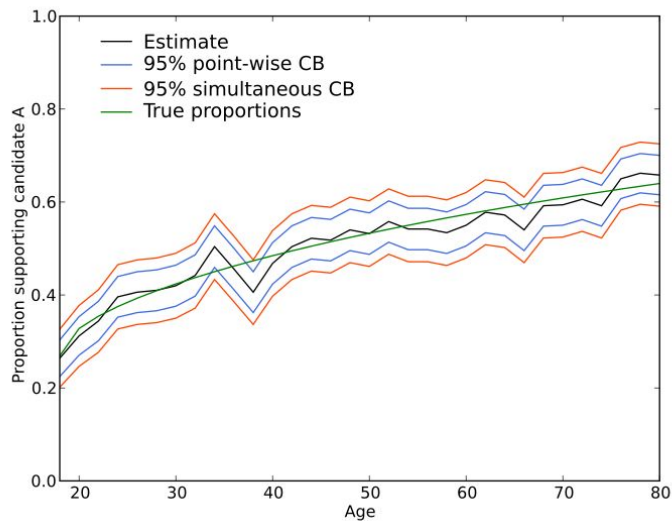


Confidence Intervals

Jakub Ryšavý
PV212
9. 3. 2023



Motivation

Solution 1:

Me: Are you 10.

π : No, I'm smaller.

Me: Are you 0.

π : No, I'm bigger.

Me: Are you 5.

π : No, I'm smaller.

Me: Are you 3.

π : No, I'm bigger.

...

Solution 2:

Me: Are you between 0 and 10.

π : Yes, I am.

“

With confidence,
you have won before you
have started.

— Marcus Garvey

@therandomvibez


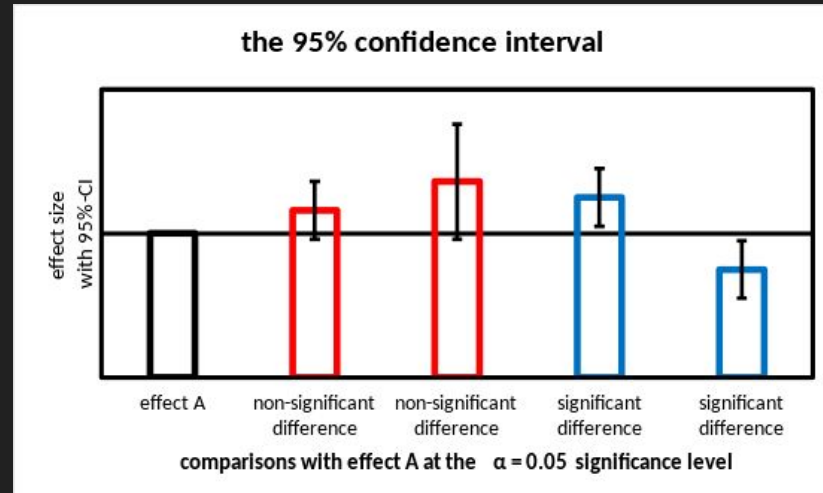


Table of Contents

- Motivation
- Table of Contents
- What is Confidence Interval (CI)?
- What is not a Confidence Interval?
- Known Methods of Calculating CI
- Factors Affecting the Width of CI
- Limitations and Challenges of CI for TS
- Tips and Best Practices
- Confidence Intervals for Time Series Forecasting
 - different approaches
- Measuring Quality of CI
- Summary

What is Confidence Interval (CI)?

- There is a 95% probability that the 95% confidence interval calculated from a given future sample will cover the true value of the population parameter.
- The 95% confidence interval represents values that are not statistically significantly different from the point estimate at the .05 level.



What is not a Confidence Interval?

- A 95% confidence level does not mean that 95% of the sample data lie within the confidence interval.
- A confidence interval is not a definitive range of plausible values for the sample parameter, though it is often heuristically taken as a range of plausible values.
- A particular confidence level of 95% calculated from an experiment does not mean that there is a 95% probability of a sample parameter from a repeat of the experiment falling within this interval.

Known Methods of Calculating CI

- Standard error
- Bootstrap resampling
- Prediction intervals
- Bayesian inference
- Jackknife resampling
- Monte Carlo simulation
- Cross-validation
- Likelihood-based methods
- Direct interval estimation

Factors Affecting the Width of CI

- Sample size
 - larger sample size → narrower the CI
- Forecast horizon
 - longer horizon → more uncertainty
- Level of confidence
- Model complexity
- Model assumptions
- Seasonality
- Autocorrelation
- Outliers and extreme values
- Non-normality

Limitations and Challenges of CI for TS

- Stationarity of the input data
 - constant mean and variance
- Autocorrelation
- Model selection
- Forecast horizon
 - longer horizon → more uncertainty
- Limited sample sizes
- Outliers and extreme values
 - widening effect
- Non-normality of the input data
- Assumptions of the model
 - residuals are normally distributed and have constant variance

Tips and Best Practices

1. **Always** report confidence intervals along with point forecasts
2. Be clear about the **level of confidence**
 - a. typically 95% or 99% level of confidence
3. Use **visual aids**
4. Discuss the **implications** of the uncertainty
5. Highlight **key drivers** of uncertainty
6. Validate **assumptions**
 - a. e.g., normally distributed residuals
7. Use multiple approaches

Confidence Intervals for Time Series Forecasting

- Consider the **task of predicting time-dependent data** (for example, hourly temperature prediction, or energy forecasting).
- In addition to the **point estimate** for (one or more) future values, we also want to obtain an **interval estimate** of the prediction.

Solutions

- Baseline
- Simple Approach
- Alternative Approaches

CI for TS Forecasting: Baseline

1. Train model on train-set.
2. Calculate error on test set.
3. Average the error over all samples = standard deviation (std).
4. Multiply std with z-score (according to required confidence level).
5. Create symmetrical confidence interval.

CI for TS Forecasting: Alternative Approach

Bootstrap approach is expensive, so we can:

- Create a model:
 - inputs: original model inputs, predictions
 - outputs: confidence bands
- Use a model predicting not only one value, but also the interval in which the prediction will be located with a given confidence.
 - predict 3 values: the prediction itself, the upper and lower bounds of the confidence interval
- Any other idea?

Measuring Quality of CI

- Coverage Probability
- Mean Interval Width
- Mean Squared Error (MSE)
- Information Criteria
- Visual Inspection

Discussion - any comments and feedback is welcome!

Sources

- <https://cdn-0.therandomvibez.com/wp-content/uploads/2022/09/quotes-about-confidence-in-self.jpg>
- https://en.wikipedia.org/wiki/Confidence_interval