Př. $\rho: \underbrace{[3,-1,0]}^{A} + s\,\underbrace{(-1,1,1)}^{u} + t\,\underbrace{(2,1,0)}^{v}$

a) $p: [7,4,2] + a(5,-2,-3)$ $\longrightarrow$ splývá $A + su + tv = B + bw = x$

b) $q: \underbrace{[1,2,3]}_{B} + b\,\underbrace{(1,5,3)}_{w}$ $su + tv - bw = B - A$

$$\left(\begin{array}{ccc|c} -1 & 2 & -1 & -2 \\ 1 & 1 & -5 & 3 \\ 1 & 0 & -3 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 2 & -1 & -2 \\ 0 & 3 & -6 & 1 \\ 0 & 2 & -4 & 1 \end{array}\right) \underset{\cdot 3}{\sim} \left(\begin{array}{ccc|c} -1 & 2 & -1 & -2 \\ 0 & 3 & -6 & 1 \\ 0 & 6 & -12 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 2 & -1 & -2 \\ 0 & 3 & -6 & 1 \\ 0 & 0 & 0 & 3 \end{array}\right)$$

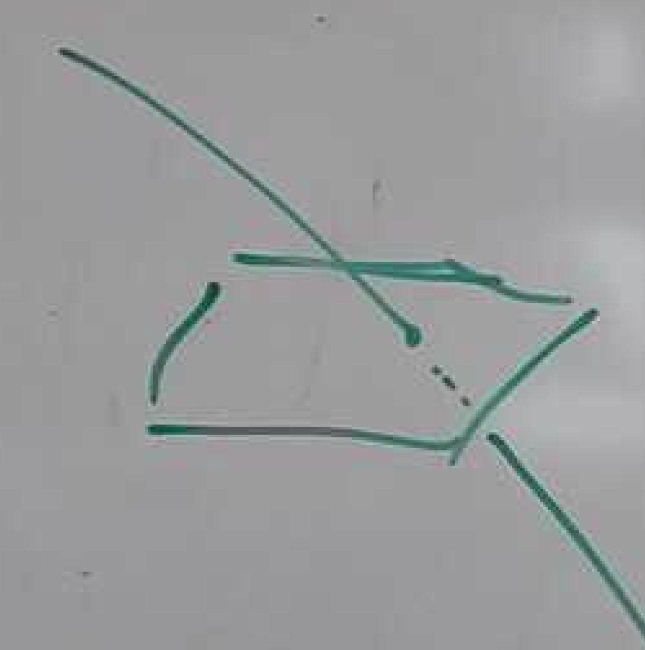
$q \cap \rho = \emptyset$ $Z(q) \subseteq Z(\rho)$ $\Longrightarrow$ q je $\parallel$ na rovinu ρ

c) $r: [1,2,3] + c(1,1,1)$

$$\left(\begin{array}{ccc|c} -1 & 2 & -1 & -2 \\ 1 & 1 & -1 & 3 \\ 1 & 0 & -1 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 2 & -1 & -2 \\ 0 & 3 & -2 & 1 \\ 0 & 2 & -2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 2 & -1 & -2 \\ 0 & 3 & -2 & 1 \\ 0 & 6 & -6 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} -1 & 2 & -1 & -2 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & -2 & 1 \end{array}\right)$$

$r \cap \rho$ = jeden bod

$\Longrightarrow$ r a ρ jsou různoběžné

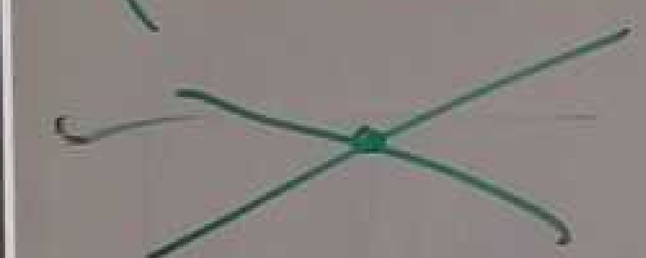


mimoběžné

$$\left(\begin{array}{cc|c} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{array}\right)$$

různoběžné

$$\left(\begin{array}{cc|c} 1 & * & * \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right)$$

