

# PV211: Introduction to Information Retrieval

<https://www.fi.muni.cz/~sojka/PV211>

## IIR 21: Link analysis Handout version

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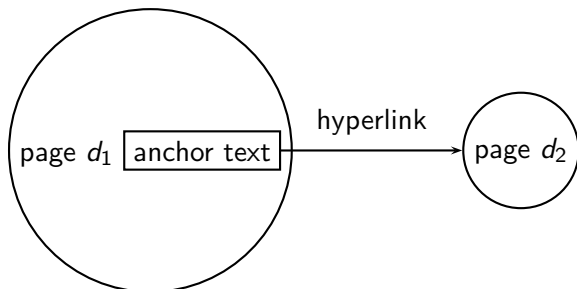
# Overview

- 1 Anchor text
- 2 Citation analysis
- 3 PageRank
- 4 HITS: Hubs & Authorities

# Take-away today

- Anchor text: What exactly are links on the web and why are they important for IR?
- Citation analysis: the mathematical foundation of PageRank and link-based ranking
- PageRank: the original algorithm that was used for link-based ranking on the web
- Hubs & Authorities: an alternative link-based ranking algorithm

# The web as a directed graph



- Assumption 1: A hyperlink is a quality signal.
  - The hyperlink  $d_1 \rightarrow d_2$  indicates that  $d_1$ 's author deems  $d_2$  high-quality and relevant.
- Assumption 2: The anchor text describes the content of  $d_2$ .
  - We use anchor text somewhat loosely here for: the text surrounding the hyperlink.
  - Example: "You can find cheap cars <a href=http://...>here</a>."
  - Anchor text: "You can find cheap cars here"



# [text of $d_2$ ] only vs. [text of $d_2$ ] + [anchor text $\rightarrow d_2$ ]

- Searching on [text of  $d_2$ ] + [anchor text  $\rightarrow d_2$ ] is often more effective than searching on [text of  $d_2$ ] only.
- Example: Query *IBM*
  - Matches IBM's copyright page
  - Matches many spam pages
  - Matches IBM Wikipedia article
  - May not match IBM home page!
  - ... if IBM home page is mostly graphics
- Searching on [anchor text  $\rightarrow d_2$ ] is better for the query *IBM*.
  - In this representation, the page with the most occurrences of *IBM* is [www.ibm.com](http://www.ibm.com). □

# Anchor text containing *IBM* pointing to [www.ibm.com](http://www.ibm.com)

[www.nytimes.com](http://www.nytimes.com): “IBM acquires Webify”

[www.slashdot.org](http://www.slashdot.org): “New IBM optical chip”

[www.stanford.edu](http://www.stanford.edu): “IBM faculty award recipients”

[www.ibm.com](http://www.ibm.com)

```
graph TD; A["www.nytimes.com: 'IBM acquires Webify'"] -.-> D["www.ibm.com"]; B["www.slashdot.org: 'New IBM optical chip'"] -.-> D; C["www.stanford.edu: 'IBM faculty award recipients'"] -.-> D;
```

# Indexing anchor text

- Thus: Anchor text is often a better description of a page's content than the page itself.
- Anchor text can be weighted more highly than document text. (based on Assumptions 1&2)

# Exercise: Assumptions underlying PageRank

- Assumption 1: A link on the web is a quality signal – the author of the link thinks that the linked-to page is high-quality.
- Assumption 2: The anchor text describes the content of the linked-to page.
- Is assumption 1 true in general?
- Is assumption 2 true in general?



# Google bombs

- A Google bomb is a search with “bad” results due to maliciously manipulated anchor text.
- Google introduced a new weighting function in 2007 that fixed many Google bombs.
- Still some remnants: [dangerous cult] on Google, Bing, Yahoo
  - Coordinated link creation by those who dislike the Church of Scientology
- Defused Google bombs: [dumb motherf. . . ], [who is a failure?], [evil empire] □

# Origins of PageRank: Citation analysis (1)

- Citation analysis: analysis of citations in the scientific literature
- Example citation: “[Miller \(2001\)](#) has shown that physical activity alters the metabolism of estrogens.”
- We can view “[Miller \(2001\)](#)” as a hyperlink linking two scientific articles.
- One application of these “hyperlinks” in the scientific literature:
  - Measure the similarity of two articles by the overlap of other articles citing them.
  - This is called [cocitation similarity](#).
  - Cocitation similarity on the web: Google’s “related:” operator, e.g. `[related:www.ford.com]` □

## Origins of PageRank: Citation analysis (2)

- Another application: Citation frequency can be used to measure the **impact** of a scientific article.
  - Simplest measure: Each citation gets one vote.
  - On the web: citation frequency = **inlink count**
- However: A high inlink count does not necessarily mean high quality ...
- ... mainly because of link spam.
- Better measure: **weighted** citation frequency or citation rank
  - An citation's vote is weighted according to its citation impact.
  - Circular? No: can be formalized in a well-defined way. □

## Origins of PageRank: Citation analysis (3)

- Better measure: weighted citation frequency or citation rank
- This is basically PageRank.
- PageRank was invented in the context of citation analysis by Pinski and Narin in the 1960s.
- Citation analysis is a big deal: The budget and salary of this lecturer are / will be determined by the impact of his publications!



