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Spatially Embedded Networks

IV124

Josef Spurný & Eva Výtvarová

Faculty of Informatics, Masaryk University

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Outline

- temporal networks demo
- miscellaneous metrics
- spatially embedded networks

Temporal Networks

Demo...

Graph Spectral Analysis

Derived from Laplacian matrix Q:

- $Q = \Delta - A$
- Δ ...degree matrix (the diagonal matrix with the nodal degrees)
- A ...adjacency matrix

- can be written in terms of their eigenvectors and corresponding eigenvalues, $Q = X \Lambda X^T$
  - X ...eigenvectors in columns
  - Λ ...diagonal matrix with corresponding eigenvalues
  - X and Λ contain the spectral information
- the spectrum of a graph regarded as a fingerprint
- eigenvalues in Λ ...precise information about network properties
- can be used to classify network topologies.

Graph Spectral Analysis

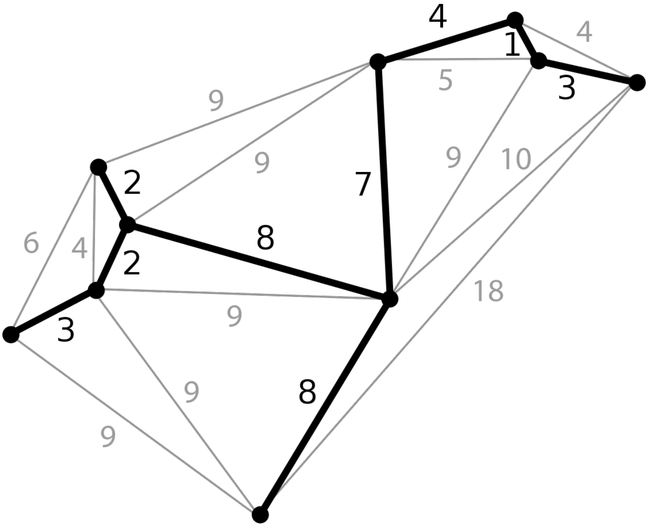
Algebraic connectivity (Fiedler eigenvalue)

- measures how difficult it is to tear a network apart (how well connected the overall graph is)
- measure for network robustness and synchronizability
- if the network is connected, the algebraic connectivity is greater than 0
- is equal to the second-smallest eigenvalue of the Laplacian matrix

Minimum Spanning Tree (MST)

- sub-network of the original weighted network
- without loops
- $M = N - 1$ edges
- Kruskal's, Prim's, Boruvka's algorithms
- leaves: nodes with one edge
- leaf number L: number of nodes with 1 edge
- MST diameter d: longest shortest path of an MST, d_{max} is related to L
- two extreme tree topologies:
- path-like configuration: leaf number = 2
- star-like configuration: leaf number = $N - 1$

Minimum Spanning Tree



Minimum Spanning Tree Topology[1]

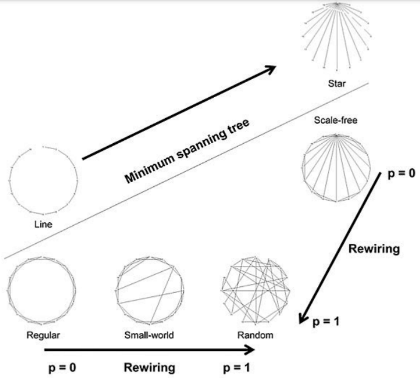


Figure 9: Schematic presentation of MST topology in relation to the topology of well known network models. The schema shows how the MST reflects the underlying network topology for regular, small-world, random, and scale-free networks. Our analyses indicate that MST metrics are strongly related to the underlying network topology. For alterations on the scale of regular to random networks, MST diameter and leaf number change similarly to the path length for the underlying network; MST diameter is positively correlated to the path length, and MST leaf number is negatively correlated to the path length. When a scale-free network is randomized, the degree of the MST reflects the degree of the underlying network.