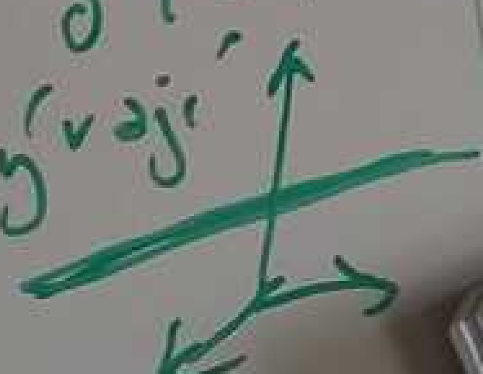


rovnoběžné

$$\left(\begin{array}{cc|c} 1 & * & * \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 1 & * & * \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

splývají

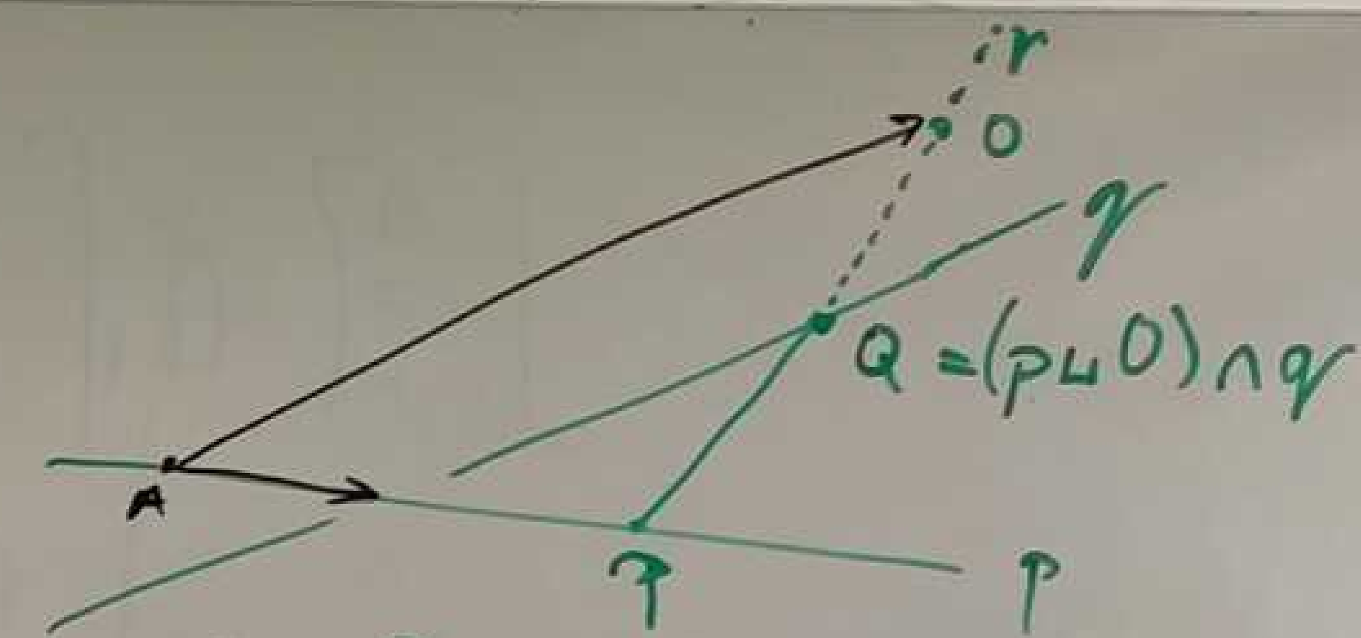


Př. Určete příčku r mimoběžek

$$p: \underbrace{[1,2,0]}_{A} + a\cdot\overbrace{(1,-1,1)}^{u}$$

$$q: \underbrace{[0,9,-2]}_{B} + b\cdot\underbrace{(1,0,0)}_{v}$$

takovou, že přímka je určená prochází bodem $O=[7,9,-5]$.