# Origins of PageRank: Summary

- We can use the same formal representation for
  - citations in the scientific literature
  - hyperlinks on the web
- Appropriately weighted citation frequency is an excellent measure of quality . . .
  - . . . both for web pages and for scientific publications.
- Next: PageRank algorithm for computing weighted citation frequency on the web

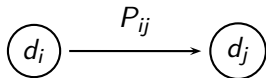


# Model behind PageRank: Random walk

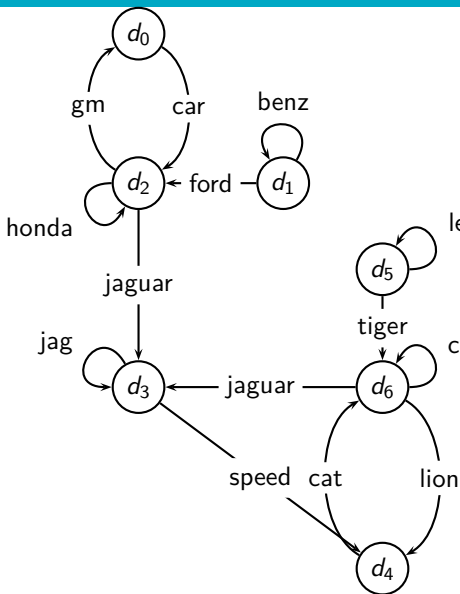
- Imagine a web surfer doing a random walk on the web
  - Start at a random page
  - At each step, go out of the current page along one of the links on that page, equiprobably
- In the steady state, each page has a **long-term visit rate**.
- This long-term visit rate is the page's **PageRank**.
- **PageRank = long-term visit rate = steady state probability** □

# Formalization of random walk: Markov chains

- A Markov chain consists of  $N$  states, plus an  $N \times N$  transition probability matrix  $P$ .
- state = page
- At each step, we are on exactly one of the pages.
- For  $1 \leq i, j \leq N$ , the matrix entry  $P_{ij}$  tells us the probability of  $j$  being the next page, given we are currently on page  $i$ .
- Clearly, for all  $i$ ,  $\sum_{j=1}^N P_{ij} = 1$



# Example web graph



## PageRank

$d_0$	0.05	$d_1$	0.04	$d_2$	0.11
$d_3$	0.25	$d_4$	0.21	$d_5$	0.04
$d_6$	0.31				

PageRank( $d_2$ ) < PageRank( $d_6$ ):  
why?

	$a$	$h$
$d_0$	0.10	0.03
$d_1$	0.01	0.04
$d_2$	0.12	0.33
$d_3$	0.47	0.18
$d_4$	0.16	0.04
$d_5$	0.01	0.04
$d_6$	0.13	0.35

highest in-degree:  $d_2, d_3, d_6$

highest out-degree:  $d_2, d_6$

highest PageRank:  $d_6$

highest hub score:  $d_6$  (close:  $d_2$ )

highest authority score:  $d_3$  □



# Link matrix for example

	$d_0$	$d_1$	$d_2$	$d_3$	$d_4$	$d_5$	$d_6$
$d_0$	0	0	1	0	0	0	0
$d_1$	0	1	1	0	0	0	0
$d_2$	1	0	1	1	0	0	0
$d_3$	0	0	0	1	1	0	0
$d_4$	0	0	0	0	0	0	1
$d_5$	0	0	0	0	0	1	1
$d_6$	0	0	0	1	1	0	1

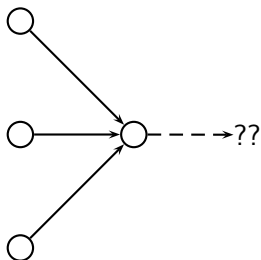
# Transition probability matrix $P$ for example

	$d_0$	$d_1$	$d_2$	$d_3$	$d_4$	$d_5$	$d_6$
$d_0$	0.00	0.00	1.00	0.00	0.00	0.00	0.00
$d_1$	0.00	0.50	0.50	0.00	0.00	0.00	0.00
$d_2$	0.33	0.00	0.33	0.33	0.00	0.00	0.00
$d_3$	0.00	0.00	0.00	0.50	0.50	0.00	0.00
$d_4$	0.00	0.00	0.00	0.00	0.00	0.00	1.00
$d_5$	0.00	0.00	0.00	0.00	0.00	0.50	0.50
$d_6$	0.00	0.00	0.00	0.33	0.33	0.00	0.33

# Long-term visit rate

- Recall: PageRank = long-term visit rate
- Long-term visit rate of page  $d$  is the probability that a web surfer is at page  $d$  at a given point in time.
- Next: what properties must hold of the web graph for the long-term visit rate to be well defined?
- The web graph must correspond to an **ergodic** Markov chain.
- First a special case: The web graph must not contain **dead ends**. □

# Dead ends



- The web is full of dead ends.
- Random walk can get stuck in dead ends.
- If there are dead ends, long-term visit rates are not well-defined (or non-sensical).



# Teleporting – to get us out of dead ends

- At a **dead end**, jump to a random web page with prob.  $1/N$ .
- At a **non-dead end**, with probability 10%, jump to a random web page (to each with a probability of  $0.1/N$ ).
- With remaining probability (90%), go out on a random hyperlink.
  - For example, if the page has 4 outgoing links: randomly choose one with probability  $(1-0.10)/4=0.225$
- 10% is a parameter, the **teleportation rate**.
- Note: “jumping” from dead end is independent of teleportation rate.



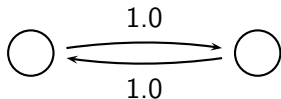
# Result of teleporting

- With teleporting, we cannot get stuck in a dead end.
- But even without dead ends, a graph may not have well-defined long-term visit rates.
- More generally, we require that the Markov chain be **ergodic**.



