Seminar 7

Confidence intervals, testing of hypothesis

1. We will work with the data sample describing the screws produced on the production line. We randomly selected 30 screws and measured them: their lengths in mm are available in the R script. Assume that our data sample is normally distributed. The producer claims, that the mean length of the screw is 20 mm, is he right?

   (a) Estimate parameters μ and σ of the normal distribution $N(\mu, \sigma)$ using the maximum likelihood method. Then construct a **two-sided** 95% **confidence interval** for parameter μ. Create a visualization of all informations you have computed (plot the histogram together with the density of the estimated normal distribution and with CI for μ).

   (b) Do the same for the **left-sided** confidence interval.

   (c) Do the same for the **right-sided** confidence interval.

2. Use the data from the previous task to test a **two-sided** hypothesis about the expected value μ of the normal distribution.

   (a) Test the null hypothesis $H_0 : \mu = 20$, against **two-sided** alternative $H_1 : \mu \neq 20$ at the significance level $\alpha = 0.05$. Compute the value of the **test statistic** and find the **critical region**. What is your conclusion?

   (b) VOLUNTARY: Create a density plot of a theoretical distribution of the test statistic (student t-distribution), visualize the critical region and the test statistic.

   (c) Find a **p-value** of the previous test and use it for decision about the null hypothesis.

   (d) VOLUNTARY: Create a density plot of a theoretical distribution of the test statistic (student t-distribution), visualize the test statistic and p-value.