Processes on A Network: Diffusion

IV124

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Processes on Networks: Diffusion

Similar principles in different contexts

- Technical networks: cascading failures
- Biological networks: epidemics
- Social networks: opinion formation, information spreading

Today: models of these processes.

Diffusion: Basic Concepts and Principles

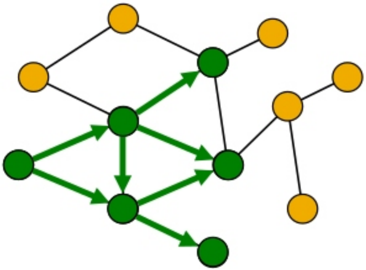
Components of the model

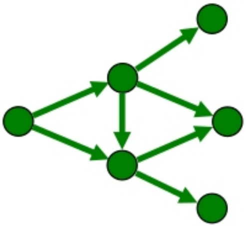
- What is spreading: infection, information, choice, etc.
- Time-point of spread: change of choice, infection, failure, etc.
- Outcome: epidemic, group-decision, excluded nodes, etc.

We consider a discrete time domain – the model evolves in iterative steps. We model a dynamic process on a static network.

Cascade on a Network

One run of the model on a network forms a directed graph – a cascade.





[1]

[1] Leskovec CS224W: Machine Learning with Graphs
http://web.stanford.edu/class/cs224w/

Forms of Diffusion/Infection

Simple spreading

- Each node infects its neighbors with a certain probability at each step

Complex spreading

- Spreading occurs only if a certain fraction of neighboring nodes are *infected*

Coordination Game on a Network

Task:

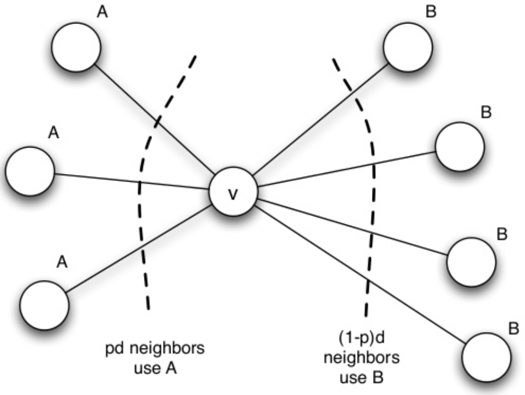
- Choose between A and B (e.g., VHS vs. Betamax, iPhone vs. Samsung)

Rewards for neighboring nodes u and v:

- Both A: payoff $a > 0$
- Both B: payoff $b > 0$
- Disagreement: no payoff

Each node plays on its own. Spreading is monotonic (nodes do not take their decision back).

Threshold for Changing Decision



[2]

[2] Kleinberg, ch. 19.

Threshold for Changing Decision

A is a better choice if

$$pda \geq (1 - p)db$$

Thus:

$$p \geq \frac{b}{a + b} = q$$

Coordination Game - Properties

Equilibrium states:

- Everyone chooses A
- Everyone chooses B
- Incomplete cascade

The initiation of a cascade depends on the topology of the network, initial conditions, and the value of q.

Cascades vs. Clusters

Clusters represent an obstacle for cascades

- dense internal connectivity
- small number of edges to the rest of the graph

Density ρ of cluster $C \subseteq G$:

- each node $u \in C$ has at least fraction ρ of edges in C

Cascades vs. density:

- cascades cannot propagate to clusters with $\rho > (1 - q)$
- conversely: if the cascade stops, there is a cluster in the graph with $\rho > (1 - q)$

Cascades vs. Weak Ties

Reminder: weak ties are bridges between communities

Role in cascades

- key for information diffusion (e.g. awareness of innovation)
- impenetrable to higher threshold phenomena (e.g. actual adoption of innovation)
- e.g. rapid global dynamics of sharing on social networks vs. slow and local dynamics of political mobilization

Extensions to Coordination Game

Bilingual nodes

- a node can choose state *AB*
- reward *AB*-*A*: a
- reward *AB*-*B*: b
- reward *AB*-*AB*: $max(a, b)$
- nodes choosing *AB* additionally pay fixed cost c

Heterogeneous thresholds

- allows to incorporate differences in susceptibility

Netlogo Demo

...