# Ergodic Markov chains

- A Markov chain is ergodic iff it is irreducible and aperiodic.
- **Irreducibility.** Roughly: there is a path from any page to any other page.
- **Aperiodicity.** Roughly: The pages cannot be partitioned such that the random walker visits the partitions sequentially.
- A non-ergodic Markov chain:



# Ergodic Markov chains

- Theorem: For any ergodic Markov chain, there is a unique long-term visit rate for each state.
- This is the **steady-state probability distribution**.
- Over a long time period, we visit each state in proportion to this rate.
- It doesn't matter where we start.
- **Teleporting makes the web graph ergodic.**
- **⇒ Web-graph+teleporting has a steady-state probability distribution.**
- **⇒ Each page in the web-graph+teleporting has a PageRank.**





# Where we are

- We now know what to do to make sure we have a well-defined PageRank for each page.
- Next: how to compute PageRank

# Formalization of “visit”: Probability vector

- A probability (row) vector  $\vec{x} = (x_1, \dots, x_N)$  tells us where the random walk is at any point.

- Example: 
$$\begin{pmatrix} 0 & 0 & 0 & \dots & 1 & \dots & 0 & 0 & 0 \\ 1 & 2 & 3 & \dots & i & \dots & N-2 & N-1 & N \end{pmatrix}$$

- More generally: the random walk is on page  $i$  with probability  $x_i$ .

- Example: 
$$\begin{pmatrix} 0.05 & 0.01 & 0.0 & \dots & 0.2 & \dots & 0.01 & 0.05 & 0.03 \\ 1 & 2 & 3 & \dots & i & \dots & N-2 & N-1 & N \end{pmatrix}$$

- $\sum x_i = 1$



# Change in probability vector

- If the probability vector is  $\vec{x} = (x_1, \dots, x_N)$  at this step, what is it at the next step?
- Recall that row  $i$  of the transition probability matrix  $P$  tells us where we go next from state  $i$ .
- So from  $\vec{x}$ , our next state is distributed as  $\vec{x}P$ . □

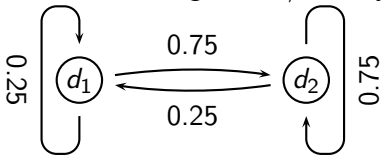
# Steady state in vector notation

- The steady state in vector notation is simply a vector  $\vec{\pi} = (\pi_1, \pi_2, \dots, \pi_N)$  of probabilities.
- (We use  $\vec{\pi}$  to distinguish it from the notation for the probability vector  $\vec{x}$ .)
- $\pi_i$  is the long-term visit rate (or PageRank) of page  $i$ .
- So we can think of PageRank as a very long vector – one entry per page.



# Steady-state distribution: Example

- What is the PageRank / steady state in this example?



# Steady-state distribution: Example

	$x_1$ $P_t(d_1)$	$x_2$ $P_t(d_2)$		
			$P_{11} = 0.25$	$P_{12} = 0.75$
			$P_{21} = 0.25$	$P_{22} = 0.75$
$t_0$	0.25	0.75	0.25	0.75
$t_1$	0.25	0.75	(convergence)	

PageRank vector =  $\vec{\pi} = (\pi_1, \pi_2) = (0.25, 0.75)$

$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$

$$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$$



# How do we compute the steady state vector?

- In other words: how do we compute PageRank?
- Recall:  $\vec{\pi} = (\pi_1, \pi_2, \dots, \pi_N)$  is the PageRank vector, the vector of steady-state probabilities ...
- ... and if the distribution in this step is  $\vec{x}$ , then the distribution in the next step is  $\vec{x}P$ .
- But  $\vec{\pi}$  is the steady state!
- So:  $\vec{\pi} = \vec{\pi}P$
- Solving this matrix equation gives us  $\vec{\pi}$ .
- $\vec{\pi}$  is the principal left eigenvector for  $P$  ...
- ... that is,  $\vec{\pi}$  is the left eigenvector with the largest eigenvalue.
- All transition probability matrices have largest eigenvalue 1.  $\square$

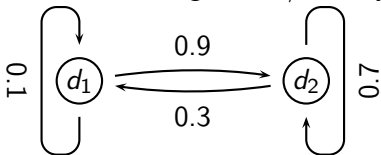
# One way of computing the PageRank $\vec{\pi}$

- Start with any distribution  $\vec{x}$ , e.g., uniform distribution
- After one step, we're at  $\vec{x}P$ .
- After two steps, we're at  $\vec{x}P^2$ .
- After  $k$  steps, we're at  $\vec{x}P^k$ .
- Algorithm: multiply  $\vec{x}$  by increasing powers of  $P$  until convergence.
- This is called the **power method**.
- Recall: regardless of where we start, we eventually reach the steady state  $\vec{\pi}$ .
- Thus: we will eventually (in asymptotia) reach the steady state. □



# Power method: Example

- What is the PageRank / steady state in this example?



- The steady state distribution (= the PageRanks) in this example are 0.25 for  $d_1$  and 0.75 for  $d_2$ .

# Computing PageRank: Power method

	$x_1$ $P_t(d_1)$	$x_2$ $P_t(d_2)$			
			$P_{11} = 0.1$ $P_{21} = 0.3$	$P_{12} = 0.9$ $P_{22} = 0.7$	
$t_0$	0	1	0.3	0.7	$= \vec{x}P$
$t_1$	0.3	0.7	0.24	0.76	$= \vec{x}P^2$
$t_2$	0.24	0.76	0.252	0.748	$= \vec{x}P^3$
$t_3$	0.252	0.748	0.2496	0.7504	$= \vec{x}P^4$
				...	...
$t_\infty$	0.25	0.75	0.25	0.75	$= \vec{x}P^\infty$

PageRank vector  $= \vec{\pi} = (\pi_1, \pi_2) = (0.25, 0.75)$

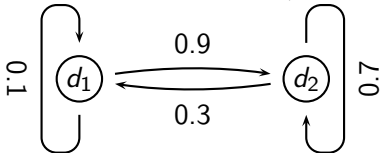
$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$

$$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$$



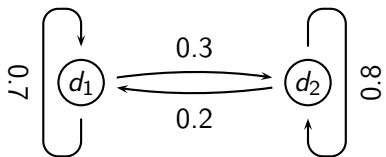
# Power method: Example

- What is the PageRank / steady state in this example?



