Seminar 5

Random variables

1. **Binomial distribution:**

   (a) Use R functions `dbinom`, `pbinom` and `qbinom` to create a density plot, distribution function plot and quantile function plot of the random variable X with the binomial distribution $X \sim Bi(100, 0.5)$.

   (b) Generate a random vector of the length 1000 with independent elements, where each element is a realization of the random variable X_i following the same binomial distribution $X_i \sim Bi(100, 0.5)$. Produce a histogram and compare it to the density plot of the same distribution.

2. Do the same as in the task number 1 for the **Poisson distribution** $Po(\lambda = 10)$. You can try other distributions as well.

3. Consider a random variable $X \sim N(20, 16)$.

   a) Compute a probability that some realization of X lies among values 12 and 28 (use the cumulative distribution function).

   b) Find the value of the random variable X for which the probability that X is lesser or equal to it is exactly 95%. Use the quantile function.

4. **Central limit theorem and Pascal triangle:**

   (a) Generate vector of 1000 random values following the binomial distribution $X \sim Bi\,(500, 0.5)$. Normalize the random values and plot the histogram. Compare it with a standardized normal distribution (*Moivre-Laplace* central limit theorem).

   (b) EXTRA TASK: Plot the probability function of the random variable X following the binomial distribution: $X \sim Bi\,(n = 20, 0.5)$ (using `dbinom()`) together with the density of the normal distribution with the same expectation value and variance like the random variable X. Try different values of n.