Pr.

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 1 & 1 & 5 & 5 & 0 & 0 & 1 & 0 \\ 2 & 1 & 6 & 4 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 5 & -1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 4 & -2 & 0 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & -1 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 6 & 4 & 0 & -3 \\ 0 & 0 & 1 & 0 & 3 & 1 & 1 & -2 \\ 0 & 0 & 0 & 1 & -2 & -1 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -5 & -2 & -2 & 4 \\ 0 & 1 & 0 & 0 & 0 & 2 & -2 & 1 \\ 0 & 0 & 1 & 0 & 3 & 1 & 1 & -2 \\ 0 & 0 & 0 & 1 & -2 & -1 & 0 & 1 \end{array}\right)$$

$$\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 3 \\ 1 & 1 & 5 & 5 \\ 2 & 1 & 6 & 4 \end{pmatrix} \cdot \begin{pmatrix} -5 & -2 & -2 & 4 \\ 0 & 2 & -2 & 1 \\ 3 & 1 & 1 & -2 \\ -2 & -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Př. Nalezněte inv. matici

$$\left(\underbrace{\begin{matrix} 1 & a & 0 & 0 & 0 \\ 0 & 1 & a & 0 & 0 \\ 0 & 0 & 1 & a & 0 \\ 0 & 0 & 0 & 1 & a \\ 0 & 0 & 0 & 0 & 1 \end{matrix}}_{A} \left| \begin{matrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{matrix} \right.\right)$$

$$\sim \left(\begin{matrix} 1 & a & & & \\ & 1 & a & & \\ & & 1 & a & \\ & & & 1 & 0 \\ 0 & & & & 1 \end{matrix} \left| \begin{matrix} 1 & & & & \\ 0 & 1 & & & \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -a \\ 0 & 0 & 0 & 0 & 1 \end{matrix} \right.\right)$$

$$\sim \left(\begin{matrix} 1 & a & & & \\ & 1 & a & & \\ & & 1 & 0 & \\ & & & 1 & 0 \\ 0 & & & & 1 \end{matrix} \left| \begin{matrix} 1 & & & & \\ & 1 & 0 & 0 & 0 \\ & & 1 & -a & a^2 \\ & & 0 & 1 & -a \\ 0 & 0 & 0 & 0 & 1 \end{matrix} \right.\right)$$

$$\sim \left(\begin{matrix} 1 & a & & & \\ & 1 & 0 & & \\ & & 1 & 0 & \\ & & & 1 & 0 \\ & & & & 1 \end{matrix} \left| \begin{matrix} 1 & 0 & & & \\ & 1 & -a & a^2 & -a^3 \\ & & 1 & -a & a^2 \\ & 0 & & 1 & -a \\ & & & & 1 \end{matrix} \right.\right)$$

$$\sim \left(E \left| \underbrace{\begin{matrix} 1 & -a & a^2 & -a^3 & a^4 \\ 0 & 1 & -a & a^2 & -a^3 \\ 0 & 0 & 1 & -a & a^2 \\ 0 & 0 & 0 & 1 & -a \\ 0 & 0 & 0 & 0 & 1 \end{matrix}}_{A^{-1}} \right.\right)$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a\cdot d - b\cdot c$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\cdot e\cdot i + d\cdot h\cdot c + g\cdot b\cdot f - c\cdot e\cdot g - f\cdot h\cdot a - i\cdot b\cdot d$$

$$\begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = 1\cdot(-1) - 2\cdot(-2) = -1+4 = 3$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 1 & 5 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 5 + (-2) + 0 - 0 - 2 - 0 = 1$$

$$\begin{vmatrix} a & & & & * \\ & b & & & \\ & & c & & \\ & & & d & \\ 0 & & & & e \end{vmatrix} = a\cdot b\cdot c\cdot d\cdot e$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ -1 & 0 & 1 \end{vmatrix} = 0 \qquad (1,2,3) = 2\cdot(0,1,2) - (-1,0,1)$$

$$\begin{vmatrix} 4 & 0 & 9 & 8 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 4 & 2 \end{vmatrix}\ |:2 \;=\; 2\cdot\begin{vmatrix} 4 & 0 & 9 & 8 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 2 & 1 \end{vmatrix} \begin{matrix} 1. \\ \\ \\ 4. \end{matrix} = 2\cdot(-1)^{1+4}\cdot\begin{vmatrix} 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 4 & 0 & 9 & 8 \end{vmatrix}\ -4\times$$

$$= -2\cdot\begin{vmatrix} 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{vmatrix} \begin{matrix} \\ 2. \\ 3. \\ \end{matrix} = -2\cdot(-1)^{2+3}\cdot\begin{vmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 4 \end{vmatrix}\ -1\times$$