- The steady state distribution (= the PageRanks) in this example are 0.25 for  $d_1$  and 0.75 for  $d_2$ .

# Exercise: Compute PageRank using power method



# Solution

	$x_1$	$x_2$		
	$P_t(d_1)$	$P_t(d_2)$		
			$P_{11} = 0.7$	$P_{12} = 0.3$
			$P_{21} = 0.2$	$P_{22} = 0.8$
$t_0$	0	1	0.2	0.8
$t_1$	0.2	0.8	0.3	0.7
$t_2$	0.3	0.7	0.35	0.65
$t_3$	0.35	0.65	0.375	0.625
				...
$t_\infty$	0.4	0.6	0.4	0.6

PageRank vector =  $\vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$

$$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$$



# PageRank summary

- Preprocessing
  - Given graph of links, build matrix  $P$
  - Apply teleportation
  - From modified matrix, compute  $\vec{\pi}$
  - $\pi_i$  is the PageRank of page  $i$ .
- Query processing
  - Retrieve pages satisfying the query
  - Rank them by their PageRank
  - Return reranked list to the user

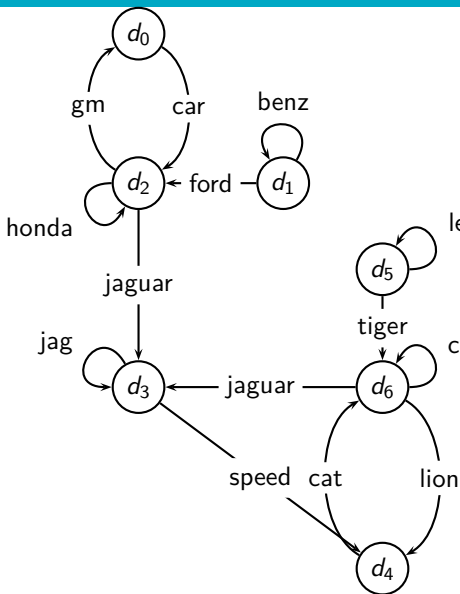


# PageRank issues

- Real surfers are not random surfers.
  - Examples of nonrandom surfing: back button, short vs. long paths, bookmarks, directories – and search!
  - → Markov model is not a good model of surfing.
  - But it's good enough as a model for our purposes.
- Simple PageRank ranking (as described on previous slide) produces bad results for many pages.
  - Consider the query [video service]
  - The Yahoo home page (i) has a very high PageRank and (ii) contains both *video* and *service*.
  - If we rank all Boolean hits according to PageRank, then the Yahoo home page would be top-ranked.
  - Clearly not desirable
- In practice: rank according to weighted combination of raw text match, anchor text match, PageRank & other factors
- → see lecture on Learning to Rank



# Example web graph



## PageRank

$d_0$	0.05	$d_1$	0.04	$d_2$	0.11
$d_3$	0.25	$d_4$	0.21	$d_5$	0.04
$d_6$	0.31				

PageRank( $d_2$ ) < PageRank( $d_6$ ):  
why?

	$a$	$h$
$d_0$	0.10	0.03
$d_1$	0.01	0.04
$d_2$	0.12	0.33
$d_3$	0.47	0.18
$d_4$	0.16	0.04
$d_5$	0.01	0.04
$d_6$	0.13	0.35

highest in-degree:  $d_2, d_3, d_6$

highest out-degree:  $d_2, d_6$

highest PageRank:  $d_6$

highest hub score:  $d_6$  (close:  $d_2$ )

highest authority score:  $d_3$



# Transition (probability) matrix

	$d_0$	$d_1$	$d_2$	$d_3$	$d_4$	$d_5$	$d_6$
$d_0$	0.00	0.00	1.00	0.00	0.00	0.00	0.00
$d_1$	0.00	0.50	0.50	0.00	0.00	0.00	0.00
$d_2$	0.33	0.00	0.33	0.33	0.00	0.00	0.00
$d_3$	0.00	0.00	0.00	0.50	0.50	0.00	0.00
$d_4$	0.00	0.00	0.00	0.00	0.00	0.00	1.00
$d_5$	0.00	0.00	0.00	0.00	0.00	0.50	0.50
$d_6$	0.00	0.00	0.00	0.33	0.33	0.00	0.33

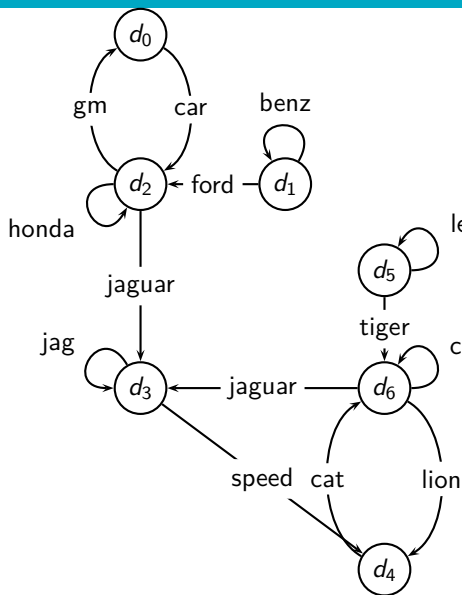
# Transition matrix with teleporting, teleportation rate=0.14