Řešení: Spočítáme rovinu $p\sqcup O$, spočítáme průnik.

$$p\sqcup O: \underbrace{[1,2,0]}_{A} + a\cdot(1,-1,1) + t\cdot\underbrace{(6,7,-5)}_{(O-A)}$$

Q:

$$(p\sqcup O)\cap q: \begin{array}{cccc} u & O-A & -v & B-A \\ \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 6 & -1 & -1 \\ -1 & 7 & 0 & 7 \\ 1 & -5 & 0 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 6 & -1 & -1 \\ 0 & 13 & -1 & 6 \\ 0 & -11 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 6 & -1 & -1 \\ 0 & 2 & 0 & 5 \\ 0 & -11 & 1 & -1 \end{array}\right) /:2 \sim$$

$$\begin{array}{ccc} a & t & b \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 6 & -1 & -1 \\ 0 & 1 & 0 & 5/2 \\ 0 & 0 & 1 & 53/2 \end{array}\right) \leftarrow -1+\frac{55}{2}$$

$$Q = [0,9,-2] + \frac{53}{2}(1,0,0) = \left(\frac{53}{2}, 9, -2\right)$$

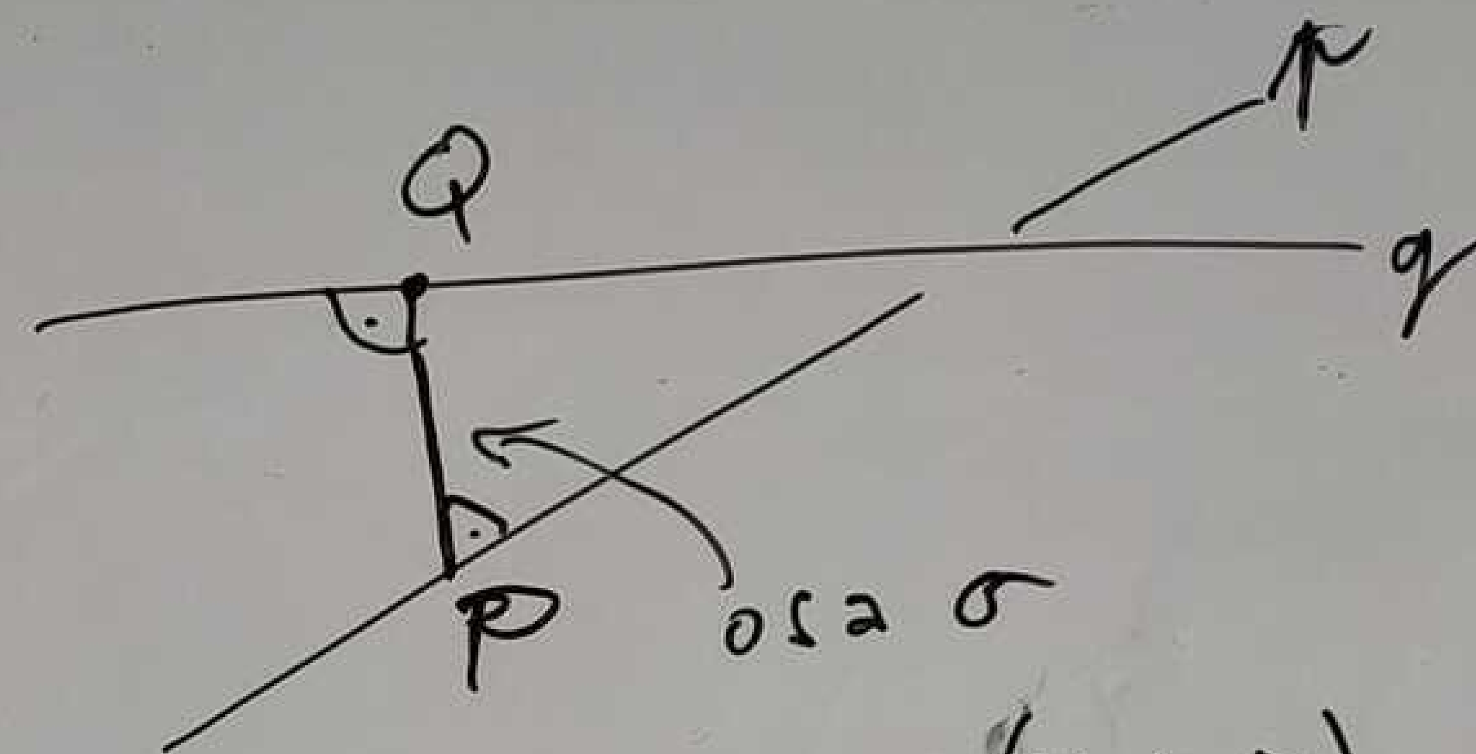
$$P = r \cap p$$

$$r: [7,9,-5] + s\overbrace{\left(\frac{-39}{2}, 0, -3\right)}^{O-Q}$$

Pr. $p: [\underbrace{1,2,3}_{A}] + a\,(\underbrace{1,2,-1}_{u})$

$q: [\underbrace{2,-3,4}_{B}] + b\,(\underbrace{2,-1,-2}_{v})$

Najděte osu p a q. a body P, Q



$\sigma: Q + t\cdot w$ $\qquad w \perp u \iff \langle u, w\rangle = 0$

$w \perp v \iff \langle v, w\rangle = 0$

$w = (x, y, z)$

$\langle (1,2,-1),(x,y,z)\rangle = 1\cdot x + 2y - 1\cdot z = 0$

$\langle (2,-1,-2),(x,y,z)\rangle = 2x - 1\cdot y - 2\cdot z = 0$

Označme si rovinu $\rho: [1,2,3] + a\,(1,2,-1) + t\cdot(1,0,1)$

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -1 \\ 0 & -5 & 0 \end{pmatrix} \qquad w = (1,0,1)$$

$Q = \rho \cap q$:

$$\begin{array}{cccc} a & t & b & B-A \end{array}$$
$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 2 & 0 & 1 & -5 \\ -1 & 1 & 2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & -2 & 5 & -7 \\ 0 & 2 & 0 & 2 \end{array}\right) \sim \begin{array}{c} \begin{array}{ccc|c} & & b & \end{array} \\ \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 5 & -5 \end{array}\right) \end{array}$$

