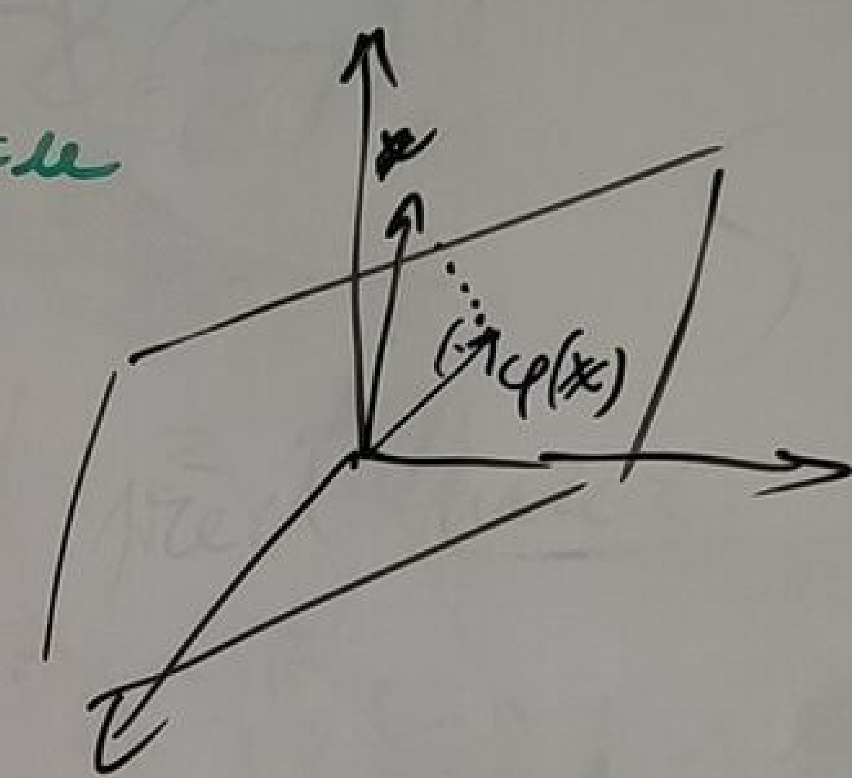
Př. $\varphi: \mathbb{R}^3 \to \mathbb{R}^3$ kolmá projekce na ρ

2 nezávislá řeš.

$\rho: 2x_1 - x_2 + 2x_3 = 0$ → $(1,0,-1) = u$, $(1,2,0) = v$



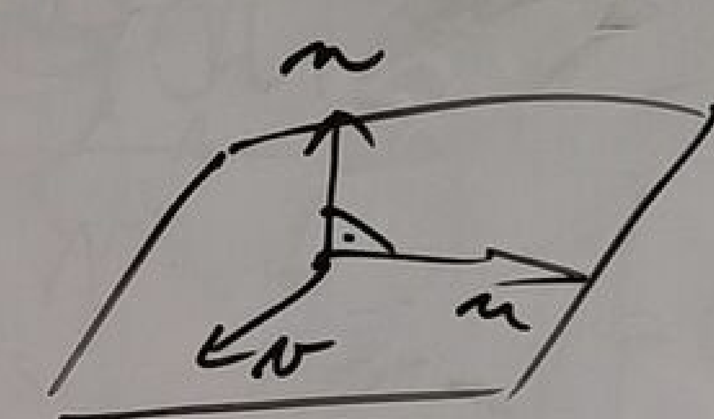
Najděte A : $\varphi(x) = Ax$

$n = (2,-1,2)$

$\varphi(n) = 0$

$\varphi(u) = u$

$\varphi(v) = v$



$$\left(\begin{array}{c|c} u & \varphi(u) \\ v & \varphi(v) \\ n & \varphi(n) \end{array}\right) = \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -1 \\ 1 & 2 & 0 & 1 & 2 & 0 \\ 2 & -1 & 2 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -1 \\ 0 & 2 & 1 & 0 & 2 & 1 \\ 0 & -1 & 4 & -2 & 0 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -1 \\ 0 & -1 & 4 & -2 & 0 & 2 \\ 0 & 0 & 9 & -4 & 2 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{5}{9} & \frac{2}{9} & -\frac{4}{9} \\ 0 & 1 & 0 & \frac{2}{9} & \frac{8}{9} & \frac{2}{9} \\ 0 & 0 & 1 & -\frac{4}{9} & \frac{2}{9} & \frac{5}{9} \end{array}\right)$$