[1] Tewarie, 2014

Minimum Spanning Tree In Multiple Sclerosis[2]

Functional networks using minimum spanning trees in MS

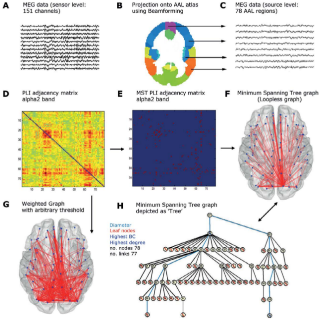


Figure 1: Overview of the applied methods. MEG data at the sensor level (A) were projected onto an AAL atlas using beamforming (B), resulting in 78 time series for each ROI (C). An 78x78 adjacency matrix is constructed for each frequency band separately based on the functional connectivity between each pair of time series (D). PLI was used as functional connectivity metric. Subsequently, Kruskal's algorithm was applied to obtain the MST matrix (E). This MST, a loopless graph, is displayed as a graph on a template (F). For illustration purposes only, a weighted graph obtained by applying an arbitraty threshold can be seen in G. This graph contains more connections and loops. F illustrates the MST as a tree structure. The node with the lowest eccentricity is placed on top. In addition, leaf nodes are depicted in red; the node with the highest degree (most connections) and BC in dark blue (node with highest BC and degree are not necessarily the same); and the diameter in light blue.

Disruption of structural and functional networks in MS

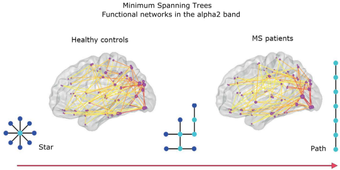


Figure 5 MST results for functional networks in the alpha2 band: For illustrative purposes the average MSTs across subjects for MS patients and healthy controls are depicted. The diameter of the circles in the glass brains is proportional to the degree of the nodes in the MST; the color of the lines connecting the circles indicates the strength of the functional connections, with warmer colors indicating stronger connections. The MST for MS patients was characterized by a shift towards a more path-like topology reflected by a lower leaf fraction, lower degree divergence and lower tree hierarchy.

[2] Tewarie, 2014

Spatially Embedded Networks

Networks with a spatial dimension:

- at the interface of geography
- node position in space plays a role

For example

- transport infrastructure
- circulatory systems (blood vessels, leaf veins)

Transport Networks – Properties

Note:

- often planar graphs – can be drawn without crossing edges
- not always: e.g., air transport

Node degree distribution $P(k)$

- establishing connections associated with costs $\implies$ cutoff in node distribution $P(k)$
- for planar networks, $P(k)$ is also very peaked due to spatial constraints

Transport Networks – Properties

Clustering coefficient C

- nearby nodes have a higher probability of connection: higher clustering coefficient

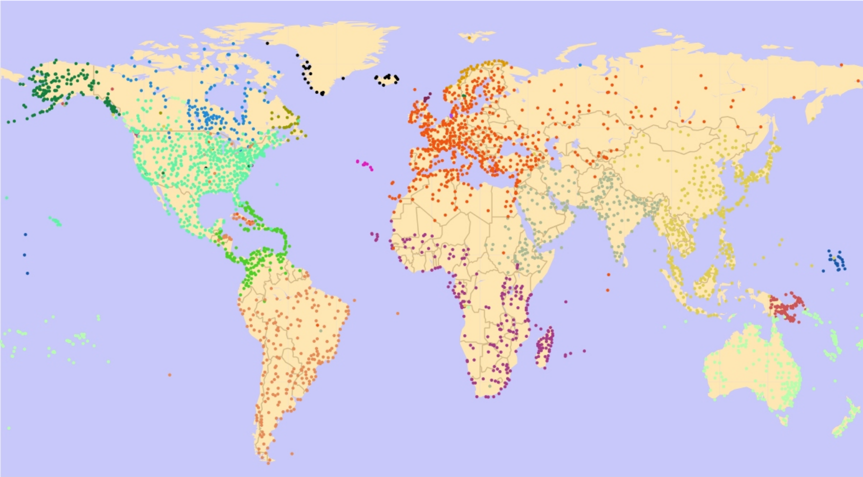
Betweenness centrality

- a natural metric of node importance
- homogeneous on the grid: grows from the periphery to the center
- shortcuts cause significant heterogeneity