	$d_0$	$d_1$	$d_2$	$d_3$	$d_4$	$d_5$	$d_6$
$d_0$	0.02	0.02	0.88	0.02	0.02	0.02	0.02
$d_1$	0.02	0.45	0.45	0.02	0.02	0.02	0.02
$d_2$	0.31	0.02	0.31	0.31	0.02	0.02	0.02
$d_3$	0.02	0.02	0.02	0.45	0.45	0.02	0.02
$d_4$	0.02	0.02	0.02	0.02	0.02	0.02	0.88
$d_5$	0.02	0.02	0.02	0.02	0.02	0.45	0.45
$d_6$	0.02	0.02	0.02	0.31	0.31	0.02	0.31

# Power method vectors $\vec{x}P^k$

	$\vec{x}$	$\vec{x}P^1$	$\vec{x}P^2$	$\vec{x}P^3$	$\vec{x}P^4$	$\vec{x}P^5$	$\vec{x}P^6$	$\vec{x}P^7$	$\vec{x}P^8$	$\vec{x}P^9$	$\vec{x}P^{10}$	$\vec{x}P^{11}$	$\vec{x}P^{12}$	$\vec{x}P^{13}$
$d_0$	0.14	0.06	0.09	0.07	0.07	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05
$d_1$	0.14	0.08	0.06	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
$d_2$	0.14	0.25	0.18	0.17	0.15	0.14	0.13	0.12	0.12	0.12	0.12	0.11	0.11	0.11
$d_3$	0.14	0.16	0.23	0.24	0.24	0.24	0.24	0.25	0.25	0.25	0.25	0.25	0.25	0.25
$d_4$	0.14	0.12	0.16	0.19	0.19	0.20	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
$d_5$	0.14	0.08	0.06	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
$d_6$	0.14	0.25	0.23	0.25	0.27	0.28	0.29	0.29	0.30	0.30	0.30	0.30	0.31	0.31

# Example web graph



PageRank

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$d_4$	0.16	0.04
$d_5$	0.01	0.04
$d_6$	0.13	0.35

highest in-degree:  $d_2, d_3, d_6$

highest out-degree:  $d_2, d_6$

highest PageRank:  $d_6$

highest hub score:  $d_6$  (close:  $d_2$ )

highest authority score:  $d_3$  □

# How important is PageRank?

- Frequent claim: PageRank is the most important component of web ranking.
- The reality:
  - There are several components that are at least as important: e.g., anchor text, phrases, proximity, tiered indexes . . .
  - Rumor has it that PageRank in its original form (as presented here) now has a negligible impact on ranking!
  - However, variants of a page's PageRank are still an essential part of ranking.
  - Addressing link spam is difficult and crucial. □

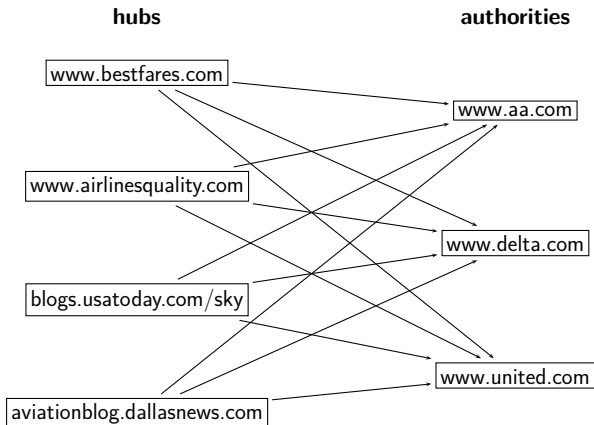
# HITS – Hyperlink-Induced Topic Search

- Premise: there are two different types of relevance on the web.
- Relevance type 1: **Hubs**. A hub page is a good list of [links to pages answering the information need].
  - E.g., for query [chicago bulls]: Bob's list of recommended resources on the Chicago Bulls sports team
- Relevance type 2: **Authorities**. An authority page is a direct answer to the information need.
  - The home page of the Chicago Bulls sports team
  - By definition: Links to authority pages occur repeatedly on hub pages.
- Most approaches to search (including PageRank ranking) don't make the distinction between these two very different types of relevance. □

# Hubs and authorities: Definition

- A good hub page for a topic **links to** many authority pages for that topic.
- A good authority page for a topic **is linked to** by many hub pages for that topic.
- Circular definition – we will turn this into an iterative computation. □

# Example for hubs and authorities

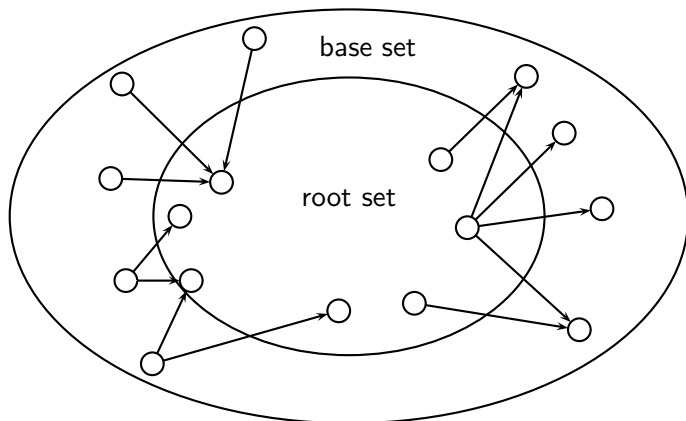




# How to compute hub and authority scores

- Do a regular web search first
- Call the search result the **root set**
- Find all pages that are linked to or link to pages in the root set
- Call this larger set the **base set**
- Finally, compute hubs and authorities for the base set (which we'll view as a small web graph) □

# Root set and base set (1)



- 1) The root set
- 2) Nodes that root set nodes link to
- 3) Nodes that link to root set nodes
- 4) The base set