$b = -1$

$Q = A + a u + t w = B + b\cdot v$

$Q = [2,-3,4] + (-1)\cdot(2,-1,-2) = [0,-2,6]$

$\sigma: [0,-2,6] + t\cdot(1,0,1)$

$P = \sigma \cap p$: $Q + t\cdot w = A + a u$

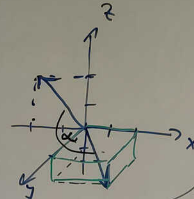
$$\begin{array}{c} \begin{array}{cc|c} w & -u & A-Q \end{array} \\ \left(\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & -2 & 4 \\ 1 & 1 & -3 \end{array}\right) \end{array} \sim \begin{array}{c} \begin{array}{cc|c} t & a & \end{array} \\ \left(\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 1 & 2 \\ \cancel{0} & \cancel{2} & \cancel{-4} \end{array}\right) \end{array}$$

$a = 2$

$P = A + 2\cdot u = [1,2,3] + 2\cdot(1,2,-1) = [3,6,1]$

Př. $u = (2,2,-1)$ Najděte velikosti u a v

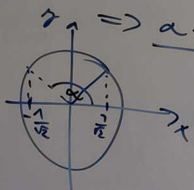
$v = (-2,0,2)$ a $\sphericalangle\, u\, v$.



$$\|u\| = \sqrt{\langle u,u\rangle} = \sqrt{2^2+2^2+(-1)^2} = \sqrt{9} = 3$$

$$\|v\| = \sqrt{\langle v,v\rangle} = \sqrt{(-2)^2+0+2^2} = \underset{\sqrt{4\cdot 2}}{\underset{\parallel}{\sqrt{8}}} = \underset{\sqrt{4}\cdot\sqrt{2}}{\underset{\parallel}{2\sqrt{2}}}$$

$$\cos\alpha = \frac{\langle u,v\rangle}{\|u\|\cdot\|v\|} = \frac{2\cdot(-2)+2\cdot 0+(-1)\cdot 2}{3\cdot 2\sqrt{2}} = \frac{-6}{6\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$



$\Rightarrow$ $\underline{\alpha = 135^\circ}$

Př. Najděte v $\mathbb{R}^3$ ortogonální (kolmý) doplněk podprostoru

$$M = [(1,2,-1),(1,-2,5)]$$

$$\langle (1,2,-1), w\rangle = 0 = \langle (1,-2,5), w\rangle$$

$$\begin{pmatrix} 1 & 2 & -1 \\ 1 & -2 & 5 \end{pmatrix} \sim \begin{array}{ccc} x & y & z \\ \end{array}\begin{pmatrix} \boxed{1} & 2 & -1 \\ 0 & \boxed{-4} & 6 \end{pmatrix}$$

$z = 4t$

$-4y = -6z$

$y = 6t$

$x = -2y + z = -8t$

$t = \frac{1}{2}: w = (-4,3,2)$

$\Uparrow$

$M^\perp = [(-4,3,2)]$

Př. Spočítejteje kolmou projekci $u=(7,-16,9)$ do M a do $M^\perp$.