$-2 + \frac{16}{9} = -\frac{2}{9}$

$2 - 4 \cdot \frac{5}{9} = \frac{2}{9}$

$$A = \left(\varphi(e_1)\ \varphi(e_2)\ \varphi(e_3)\right) = \frac{1}{9}\begin{pmatrix} 5 & 2 & -4 \\ 2 & 8 & 2 \\ -4 & 2 & 5 \end{pmatrix}$$

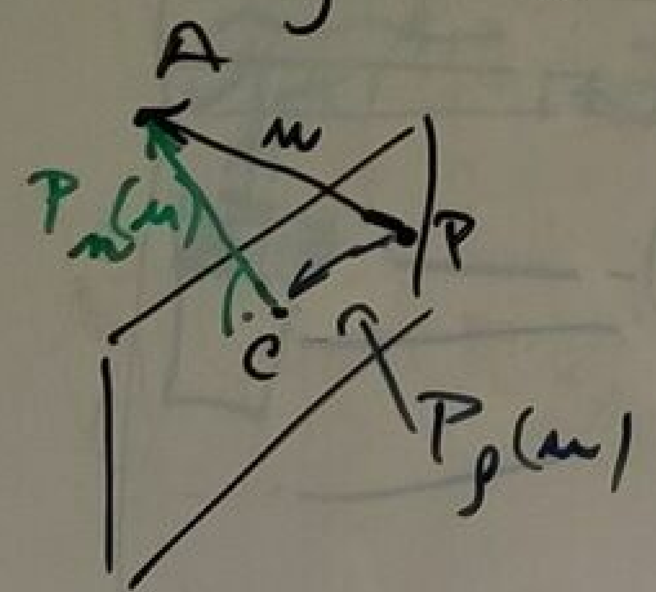
$-\frac{4}{9}$

$+\frac{2}{9}$

$\frac{5}{9}$

Př. V $\mathcal{E}_3$ spočítejte vzdálenost bodu $A=[5,7,1]$ od roviny ρ

$\rho: x_1+3x_2-2x_3+4=0 \longrightarrow n=(1,3,-2)$



$\rho \ni P=[1,1,4]$

$u=\overrightarrow{PA}=A-P=(4,6,-3)$

$\underbrace{u-P_n(u)}_{P_\rho(u)} \perp n$

$0=\langle u-k\cdot n, n\rangle = \langle u,n\rangle - \langle k\cdot n, n\rangle$
$= \langle u,n\rangle - k\cdot\langle n,n\rangle$

$k=\dfrac{\langle u,n\rangle}{\langle n,n\rangle}=\dfrac{\langle(4,6,-3),(1,3,-2)\rangle}{\langle(1,3,-2),(1,3,-2)\rangle}=\dfrac{28}{14}=2$

$\text{dist}(A,\rho)=\|P_n(u)\|=\|2\cdot(1,3,-2)\|=\|(2,6,-4)\|$
$=\sqrt{4+36+16}=\sqrt{56}$

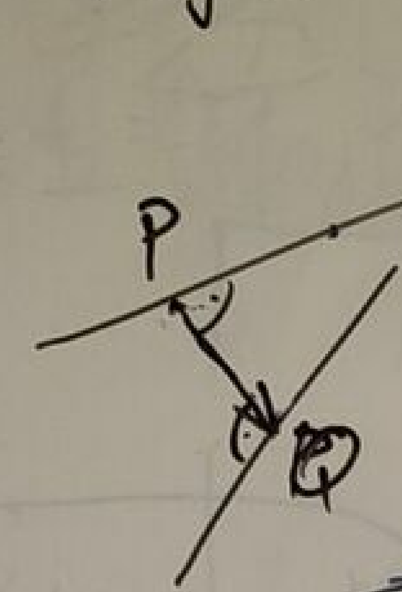
Najděte bod C takový, že $\text{dist}(A,\rho)=\text{dist}(A,C)$.

