IA169: Model Checking

Seminar 1

Exercise 1 *Let AP be a set of atomic propositions such that $p, q, r \in AP$. For each of the following LTL formulas over the set of atomic propositions AP, find at least two words $w_1, w_2 \in (2^{AP(\varphi)})^\omega$ that satisfy the formula and at least two words $w_3, w_4 \in (2^{AP(\varphi)})^\omega$ that do not satisfy it. What is the intuitive meaning of the formulas?*

1. $\mathsf{G}p \wedge \mathsf{F}q$
2. $\mathsf{F}(p \wedge Xq)$
3. $\mathsf{G}(p \rightarrow Xq)$
4. $\mathsf{G}(p \rightarrow Fq)$
5. $\mathsf{G}(p \rightarrow Gq)$
6. $(\neg p \rightarrow Xq)\mathsf{U}r$

Exercise 2 *Write LTL formulas for the following properties*

1. *every state of the run that satisfies p also satisfies q*
2. *the run contains at least two states that satisfy p*
3. *the run contains infinitely many states that satisfy p*
4. *the run contains exactly two states that satisfy p*

Exercise 3 *Let us have two fish, Alice (A) and Bob (B). There is an aquarium divided into two parts: left (L) and right (R). Both fish start on the right side of the aquarium and repeat the following sequence of steps (at the beginning they start moving independently): They move left, eat, and move back right.*

Write (or draw) a Kripke structure that corresponds to this system. What are the atomic propositions?

Then formulate the following properties using LTL and decide whether the system satisfies each property.

1. *Whenever Alice eats, she is on the left.*
2. *Whenever Alice is on the left, she will eat eventually.*
3. *Whenever Alice eats, she will immediately go to the left.*
4. *Alice will not eat before Bob.*
5. *If Alice does not eat before Bob, she will never eat.*
6. *Alice and Bob will never be on the same side from some point.*

7. *Bob chases Alice until they both eat together.*

Exercise 4 *Design non-deterministic Buchi automata for languages of words over alphabet $\Sigma = \{a, b, c\}$ that:*

1. *contain infinitely many letters a*
2. *contain infinitely many letters a and b*
3. *contain finitely many letters a and b*
4. *each a is immediately followed by b*
5. *each a is (possibly not immediately) followed by b*

Exercise 5 *Consider again the model from Exercise 3, but now with only one fish. Draw the Kripke structure and convert it to the corresponding Buchi automaton.*