Models of Spreading: Epidemics

So far: Complex spreading

- Spreading via the majority of neighbors
- Sociological applications

Now: **Simple spreading**

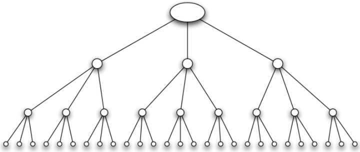
- Stochastic model
- Applications in biological and technical networks

Note on Stochastic Models

Simple deterministic models can be made more complex (by adding rules)

- Increases the range of possible behaviors
- Analysis becomes more demanding
- At some point, it becomes easier to summarize a large number of real-world events into a single variable

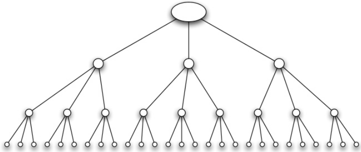
Branching Processes



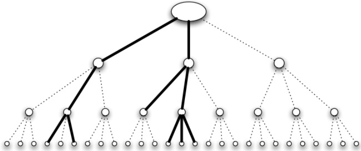
(a) The contact network for a branching process

The simplest tree model

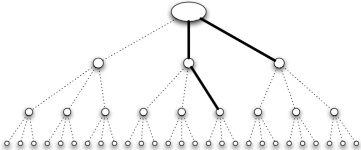
- Patient zero enters population and meets k individuals
- Probability of transmission upon meeting is p
- k and p remain constant in every subsequent wave
- Results in a tree of contacts between potentially infected individuals and the subtree of actual infection



(a) *The contact network for a branching process*



(b) *With high contagion probability, the infection spreads widely*



(c) *With low contagion probability, the infection is likely to die out quickly* [3]

[3] Easley and Kleinberg 2010

Branching Processes

Possible outcomes

- Infection stops after a while (dies out)
- Large epidemic

Reproduction number R_0

- Expected number of new infections caused by a single individual
- Describes the viability and aggressiveness of the infection
- Here, $R_0 = pk$

Branching Processes

Development depending on R_0:

- $R_0 \ll 1$: rapid end of spread
- $R_0 \gg 1$: aggressive epidemic
- $R_0 \approx 1$: the extent of the infection can vary significantly between runs; even small changes in the spread mechanism determine the outbreak of the epidemic

SIR Model

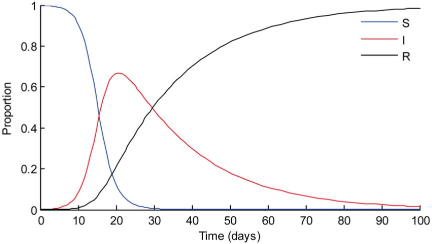
Three consecutive states of a node:

1. **S**usceptible: susceptible to infection from neighbors
2. **I**nfectious: infected node spreading the disease for t_I steps
3. **R**emoved (**R**ecovered): immune/dead node

In each step, nodes in state I transmit the disease to all their neighbors with probability p.

Classic Epidemiological Models: SIR model

Assumes possible contact between subject and any other population member. Defined by differential equations:

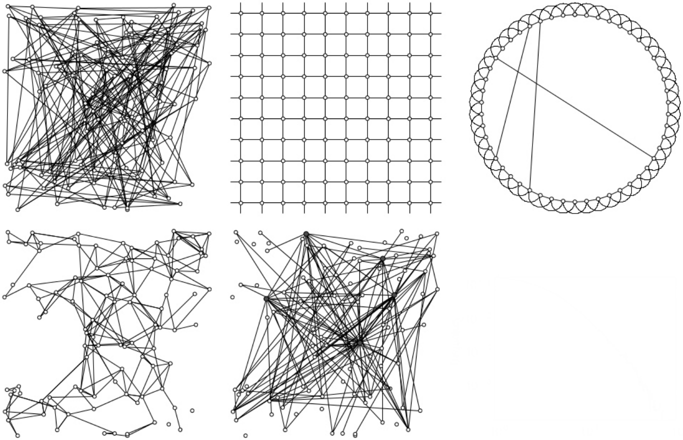


[a]

$$\frac{dS}{dt} = -\frac{\beta SI}{N}$$
$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