Air Transport – Properties

- nodes: airports, edges: direct flights
- the network is spatial, not planar
- scale-free and small-world
- cutoff $P(k)$ – physical limits of airport capacity
- strong correlation between node degree and traffic volume
- strong correlation between node degree and range of transport
- betweenness and degree do not correlate
- communities determined by geographical and political factors

Airports

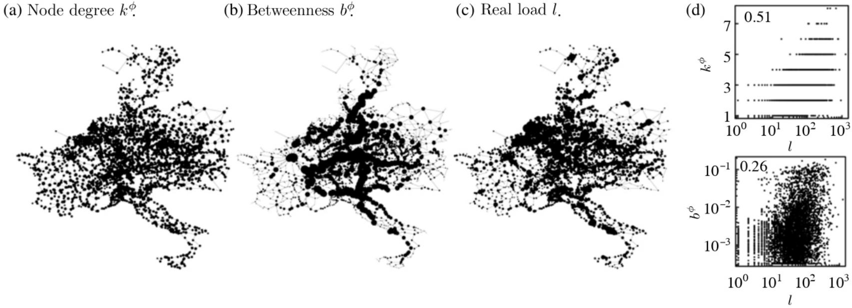


Airline Network[3]

[3] Sun, 2018

Public Transport

A real load may not correspond to betweenness:



Commuting And Migration

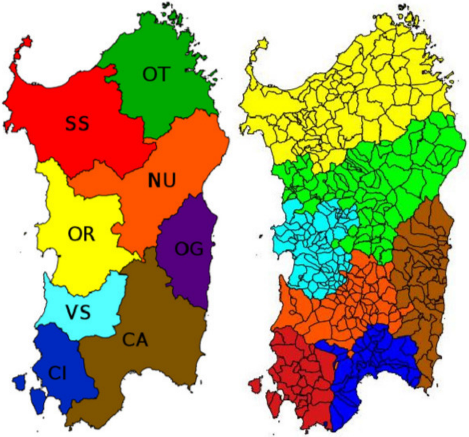
Population movements:

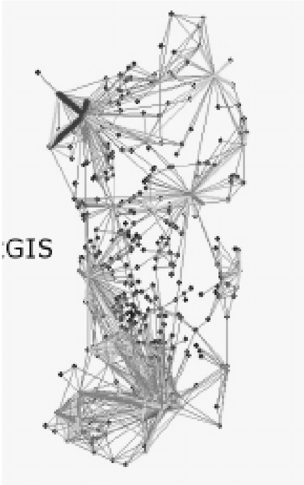
- nodes: destinations, weighted edges: movement of people
- a very interesting topic with overlaps in economics and sociology

Gravity law: $T_{ij} = K \frac{P_i P_j}{d_{ij}^{\sigma}}$

- where d_{ij} is physical distance, P is population, and σ depends on the system and T_{ij} is the migration rate
- verified on many datasets

Commuting And Migration – Communities[4]





[4]Sardinia; Caschili, 2010

Energy Infrastructure

Power distribution network

- very important and extensive network
- gradual development – complex system
- incomplete understanding of behavior, the possibility of cascading failures
- exponential $P(k)$, high clustering

Water distribution

- sparse planar network
- very peaked $P(k)$

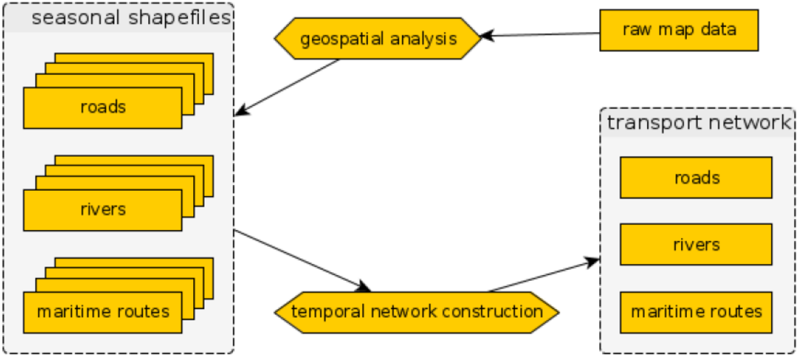
Internet

Global information infrastructure

- nodes: autonomous systems or routers
- edges: connections at L1-L3 ISO OSI
- one level higher: a network of hyperlink references