$$= 2\cdot\begin{vmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 2\cdot 1\cdot 1\cdot 1\cdot 1 = \underline{\underline{2}}$$

$$\begin{vmatrix} 1 & & & 0 \\ 10 & 2 & & \\ 100 & 53 & 3 & \\ 1 & -20 & 1001 & 4 \end{vmatrix} = 1\cdot 2\cdot 3\cdot 4 = 24$$

$$\begin{vmatrix} A & D & E \\ 0 & B & F \\ 0 & 0 & C \end{vmatrix} = |A|\cdot|B|\cdot|C|$$

$$\begin{vmatrix} 1 & 0 & 101 & -5 & -2 \\ 2 & 3 & 201 & -3 & -1 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix}\cdot\begin{vmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 0 & 1 & 5 \end{vmatrix}$$

6x6

Př.

$$\begin{vmatrix} 2 & -1 & 0 & 3 \\ 1 & 0 & -2 & 0 \\ -1 & 1 & 2 & 1 \\ -3 & -2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & -1 & 0 & 3 \\ 1 & 0 & -2 & 0 \\ 1 & 0 & 2 & 4 \\ -7 & 0 & 1 & -5 \end{vmatrix} = \begin{vmatrix} 2 & -1 & 0 & 3 \\ 1 & 0 & -2 & 0 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & -13 & -5 \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix}\cdot\begin{vmatrix} 4 & 4 \\ -13 & -5 \end{vmatrix} = 1\cdot(-20+52) = 32$$

($+1\times$, $-2\times$)

ř. sl.

$$= (-1)^{1+2}\cdot(-1)\cdot\begin{vmatrix} 1 & -2 & 0 \\ 1 & 2 & 4 \\ -7 & 1 & -5 \end{vmatrix} = \begin{vmatrix} 1 & -2 & 0 \\ 1 & 2 & 4 \\ -7 & 1 & -5 \end{vmatrix} = -10 + 0 + 56 - 0 - 4 - 10 = 32$$

Laplaceův rozvoj

$$\begin{vmatrix} 2 & -1 & 0 & 3 \\ 1 & 0 & 2 & 0 \\ -1 & 1 & 2 & 1 \\ -3 & -2 & 1 & 1 \end{vmatrix} = (-1)^{2+1}\cdot 1\cdot\begin{vmatrix} -1 & 0 & 3 \\ 1 & 2 & 1 \\ -2 & 1 & 1 \end{vmatrix} + 0 + (-1)^{2+3}\cdot(-2)\cdot\begin{vmatrix} 2 & -1 & 3 \\ -1 & 1 & 1 \\ -3 & -2 & 1 \end{vmatrix} + 0 = -1\cdot 14 + 2\cdot 23 = 32$$

Př:

$$\begin{vmatrix} 2 & -1 & 0 & 3 \\ 1 & 0 & -2 & 0 \\ -1 & 1 & 2 & 1 \\ -3 & -2 & 1 & 1 \end{vmatrix} = -\begin{vmatrix} 1 & 0 & -2 & 0 \\ 2 & -1 & 0 & 3 \\ -1 & 1 & 2 & 1 \\ -3 & -2 & 1 & 1 \end{vmatrix} = -\begin{vmatrix} 1 & 0 & -2 & 0 \\ 0 & -1 & 4 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & -2 & -5 & 1 \end{vmatrix} = (-1)\cdot 1 \cdot \begin{vmatrix} -1 & 4 & 3 \\ 1 & 0 & 1 \\ -2 & -5 & 1 \end{vmatrix} = (-1)\cdot(-15-8-5-4) = 32$$

Laplaceův rozvoj

1. ř. sloupec 2.

$$\begin{vmatrix} 1 & 2 & -1 & 3 \\ -1 & 5 & 0 & 1 \\ -1 & 1 & -1 & 1 \\ -6 & 1 & 0 & 1 \end{vmatrix} = (-1)^{2+1}\cdot 1 \cdot \begin{vmatrix} 1 & 2 & -1 & 3 \\ -1 & 5 & 0 & 1 \\ -1 & 1 & -1 & 1 \\ -6 & 1 & 0 & 1 \end{vmatrix} + (-1)^{2+2}\cdot 5 \cdot \begin{vmatrix} 1 & -1 & 3 \\ -1 & -1 & 1 \\ -6 & 0 & 1 \end{vmatrix} + (-1)^{2+3}\cdot 0 \cdot |\ \dots\ | + (-1)^{2+4}\cdot 1 \cdot \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & -1 \\ -6 & 1 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & -1 & 3 \\ -1 & 5 & 0 & 1 \\ -1 & 1 & -1 & 1 \\ -6 & 1 & 0 & 1 \end{vmatrix} = (-1)^{1+3}\cdot(-1)\cdot \begin{vmatrix} -1 & 5 & 1 \\ -1 & 1 & 1 \\ -6 & 1 & 1 \end{vmatrix} + (-1)^{3+3}\cdot(-1)\cdot \begin{vmatrix} 1 & 2 & 3 \\ -1 & 5 & 1 \\ -6 & 1 & 1 \end{vmatrix} = (-1)\cdot(-20) + (-1)\cdot 81 = 20-81 = -61$$