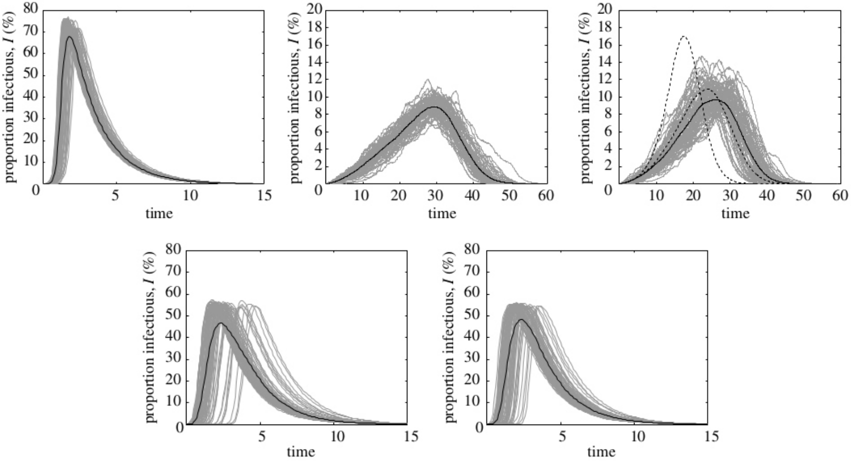
[a] Luz, P., et al (2010)

SIR vs. Networks[4]



[4] Keeling & Eames 2005

SIR vs. Networks[5]



[5]Keeling & Eames 2005

SIR Model: Extensions

The dynamics are simple (branching process on a network)

Possible extensions:

- Weighted graph: non-homogeneous transmission probability p
- Non-homogeneous I_t
- Division of I into more detailed categories: infectious incubation, less infectious period with symptoms, ...

SIS Model

We allow for repeated infection.

1. **S**usceptible: susceptible to infection from neighbors
2. **I**nfectious: infected node spreading infection for t_I steps
3. **S**usceptible

Compared to the SIR model, SIS allows for very long runs on a finite network.

SIRS Model

In the occurrence of real diseases, we observe significant oscillations, which are not captured even by the SIS model.

We add time-limited immunity.

1. **S**usceptible: susceptible to infection from neighbors
2. **I**nfectious: infected node spreading infection for t_I steps
3. **R**ecovery: recovered and immune node for t_R steps
4. **S**usceptible

Global vs. Local Oscillations

SIRS exhibits local oscillations on a general network.

Global oscillations

- require homophilic (local) links and long-range shortcuts
- correspond to the characteristic of small worlds
- the specific dynamics is closely related to the network topology

SIRS and Small Worlds

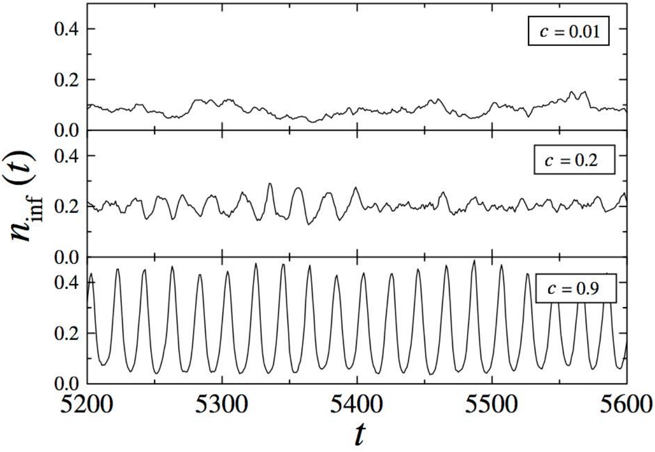
Small-world Watts-Strogatz network model (reminder):

- ring with local connections; with probability c, the edges are rewired to a random target

SIRS dynamics

- global oscillations (synchronization) depend on the number of *shortcuts* – weak links
- small c – local infection, large c – global oscillations

SIRS and Small Worlds



[6]

[6] Kiperman et al. 2001

Demos

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