Seminar 7

1. Use the data sample from the previous seminar describing the lengths of 30 screws (in mm). We tested the hypothesis $H_0 : \mu = 20$ against different alternatives under the assumption of normality. But was this assumption reasonable? Can we really claim that our data sample is normally distributed (lets say at the significance level $\alpha = 0.05$)? Use both graphical methods and numerical approach.

   (a) Firstly, estimate parameters μ and σ of the normal distribution $N(\mu, \sigma)$ using the maximum likelihood method. Then plot the histogram of our data together with the density of the normal distribution with the estimated parameters.

   (b) Create the Q-Q plot of the theoretical and empirical quantiles against each other. Do you think the data are normally distributed?

   (c) Plot the empirical cumulative distribution function of the data together with the theoretical distribution function of the normal distribution with the estimated parameters.

   (d) Create the P-P plot of the theoretical and empirical probability distribution functions against each other. Compute the difference of the theoretical and empirical distributions and plot it.

   (e) Use some of the normality tests (from the lecture) to decide about the normality of our data at the significance level $\alpha = 0.05$.

2. Use the data sample `toss_a_coin.RData` from the 5th seminar describing the number of tossed heads from 100 tosses. Can we claim the coin was *unspoiled* (fair)?

   (a) Test the null hypothesis $H_0 : \mu = 50$ against 2-sided alternative at the significance level $\alpha = 0.05$. Use the test without the assumption of normality (see the lecture 6 slides).

   (b) Can we consider our data as normally distributed (based on the central limit theorem)? Check it (choose any appropriate method).

   (c) Test the null hypothesis $H_0 : \mu = 50$ against the right-sided alternative $H_1 : \mu > 50$ at the significance level $\alpha = 0.05$. Use the test with the assumption of normality (one-sample t-test).