$C=A-P_n(u)$
$=[5,7,1]-2\cdot(1,3,-2)=\boxed{[3,1,5]}$

Př. Spočítejte vzdálenost přímek

$p: \underbrace{[4,4,4]}_{A}+a\overbrace{(2,1,-1)}^{u}, \quad q: \overbrace{[1,15,12]}^{B}+b\overbrace{(1,-2,1)}^{v}$

Najděte body P, Q, v nichž se vzdálenost realizuje.



$w\perp(2,1,-1)$
$(x,y,z)=w\perp(1,-2,1) \longleftrightarrow \begin{pmatrix}2&1&-1\\1&-2&1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=0$

$\begin{pmatrix}2&1&-1\\1&-2&1\end{pmatrix}\sim\begin{pmatrix}1&-2&1\\0&5&-3\end{pmatrix}$

$[4,9,15]$ $\quad w=(1,3,5)$

$\overrightarrow{PQ}\perp(2,1,-1)$
$\perp(1,-2,1)$

$Q=[1,15,12]+b_0(1,-2,1)$

$P=[4,4,4]+a_0(2,1,-1)=[2,3,5]$

$\overrightarrow{PQ}=\underbrace{(-3,11,8)}_{B-A}-a_0\underbrace{(2,1,-1)}_{u}+b_0\underbrace{(1,-2,1)}_{v}$

$0=\langle(B-A)-a_0u+b_0v, u\rangle=\langle B-A,u\rangle-a_0\langle u,u\rangle+b_0\langle v,u\rangle$

$\langle \quad , v\rangle=$

$(2,6,10)=\overrightarrow{PQ}=k\,w \longleftrightarrow -a_0u+b_0v-k\,w=-(B-A)$

$\left(\begin{array}{ccc|c}-2&1&-1&3\\-1&-2&-3&-11\\+1&1&-5&-8\end{array}\right)$

$\sim\left(\begin{array}{ccc|c}1&1&-5&-8\\0&-1&-8&-19\\0&3&-11&-13\end{array}\right)\sim\left(\begin{array}{ccc|c}1&1&-5&-8\\0&1&8&19\\0&0&-35&-70\end{array}\right)$

$a_0=-1$

$b_0=19-8\cdot2=3$

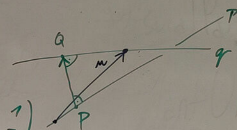
$k=\frac{70}{35}=2$

$\|\overrightarrow{PQ}\|=\sqrt{140}$

Alternativní výpočet příkladu na minulé straně.
Zde děláme projekci libovolného vektoru směřujícího z p do q, např. $\vec{PQ}$, na přímku generovanou vektorem kolmým k p i q.
Výhoda: vzdálenost najdu hned. Pro výpočet bodů P, Q musím udělat skoro stejný výpočet, jako na minulé straně.

Př. Vzdálenost přímek p, q a body P, Q v nichž se realizuje.

$p: [4,4,4] + a\underbrace{(2,1,-1)}_{Z(p)}$ $\quad q: [1,15,12] + b\underbrace{(1,-2,1)}_{Z(q)}$



$\vec{PQ} \perp (2,1,-1)$
$\quad \perp (1,-2,1)$

$w \perp Z(p)$
$\quad \perp Z(q)$

$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & -2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 \\ 0 & 5 & -3 \end{pmatrix}$$

$w = (1, 3, 5) \in (Z(p) + Z(q))^{\perp}$

$\vec{PQ} = k \cdot w$

$\mathrm{dist}(p,q) = \| P_w(u) \| = \sqrt{2^2+6^2+10^2} = \sqrt{140}$

$u = [1,15,12] - [4,4,4] = (-3, 11, 8)$

$\langle u - k w, w \rangle = 0$

$$k = \frac{\langle u, w\rangle}{\langle w, w\rangle} = \frac{(-3)\cdot 1 + 11 \cdot 3 + 8 \cdot 5}{1^2+3^2+5^2} = \frac{70}{35} = 2 \Rightarrow P_w(u) = 2\cdot(1,3,5) = (2,6,10) = \vec{PQ}$$