Př. Zjistěte pro které $a, b, c \in \mathbb{R}$ je soustava

$$\begin{aligned} a x_1 + b x_2 &= c \\ c x_1 + a x_3 &= b \\ c x_2 + b x_3 &= a \end{aligned}$$

$$\left(\begin{array}{ccc|c} a & b & 0 & c \\ c & 0 & a & b \\ 0 & c & b & a \end{array}\right)$$

jednoznačně řešitelná. $\Leftrightarrow \begin{vmatrix} a & b & 0 \\ c & 0 & a \\ 0 & c & b \end{vmatrix} = -a^2c - b^2c \neq 0$

$= -c \cdot (a^2 + b^2)$

$\Leftrightarrow c \neq 0 \wedge (a \neq 0 \vee b \neq 0)$

Najděte řeš.

$*$ .. cokoli nenul.

$\left(\begin{array}{ccc|c} * & * & * & * \\ 0 & * & * & * \\ 0 & 0 & 0 & * \end{array}\right)$ nemá řeš.

$\left(\begin{array}{ccc|c} * & * & * & * \\ 0 & * & * & * \\ 0 & 0 & 0 & 0 \end{array}\right)$ má nek. mn. řeš.

$\left(\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \end{array}\right)$ má jednozn. řeš.

Cramerovo pravidlo

řešení $x_i = \dfrac{\text{det., kde v i-tém sl. je pravá strana}}{\text{det. mat. na levé straně}}$

$$x_1 = \frac{\begin{vmatrix} c & b & 0 \\ b & 0 & a \\ a & c & b \end{vmatrix}}{\begin{vmatrix} a & b & 0 \\ c & 0 & a \\ 0 & c & b \end{vmatrix}} = \frac{a^2b - ac^2 - b^3}{-c\cdot(a^2+b^2)}$$

$$x_2 = \frac{\begin{vmatrix} a & c & 0 \\ c & b & a \\ 0 & a & b \end{vmatrix}}{\begin{vmatrix} a & b & 0 \\ c & 0 & a \\ 0 & c & b \end{vmatrix}} = \frac{ab^2 - a^3 - bc^2}{-c\cdot(a^2+b^2)}$$

$$x_3 = \frac{\begin{vmatrix} a & b & c \\ c & 0 & b \\ 0 & c & a \end{vmatrix}}{\begin{vmatrix} a & b & 0 \\ c & 0 & a \\ 0 & c & b \end{vmatrix}} = \frac{c^3 - 2\cdot abc}{-c(a^2+b^2)}$$

Pr.

$$\begin{vmatrix} a & 1 & 1 & 1 & 1 \\ 1 & a & 1 & 1 & 1 \\ 1 & 1 & a & 1 & 1 \\ 1 & 1 & 1 & a & 1 \\ 1 & 1 & 1 & 1 & a \end{vmatrix} = \begin{vmatrix} a+4 & a+4 & a+4 & a+4 & a+4 \\ 1 & a & 1 & 1 & 1 \\ 1 & 1 & a & 1 & 1 \\ 1 & 1 & 1 & a & 1 \\ 1 & 1 & 1 & 1 & a \end{vmatrix} = (a+4) \cdot \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & a & 1 & 1 & 1 \\ 1 & 1 & a & 1 & 1 \\ 1 & 1 & 1 & a & 1 \\ 1 & 1 & 1 & 1 & a \end{vmatrix}$$

$$= (a+4) \cdot \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & a-1 & 0 & 0 & 0 \\ 0 & 0 & a-1 & 0 & 0 \\ 0 & 0 & 0 & a-1 & 0 \\ 0 & 0 & 0 & 0 & a-1 \end{vmatrix} = (a+4) \cdot (a-1)^4$$

$$\begin{vmatrix} 2 & 4 & 6 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{vmatrix} /{:}2 = \frac{1}{2} \cdot \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{vmatrix} = \frac{1}{2} \cdot (-1)^{2+3} \cdot \begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$