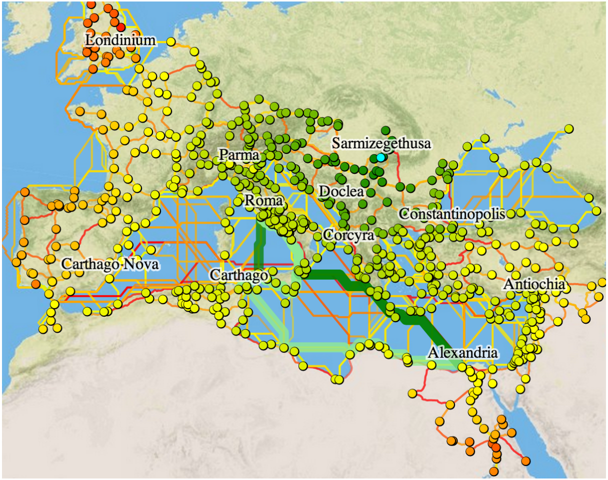
- scale-free network
- router placement correlates with population density
- generative model includes a combination of spatial factors and preferential attachment

Historic Networks[5]



[5] Fousek,2016

Historic Networks[6]



[6] Meeks, 2013, https://digitalhumanities.stanford.edu/orbis-v2/

Historic Networks[7]

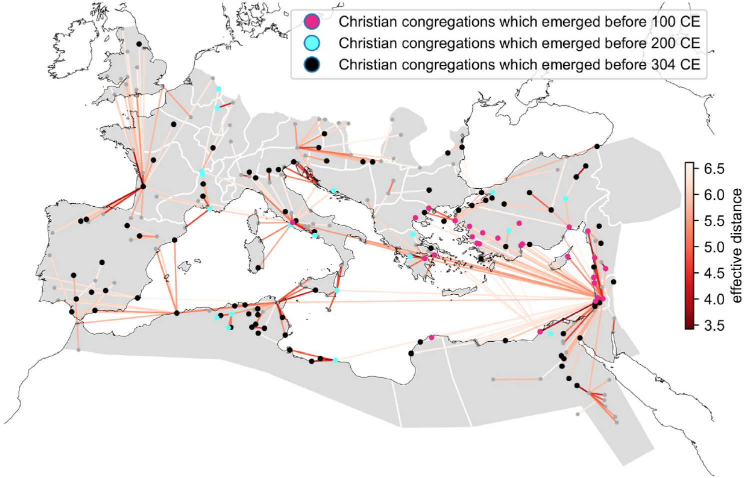
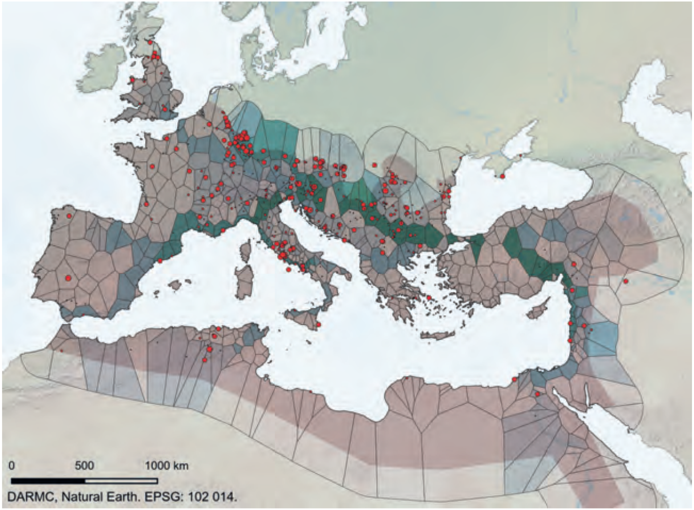


Fig 3. A geographical view of the shortest effective distance tree from Jerusalem.

[7] Fousek, 2018

Historic Networks[8]



[8] Chalupa, 2021

Recommended reading

Barthélemy, M. (2011). Spatial networks. Physics Reports, 499(1), 1-101.

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