modularity



pink ≈ 53 %, green ≈ 47 %

Available tools

- Matlab or R
- Networkx for Python
- Open-Source Desktop Apps:
  - CytoScape (bioinformatics)
  - Gephi (swiss knife)
  - SocNetV (built-in web crawler)
- Specific purpose: Netlytic
  - web-based
  - text and network analysis of public discussion media (Reddit, Twitter, YouTube, etc.)
- Many more...

Gephi Demo

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