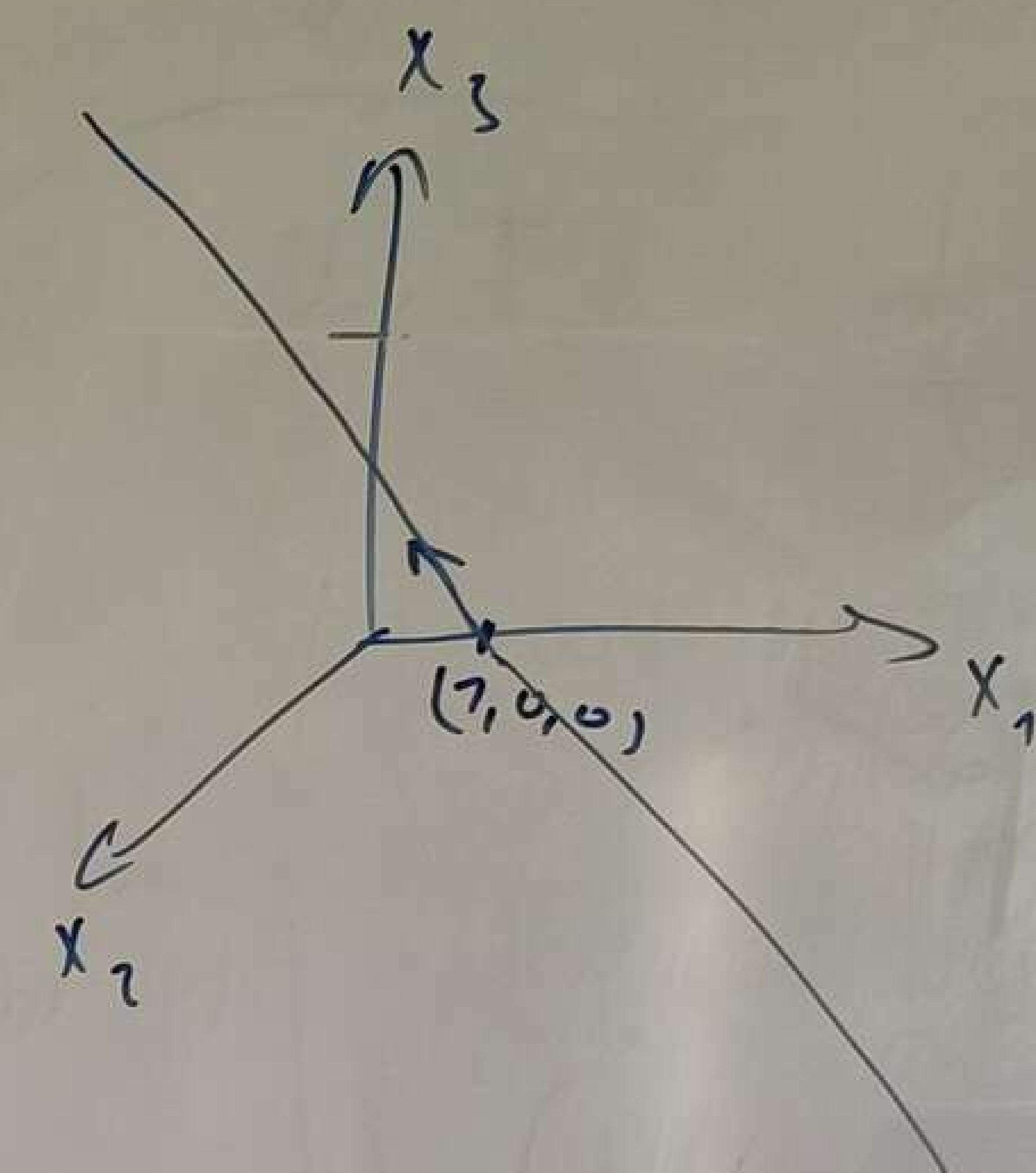
$$\begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & -2 & 2 & 2 \\ 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 \end{pmatrix}$$

$4 \times 1 \qquad 1 \times 4 \qquad 4 \times 4$



$$x_1 + 2x_2 + x_3 = 1$$
$$2x_1 + x_2 + 2x_3 = 2$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & 1 & 2 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -3 & 0 & 0 \end{array}\right) \sim \begin{array}{c} x_1 \; x_2 \; x_3 \\ \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{array}\right) \end{array}$$

$x_2 = 0$
$x_3 = t$
$x_1 = 1 - x_3 = 1 - t$

$$(x_1 \; x_2 \; x_3) = (1-t, 0, t) = (1,0,0) + t(-1,0,1)$$

mod 3 $\qquad \mathbb{Z}_3$

$x_1 + 2x_2 + x_3 = 1$

$2x_1 + x_2 + 2x_3 = 2$

$\begin{pmatrix} [1]_3 & [2]_3 & 1 & | & 1 \\ 2 & 1 & 2 & | & 2 \end{pmatrix} \overset{/\cdot 2}{\sim} \begin{pmatrix} 2 & 4 & 2 & | & 2 \\ 2 & 1 & 2 & | & 2 \end{pmatrix}$