$u = \underset{\underset{M}{\in}}{v} + \underset{\underset{M^\perp}{\in}}{v_\perp}$

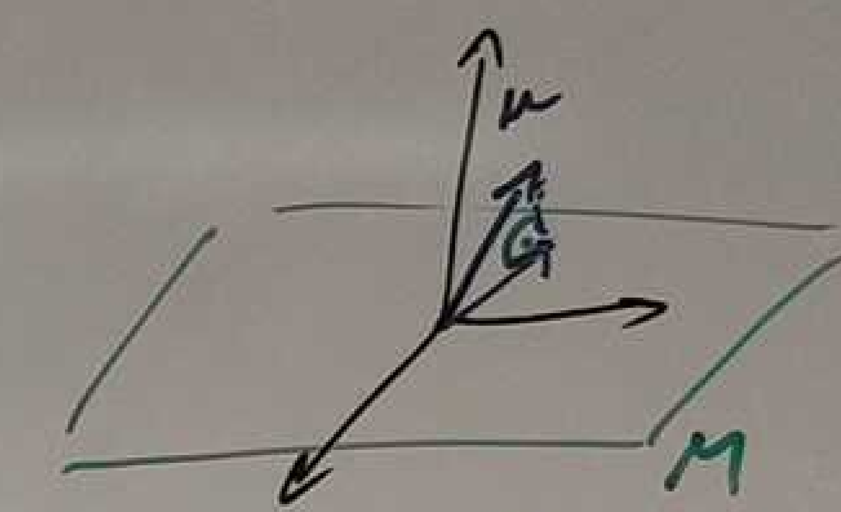
$u - \overset{P_M}{v} \perp M$ 1)

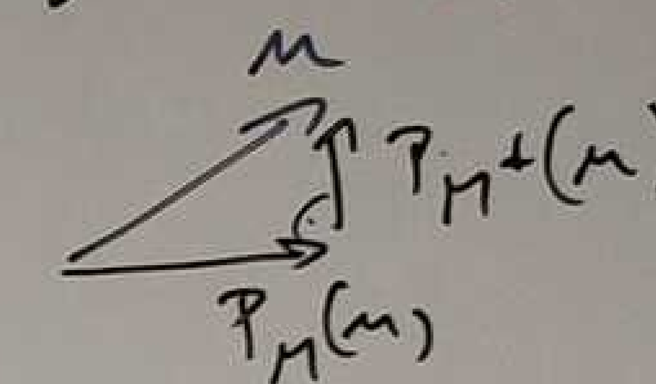
$u - \underset{\underset{M^\perp}{\in}}{v_\perp} \perp M^\perp$ 2)

$u = P_M(u) + P_{M^\perp}(u)$

(3) $u - P_M(u) \in M^\perp$

$u - a(\ldots) - b(\ldots) = t \cdot w$)





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1) $P_M(u) = a(1,2,-1) + b(1,-2,5)$

$0 = \langle u - a(1,2,-1) - b(1,-2,5), (1,2,-1)\rangle = \langle u,(1,2,-1)\rangle - a\langle(1,2,-1),(1,2,-1)\rangle$

$0 = \qquad (1,-2,5) \qquad - b\langle(1,-2,5),(1,2,-1)\rangle$

$\boxed{\langle a\cdot u + b v, w\rangle = a\langle u,w\rangle + b\langle v,w\rangle}$

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2) $P_{M^\perp}(u) = t w = t\cdot(-4,3,2)$

$0 = \langle u - t\cdot(-4,3,2), (-4,3,2)\rangle = \langle(7,-16,9),(-4,3,2)\rangle - t\cdot\langle(-4,3,2),(-4,3,2)\rangle = -58 - t\cdot 29$

$t = -\frac{58}{29} = -2$

$\underline{P_{M^\perp}(u)} = -2\cdot(-4,3,2) = \underline{(8,-6,-4)}$

$\underline{P_M(u)} = u - P_{M^\perp}(u) = (7,-16,9) - (8,-6,-4) = \underline{(-1,-10,13)}$