Pro body P, Q platí
$P + \vec{PQ} = Q$
$P = [4,4,4] + a_0(2,1,-1), \quad Q = [1,15,12] + b_0(1,-2,1)$
$[4,4,4] + a_0(2,1,-1) + (2,6,10) = [1,15,12] + b_0(1,-2,1)$
$a_0(2,1,-1) + b_0(-1,2,-1) = (-3,11,8) - (2,6,10) = (-5,5,-2)$

$$\left(\begin{array}{cc|c} 2 & -1 & -5 \\ 1 & 2 & 5 \\ -1 & -1 & -2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & -5 & -15 \\ 0 & 1 & 3 \end{array}\right) \sim \begin{array}{c} \left(\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right) \\ \begin{array}{ccc} a_0 & b_0 & \end{array} \end{array} \quad \begin{array}{l} b_0 = 3 \\ a_0 = 5 - 2\cdot b_0 = -1 \end{array}$$

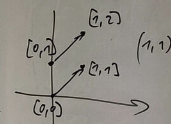
$Q = [1,15,12] + 3\cdot(1,-2,1) = [4,9,15]$

$P = [4,4,4] + (-1)(2,1,-1) = [2,3,5]$

Př. Odchy…

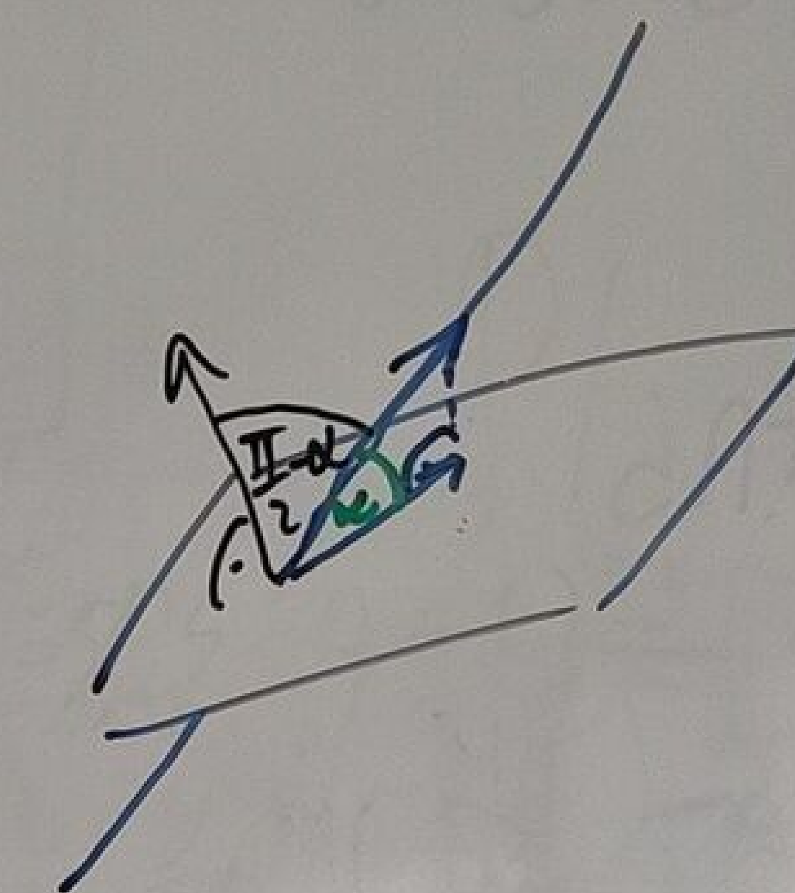
$p: [1, …$
$p: [3, …$

Najdeme nor…



Př. Určete odchylku roviny ρ od přímky p

$\rho: [1,3,5] + a(1,1,1) + b(1,3,2)$ $\qquad p: [-3,1,7] + c(1,0,-1)$

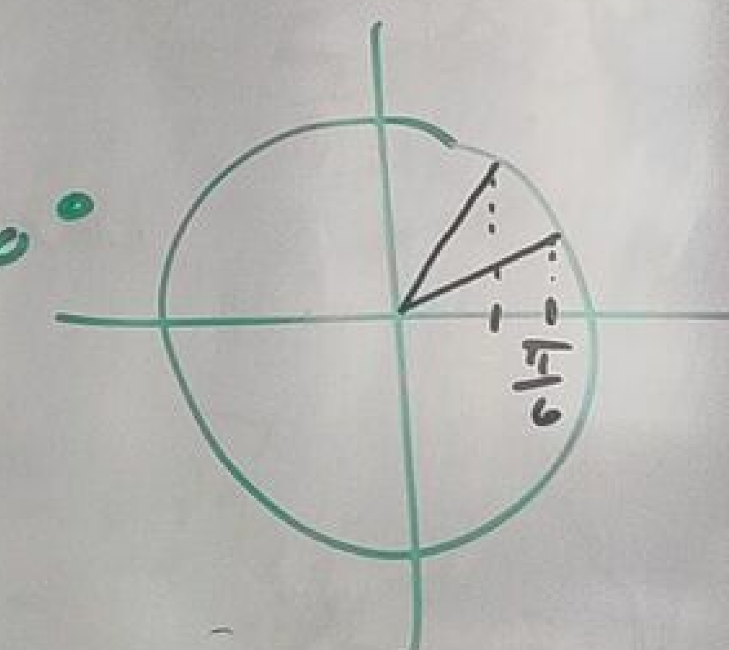


Najdeme normálu k ρ