$\equiv \begin{pmatrix} 2 & 1 & 2 & | & 2 \\ 2 & 1 & 2 & | & 2 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 2 & | & 2 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} /\cdot 2$

$\sim \begin{pmatrix} 1 & 2 & 1 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$ $\quad x_3 \equiv t \quad t \in \mathbb{Z}_3 = \{[0]_3, [1]_3, [2]_3\}$

$x_2 \equiv s \quad s$ —— 1 ——

$x_1 \equiv 1 + s - t$

$(x_1, x_2, x_3) = (1 + s - t, s, t) = (1, 0, 0) + s(1, 1, 0) + t(-1, 0, 1)$

<u>Př.</u> $x_1 + 2x_2 = 1$

$2x_1 + 4x_2 = 0$

$\begin{pmatrix} 1 & 2 & | & 1 \\ 2 & 4 & | & 0 \end{pmatrix} \sim$

$\sim \begin{pmatrix} 1 & 2 & | & 1 \\ 0 & 0 & | & -2 \end{pmatrix}$

$\Rightarrow$ nemá řešení

$$A = \begin{pmatrix} 3 & 5 & 6 & 7 \\ -2 & 1 & 0 & 5 \\ 1 & 3 & -2 & 1 \end{pmatrix}$$

e... postupná aplikace EŘO:

1) vynásobení 2. řádku č. 3

2) výměna 1. a 3. ř.

3) ke 3. řádku přičteme 2x 1. řádek

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$E \cdot A = A \cdot E = A$$

Ukažte, že $e(E) \cdot A = e(A)$ ✓

$$e(E): \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix} = e(E)$$

$$e(E) \cdot A = \underbrace{\begin{pmatrix} 0 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}}_{3 \times 3} \cdot \underbrace{\begin{pmatrix} 3 & 5 & 6 & 7 \\ -2 & 1 & 0 & 5 \\ 1 & 3 & -2 & 1 \end{pmatrix}}_{3 \times 4} = \begin{pmatrix} 1 & 3 & -2 & 1 \\ -6 & 3 & 0 & 15 \\ 5 & 11 & 2 & 9 \end{pmatrix}$$

$$e(A): \begin{pmatrix} 3 & 5 & 6 & 7 \\ -2 & 1 & 0 & 5 \\ 1 & 3 & -2 & 1 \end{pmatrix} \sim \begin{pmatrix} 3 & 5 & 6 & 7 \\ -6 & 3 & 0 & 15 \\ 1 & 3 & -2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & -2 & 1 \\ -6 & 3 & 0 & 15 \\ 3 & 5 & 6 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & -2 & 1 \\ -6 & 3 & 0 & 15 \\ 5 & 11 & 2 & 9 \end{pmatrix} = e(A)$$

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}\cdot\begin{pmatrix}a&b&c\\d&e&f\\g&h&i\end{pmatrix}=\begin{pmatrix}d&e&f\\a&b&c\\g&h&i\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&x\end{pmatrix}\cdot\begin{pmatrix}a&b&c\\d&e&f\\g&h&i\end{pmatrix}=\begin{pmatrix}a&b&c\\d&e&f\\x\cdot g&x\cdot h&x\cdot i\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\0&1&0\\0&x&1\end{pmatrix}\cdot\begin{pmatrix}\ &\ &\ \end{pmatrix}=\begin{pmatrix}a&b&c\\d&e&f\\x\cdot d+g&x\cdot e+h&x\cdot f+i\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\0&1&0\\2&0&1\end{pmatrix}\cdot\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}\cdot\begin{pmatrix}1&0&0\\0&3&0\\0&0&1\end{pmatrix}A=e(A)$$

$$(A\,|\,E)\sim(E\,|\,A^{-1})$$

e … EŘO, které udělají z A matici E

$$e(E)\cdot A=e(A)=E$$

Spočtěte inverzní matici k matici a udělejte zkoušku.

$$A=\begin{pmatrix}1&1&2\\1&-1&-3\\2&1&2\end{pmatrix}\cdot\begin{pmatrix}-1&0&1\\8&2&-5\\-3&-1&2\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\ \checkmark$$

$-1x$, $-2x$

$$\left(\begin{array}{ccc|ccc}1&1&2&1&0&0\\1&-1&-3&0&1&0\\2&1&2&0&0&1\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1&1&2&1&0&0\\0&-2&-5&-1&1&0\\0&-1&-2&-2&0&1\end{array}\right)\cdot(-1)$$

$-1x$, $+2x$

$$\sim\left(\begin{array}{ccc|ccc}1&1&2&1&0&0\\0&1&2&2&0&-1\\0&-2&-5&-1&1&0\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1&0&0&-1&0&1\\0&1&2&2&0&-1\\0&0&-1&3&1&-2\end{array}\right)$$