## Root set and base set (2)

- Root set typically has 200–1,000 nodes.
- Base set may have up to 5,000 nodes.
- Computation of base set, as shown on previous slide:
  - Follow outlinks by parsing the pages in the root set
  - Find  $d$ 's inlinks by searching for all pages containing a link to  $d$



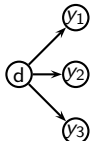
# Hub and authority scores

- Compute for each page  $d$  in the base set a **hub score**  $h(d)$  and an **authority score**  $a(d)$
- Initialization: for all  $d$ :  $h(d) = 1$ ,  $a(d) = 1$
- Iteratively update all  $h(d)$ ,  $a(d)$
- After convergence:
  - Output pages with highest  $h$  scores as top hubs
  - Output pages with highest  $a$  scores as top authorities
  - So we output **two** ranked lists

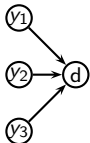


# Iterative update

- For all  $d$ :  $h(d) = \sum_{d \mapsto y} a(y)$



- For all  $d$ :  $a(d) = \sum_{y \mapsto d} h(y)$



- Iterate these two steps until convergence



# Details

- Scaling
  - To prevent the  $a()$  and  $h()$  values from getting too big, can scale down after each iteration
  - Scaling factor doesn't really matter.
  - We care about the **relative** (as opposed to absolute) values of the scores.
- In most cases, the algorithm converges after a few iterations. □

# Authorities for query [Chicago Bulls]

- 0.85 [www.nba.com/bulls](http://www.nba.com/bulls)
- 0.25 [www.essex1.com/people/jmiller/bulls.htm](http://www.essex1.com/people/jmiller/bulls.htm)  
“da Bulls”
- 0.20 [www.nando.net/SportServer/basketball/nba/chi.html](http://www.nando.net/SportServer/basketball/nba/chi.html)  
“The Chicago Bulls”
- 0.15 [users.aol.com/rynocub/bulls.htm](http://users.aol.com/rynocub/bulls.htm)  
“The Chicago Bulls Home Page”
- 0.13 [www.geocities.com/Colosseum/6095](http://www.geocities.com/Colosseum/6095)  
“Chicago Bulls”

(Ben-Shaul et al, WWW8)

# The authority page for [Chicago Bulls]

NBA D-LEAGUE WNBA GLOBAL TEAMS MOBILE NBA TICKETS FANTASY NBATV STORE VIDEO

**bills.com** THE OFFICIAL SITE OF THE CHICAGO BULLS  
Delivered by at&t

NEWSLETTER CONTACT US

TICKETS TEAM NEWS SCHEDULE FEATURES GAME NIGHT INSIDE THE BULLS HISTORY STORE  SEARCH

**Fore!!! Golf with the Bulls!**  
Tickets for the Chicago Bulls/Verizon Wireless *Cherry Red Series* are now on sale! Join Bulls' personalities including current players, coaches, legends, broadcasters and entertainment teams on August 17 at the White Pine Golf Club in Bensenville, Ill.

- 2009.10: [Season & Game Tickets](#)
- [Mobile Alerts](#) | [Facebook](#) | [Twitter](#)
- [RSS](#) | [News Clips](#) | [myBulls](#) | [Sam Smith](#)

- Bulls to compete in NBA Summer League
- Chicago Bulls | Draft Central 2009
- Pre-draft Ask Sam mailbag special
- Pre-draft interview: Wake's Jeff Teague
- Pre-draft interview: VCU's Eric Maynor
- Pre-draft interview: Wake's James Johnson
- Pre-draft interview: UNC's Wayne Ellington

**Draft Workouts**

**BULLS EYE** POWERED BY KIA KIA MOTORS

CALENDAR	TICKETS
SEASON TICKETS	TICKETEXCHANGE
GROUP TICKETS	E-NEWSLETTER

**SEASON TICKETS**

CHICAGO BULLS PRESENTED BY **HARRIS**

verizon wireless FAN POLL



## Hubs for query [Chicago Bulls]

- 1.62 [www.geocities.com/Colosseum/1778](http://www.geocities.com/Colosseum/1778)  
“Unbelieveabulls!!!!!”
- 1.24 [www.webring.org/cgi-bin/webring?ring=chbulls](http://www.webring.org/cgi-bin/webring?ring=chbulls)  
“Erin’s Chicago Bulls Page”
- 0.74 [www.geocities.com/Hollywood/Lot/3330/Bulls.html](http://www.geocities.com/Hollywood/Lot/3330/Bulls.html)  
“Chicago Bulls”
- 0.52 [www.nobull.net/web\\_position/kw-search-15-M2.htm](http://www.nobull.net/web_position/kw-search-15-M2.htm)  
“Excite Search Results: bulls”
- 0.52 [www.halcyon.com/wordsltd/bball/bulls.htm](http://www.halcyon.com/wordsltd/bball/bulls.htm)  
“Chicago Bulls Links”

(Ben-Shaul et al, WWW8)

# A hub page for [Chicago Bulls]



**COAST TO COAST TICKETS**  
great tickets from nice people

Returning Customer

City Guide | \

Minnesota Timberwolves Tickets  
New Jersey Nets Tickets  
New Orleans Hornets Tickets  
New York Knicks Tickets  
Oklahoma City Thunder Tickets  
Orlando Magic Tickets  
Philadelphia 76ers Tickets  
Phoenix Suns Tickets  
Portland Trail Blazers Tickets  
Sacramento Kings Tickets  
San Antonio Spurs Tickets  
Toronto Raptors Tickets  
Utah Jazz Tickets  
Washington Wizards Tickets  
**NBA All-Star Weekend**  
**NBA Finals Tickets**  
**NBA Playoffs Tickets**