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \end{pmatrix} \quad n = (1,1,-2)$$

$$\cos\left(\frac{\pi}{2}-\alpha\right) = \frac{|\langle (1,1,-2),(1,0,-1)\rangle|}{\sqrt{1+1+4}\cdot\sqrt{1+0+1}} = \frac{3}{\sqrt{12}} = \frac{3}{\sqrt{4\cdot 3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \longrightarrow \frac{\pi}{2}-\alpha = \frac{\pi}{6}$$

$$\Rightarrow \alpha = \frac{\pi}{3} = 60^\circ$$



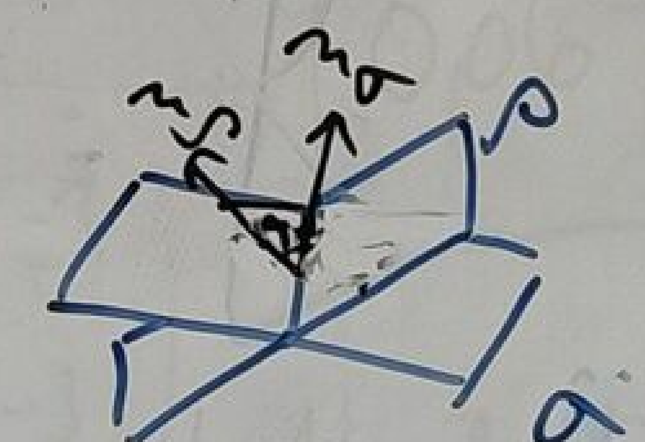
Př. Určete odchylku rovin ρ a σ

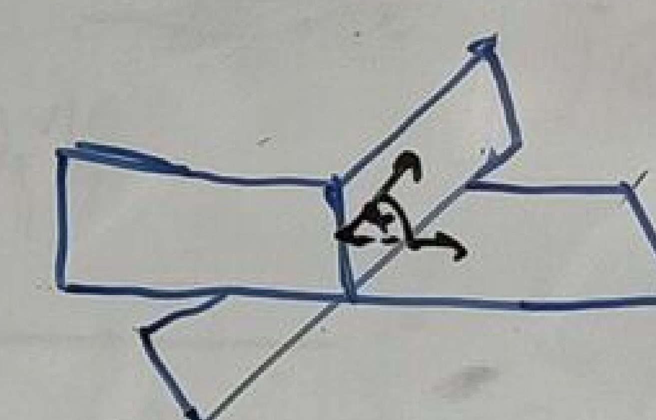
$\rho: [2,3,4] + a(2,2,1) + b(3,3,-2)$

$$\begin{pmatrix} 2 & 2 & 1 \\ 3 & 3 & -2 \end{pmatrix} \sim \begin{pmatrix} 2 & 2 & 1 \\ 1 & 1 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -3 \\ 0 & 0 & 7 \end{pmatrix} \quad n_\rho = (1,-1,0)$$

$\sigma: x_1 - 2x_2 + x_3 = 4$

$n_\sigma = (1,-2,1)$





$$\cos(\alpha) = \frac{|\langle n_\rho, n_\sigma\rangle|}{\|n_\rho\|\cdot\|n_\sigma\|} = \frac{1+2+0}{\sqrt{2}\cdot\sqrt{6}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = \frac{\pi}{6}.$$