$+2x$, $\cdot(-1)$

$$\sim\left(\begin{array}{ccc|ccc}1&0&0&-1&0&1\\0&1&0&8&2&-5\\0&0&1&-3&-1&2\end{array}\right)$$

A^{-1}

(A)

$x_1 + 2x_2 = 1$

$x_1 - x_2 = 0$

$$\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$A \cdot \mathbf{x} = \mathbf{b} \quad / \cdot A^{-1}$

Předp., že $\exists A^{-1}$:

$\underbrace{A^{-1} \cdot A} \cdot \mathbf{x} = A^{-1} \cdot \mathbf{b}$

$\mathbf{x} = E \cdot \mathbf{x} = A^{-1} \cdot \mathbf{b}$

$\Rightarrow \exists!$ řešení soustavy r.

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -3 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & +\frac{1}{3} & -\frac{1}{3} \end{array}\right)$$

$$\sim \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} \end{array}\right)$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} +\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{-1}{3} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$$

předp.

$\underline{i < j}$

$$(A \cdot B)_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} = \sum_{k=1}^{i} \underbrace{a_{ik}}_{=1} \cdot \underbrace{b_{kj}}_{=1} +$$

$$+ \sum_{k=i+1}^{j} \overbrace{a_{ik}}^{=2} \overbrace{b_{kj}}^{1} + \sum_{k=j+1}^{n} \overbrace{a_{ik}}^{=2} \overbrace{b_{kj}}^{=3} = i + (j-i) \cdot 2 + (n-j) \cdot 6$$

podobně případy $i = j$, $i > j$.

A, B

$A \cdot B = ?$

$$A_{ij} = \begin{cases} 1 & i \geq j \\ 2 & i < j \end{cases}$$

$$B_{ij} = \begin{cases} 1 & i \leq j \\ 3 & i > j \end{cases}$$

$$A = \begin{pmatrix} 1 & 2 & 2 & \cdots & 2 \\ 1 & 1 & 2 & & \vdots \\ 1 & 1 & 1 & \ddots & \\ \vdots & & & \ddots & 2 \\ 1 & \cdots & & & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 3 & 1 & 1 & & \vdots \\ 3 & 3 & 1 & & \\ \vdots & & \ddots & \ddots & \\ 3 & \cdots & & 3 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \\ 3 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 13 & 9 & 5 \\ 10 & 8 & 4 \\ 7 & 5 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1+6+6 & 1+2+6 & 1+2+2 \\ 1+3+6 & 1+1+6 & 1+1+2 \\ 1+3+3 & 1+1+3 & 1+1+1 \end{pmatrix}$$

v lib. prvku matice: $a \cdot 1 + b \cdot 2 + c \cdot 3 + d \cdot 6$

$a + b + c + d = n$

$$A \cdot B = \begin{pmatrix} 1+(n-1)\cdot 6 & 1+2+(n-2)\cdot 6 & 1+2\cdot 2+(n-3)6 & \cdots & 1+(n-1)\cdot 2 \\ 1+3+(n-2)\cdot 6 & 2+(n-2)\cdot 6 & 2\cdot 1+1\cdot 2+(n-3)\cdot 6 & \cdots & 2+(n-2)\cdot 2 \\ \vdots & & 3+(n-3)\cdot 6 & & \vdots \\ & & & \ddots & \\ 1+(n-1)\cdot 3 & & & & n \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{31} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + a_{13} \cdot b_{31} & & \\ & & \\ & & \end{pmatrix}$$

$$(A \cdot B)_{ij} = \sum_{k=1}^{n} a_{ik} \cdot b_{kj}$$

$\underline{i<j}$

$$(A\cdot B)_{ij}=\sum_{k=1}^{n} a_{ik}b_{kj}=\sum_{k=1}^{i}\overbrace{a_{ik}}^{=1}\underbrace{b_{kj}}_{1}+\sum_{k=i+1}^{j}\underbrace{a_{ik}}_{2}\overbrace{b_{kj}}^{=1}+\sum_{k=j+1}^{n}\overbrace{a_{ik}}^{=2}\underbrace{b_{kj}}_{3}$$

$$=i\cdot 1+(j-i)\cdot 2+(n-j)\cdot 6$$

$i=j$

$$(A\cdot B)_{ii}=\sum_{k=1}^{n} a_{ik}b_{ki}=\sum_{k=1}^{i}\underbrace{a_{ik}}_{1}\overbrace{b_{ki}}^{1}+\sum_{k=i+1}^{n}\overbrace{a_{ik}}^{2}\underbrace{b_{kj}}_{3}=i\cdot 1+(n-i)\cdot 6$$

$i>j$

$$(A\cdot B)_{ij}=j\cdot 1+(i-j)\cdot 3+(n-i)\cdot 6$$