[All NBA Tickets](#)

## Event Selections

### Sporting Events

**MLB Baseball Tickets**

**NFL Football Tickets**

**NBA Basketball Tickets**

**NHL Hockey Tickets**

**NASCAR Racing Tickets**

**PGA Golf Tickets**

**Tennis Tickets**

**NCAA Football Tickets**

### Official Website Links:

[Chicago Bulls \(official site\)](#)  
<http://www.nba.com/bulls/>

### Fan Club - Fan Site Links:

[Chicago Bulls](#)  
Chicago Bulls Fan Site with Bulls Blog, News, Bulls Forum, Wallpapers and all your basic Chicago Bulls essentials!!  
<http://www.bullscentral.com>

[Chicago Bulls Blog](#)  
The place to be for news and views on the Chicago Bulls and NBA Basketball  
<http://chi-bulls.blogspot.com>

### News and Information Links:

[Chicago Sun-Times \(local newspaper\)](#)  
<http://www.suntimes.com/sports/basketball/bulls/index.html>

[Chicago Tribune \(local newspaper\)](#)  
<http://www.chicagotribune.com/sports/basketball/bulls/>

[Wikipedia - Chicago Bulls](#)  
All about the Chicago Bulls from Wikipedia, the free online encyclopedia.  
[http://en.wikipedia.org/wiki/Chicago\\_Bulls](http://en.wikipedia.org/wiki/Chicago_Bulls)

### Merchandise Links:

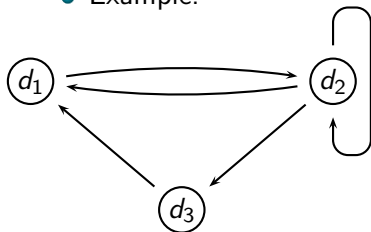
[Chicago Bulls watches](#)  
[http://www.sportimewatches.com/NBA\\_watches/Chicago-Bulls-watches.html](http://www.sportimewatches.com/NBA_watches/Chicago-Bulls-watches.html)

# Hubs & Authorities: Comments

- HITS can pull together good pages regardless of page content.
- Once the base set is assembled, we only do link analysis, no text matching.
- Pages in the base set often do not contain any of the query words.
- In theory, an English query can retrieve Japanese-language pages!
  - If supported by the link structure between English and Japanese pages
- Danger: **topic drift** – the pages found by following links may not be related to the original query. □

# Proof of convergence

- We define an  $N \times N$  **adjacency matrix**  $A$ . (We called this the link matrix earlier.)
- For  $1 \leq i, j \leq N$ , the matrix entry  $A_{ij}$  tells us whether there is a link from page  $i$  to page  $j$  ( $A_{ij} = 1$ ) or not ( $A_{ij} = 0$ ).
- Example:



	$d_1$	$d_2$	$d_3$
$d_1$	0	1	0
$d_2$	1	1	1
$d_3$	1	0	0

# Write update rules as matrix operations

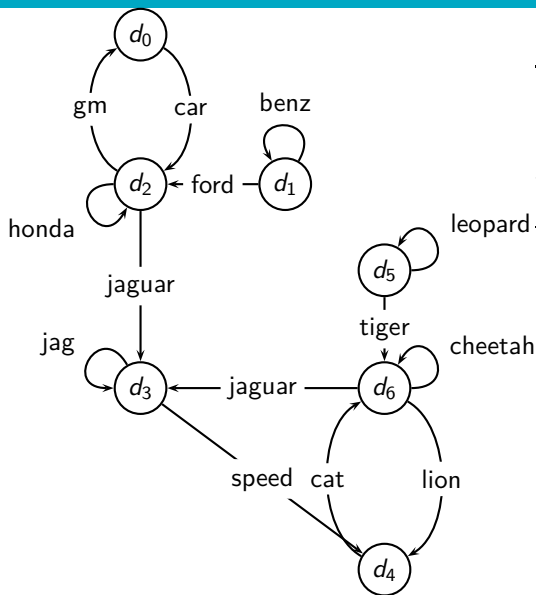
- Define the hub vector  $\vec{h} = (h_1, \dots, h_N)$  as the vector of hub scores.  $h_i$  is the hub score of page  $d_i$ .
- Similarly for  $\vec{a}$ , the vector of authority scores
- Now we can write  $h(d) = \sum_{d \rightarrow y} a(y)$  as a matrix operation:  
$$\vec{h} = A\vec{a} \dots$$
- ... and we can write  $a(d) = \sum_{y \rightarrow d} h(y)$  as  $\vec{a} = A^T \vec{h}$
- HITS algorithm in matrix notation:
  - Compute  $\vec{h} = A\vec{a}$
  - Compute  $\vec{a} = A^T \vec{h}$
  - Iterate until convergence



# HITS as eigenvector problem

- HITS algorithm in matrix notation. Iterate:
  - Compute  $\vec{h} = A\vec{a}$
  - Compute  $\vec{a} = A^T\vec{h}$
- By substitution we get:  $\vec{h} = AA^T\vec{h}$  and  $\vec{a} = A^TA\vec{a}$
- Thus,  $\vec{h}$  is an eigenvector of  $AA^T$  and  $\vec{a}$  is an eigenvector of  $A^TA$ .
- So the HITS algorithm is actually a special case of the power method and hub and authority scores are eigenvector values.
- HITS and PageRank both formalize link analysis as eigenvector problems. □

# Example web graph



PageRank

$d_0$	0.05	$d_1$	0.04	$d_2$	0.11
$d_3$	0.25	$d_4$	0.21	$d_5$	0.04
$d_6$	0.31				

PageRank( $d_2$ ) < PageRank( $d_6$ ):  
why?

	$a$	$h$
$d_0$	0.10	0.03
$d_1$	0.01	0.04
$d_2$	0.12	0.33
$d_3$	0.47	0.18
$d_4$	0.16	0.04
$d_5$	0.01	0.04
$d_6$	0.13	0.35

highest in-degree:  $d_2, d_3, d_6$

highest out-degree:  $d_2, d_6$

highest PageRank:  $d_6$

highest hub score:  $d_6$  (close:  $d_2$ )

highest authority score:  $d_3$  □

# Raw matrix $A$ for HITS

We double-weight links whose anchors contain query word:

	$d_0$	$d_1$	$d_2$	$d_3$	$d_4$	$d_5$	$d_6$
$d_0$	0	0	1	0	0	0	0
$d_1$	0	1	1	0	0	0	0
$d_2$	1	0	1	2	0	0	0
$d_3$	0	0	0	1	1	0	0
$d_4$	0	0	0	0	0	0	1
$d_5$	0	0	0	0	0	1	1
$d_6$	0	0	0	2	1	0	1



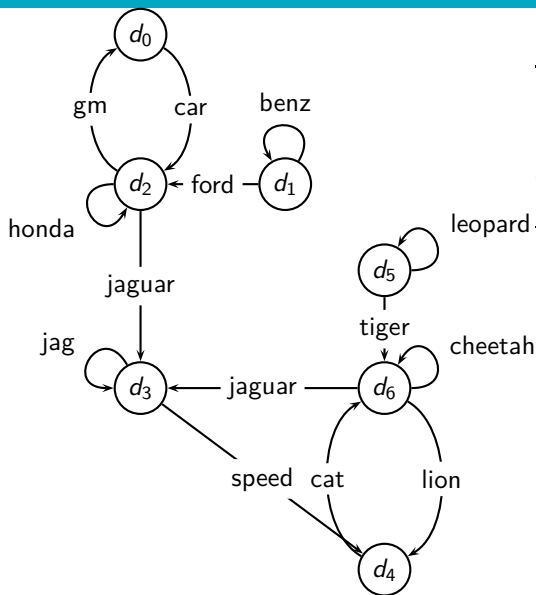
Hub vectors  $h_0, \vec{h}_i = \frac{1}{d_i} A \cdot \vec{a}_i, i \geq 1$

	$\vec{h}_0$	$\vec{h}_1$	$\vec{h}_2$	$\vec{h}_3$	$\vec{h}_4$	$\vec{h}_5$
$d_0$	0.14	0.06	0.04	0.04	0.03	0.03
$d_1$	0.14	0.08	0.05	0.04	0.04	0.04
$d_2$	0.14	0.28	0.32	0.33	0.33	0.33
$d_3$	0.14	0.14	0.17	0.18	0.18	0.18
$d_4$	0.14	0.06	0.04	0.04	0.04	0.04
$d_5$	0.14	0.08	0.05	0.04	0.04	0.04
$d_6$	0.14	0.30	0.33	0.34	0.35	0.35

Authority vectors  $\vec{a}_i = \frac{1}{c_i} A^T \cdot \vec{h}_{i-1}, i \geq 1$

	$\vec{a}_1$	$\vec{a}_2$	$\vec{a}_3$	$\vec{a}_4$	$\vec{a}_5$	$\vec{a}_6$	$\vec{a}_7$
$d_0$	0.06	0.09	0.10	0.10	0.10	0.10	0.10
$d_1$	0.06	0.03	0.01	0.01	0.01	0.01	0.01
$d_2$	0.19	0.14	0.13	0.12	0.12	0.12	0.12
$d_3$	0.31	0.43	0.46	0.46	0.46	0.47	0.47
$d_4$	0.13	0.14	0.16	0.16	0.16	0.16	0.16
$d_5$	0.06	0.03	0.02	0.01	0.01	0.01	0.01
$d_6$	0.19	0.14	0.13	0.13	0.13	0.13	0.13

# Example web graph



PageRank

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highest in-degree:  $d_2, d_3, d_6$

highest out-degree:  $d_2, d_6$

highest PageRank:  $d_6$

highest hub score:  $d_6$  (close:  $d_2$ )

highest authority score:  $d_3$

# PageRank vs. HITS: Discussion

- PageRank can be precomputed, HITS has to be computed at query time.
  - HITS is too expensive in most application scenarios.
- PageRank and HITS make two different design choices concerning (i) the eigenproblem formalization (ii) the set of pages to apply the formalization to.
- These two are orthogonal.
  - We could also apply HITS to the entire web and PageRank to a small base set.
- Claim: On the web, a good hub almost always is also a good authority.
- The actual difference between PageRank ranking and HITS ranking is therefore not as large as one might expect. □

# Exercise

- Why is a good hub almost always also a good authority?

# Take-away today

- Anchor text: What exactly are links on the web and why are they important for IR?
- Citation analysis: the mathematical foundation of PageRank and link-based ranking
- PageRank: the original algorithm that was used for link-based ranking on the web
- Hubs & Authorities: an alternative link-based ranking algorithm

# Resources

- Chapter 21 of IIR
- Resources at <https://www.fi.muni.cz/~sojka/PV211/> and <http://cislmu.org>, materials in MU IS and FI MU library
  - American Mathematical Society article on PageRank (popular science style)
  - Jon Kleinberg's home page (main person behind HITS)
  - A Google bomb and its defusing
  - Google's official description of PageRank: *PageRank reflects our view of the importance of web pages by considering more than 500 million variables and 2 billion terms. Pages that we believe are important pages receive a higher PageRank and are more likely to appear at the top of the search results.*